I. Basic Cosmology Elements

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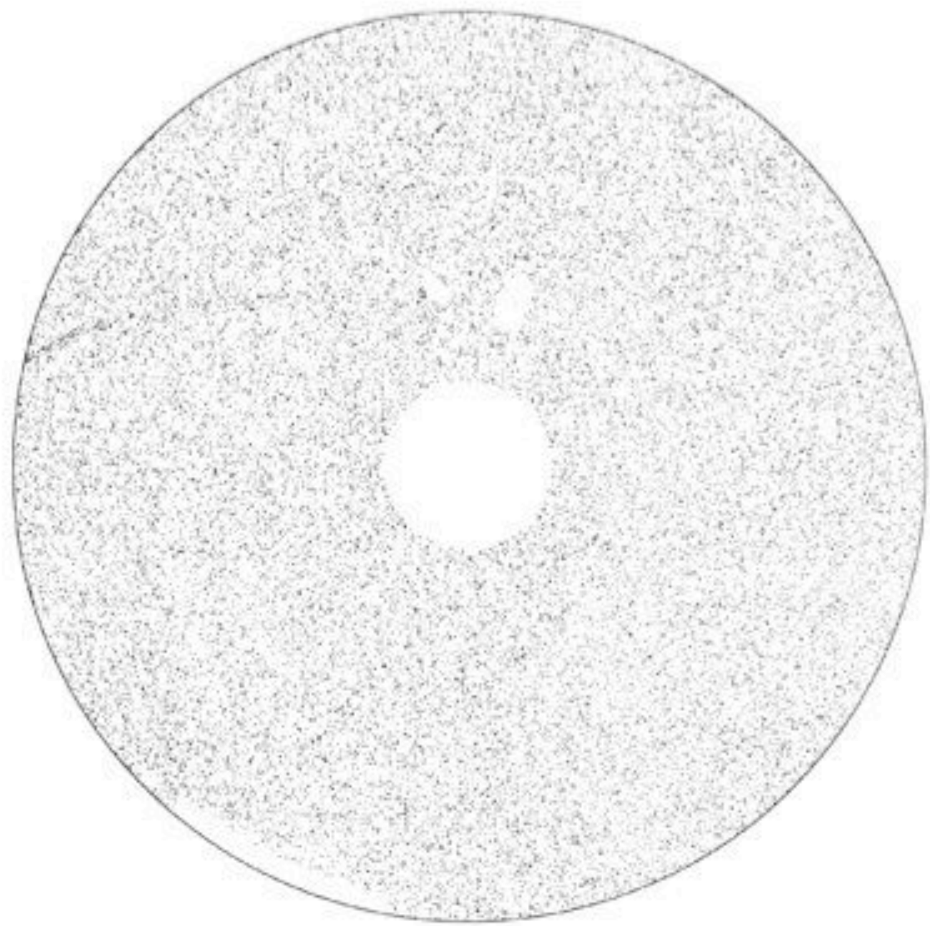
Cosmological Principle

"The Universe is homogeneous and isotropic on large-scales"

As can be seen by the position of extragalactic radio-sources

Angular distribution of the $\sim$ 31 000 brightest 6cm radio sources in the sky

(Peebles 1993)

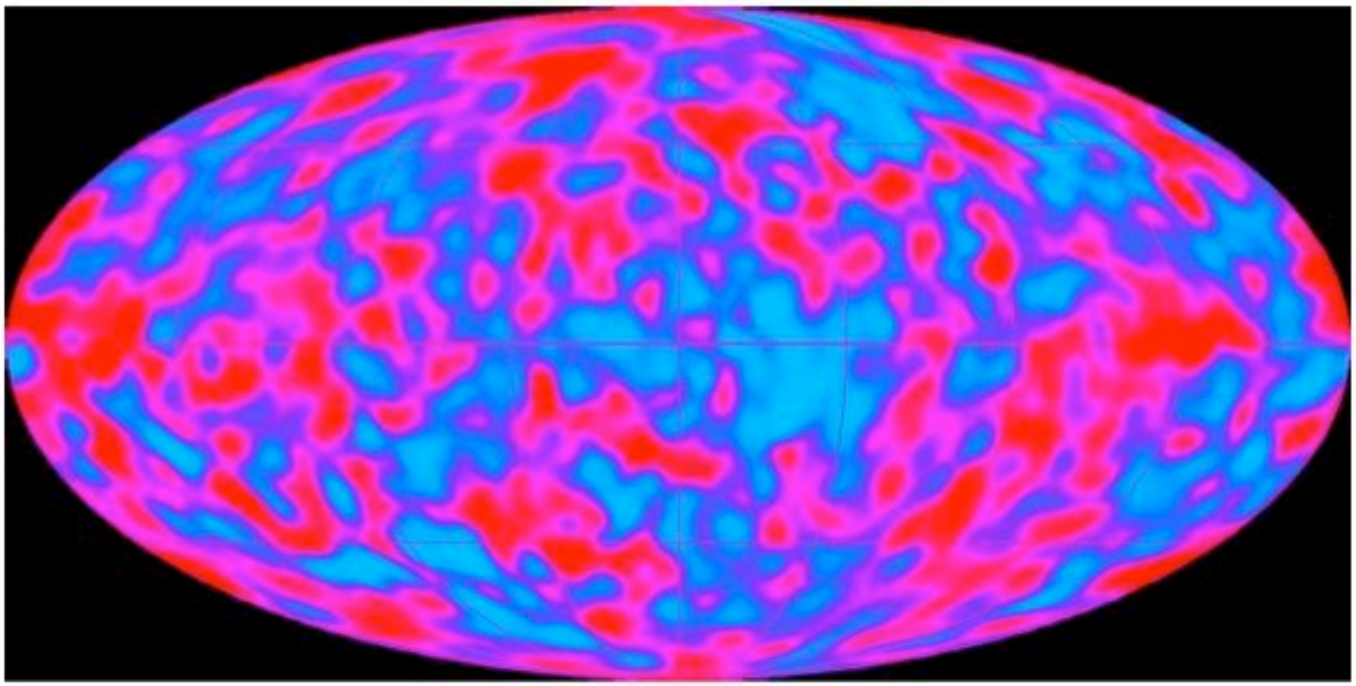


(From R. Bender's notes)

Cosmological Principle

"The Universe is homogeneous and isotropic on large-scales"

As can be seen by Cosmic Microwave Background (CMB) radiation



Temperature fluctuations in the Cosmic Microwave Background as measured by the COBE satellite. The amplitude of the fluctuations is only $\Delta T/T \simeq 10^{-5}$ and reflects density inhomogeneities in the baryons of the same order about 100 000 years after the big bang.

(From R. Bender's notes)

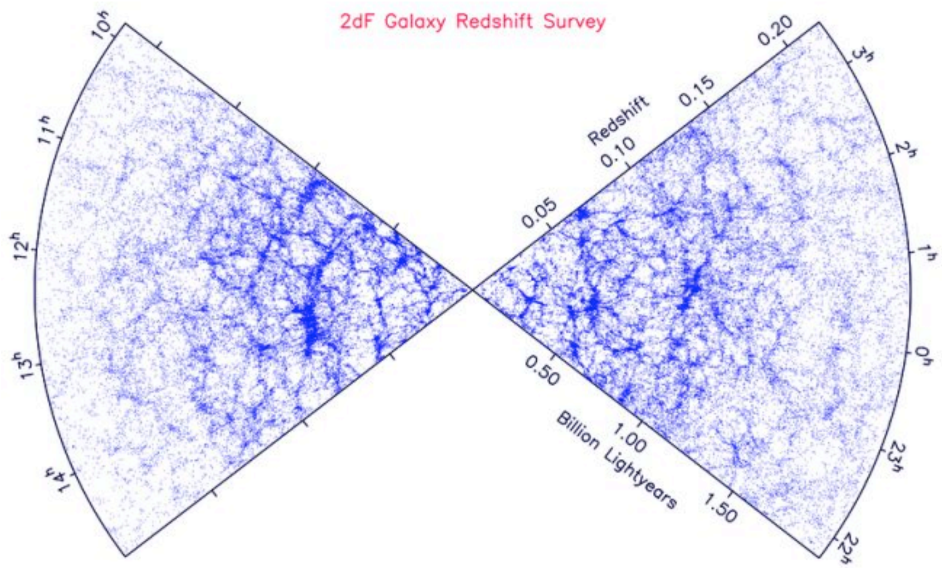
Cosmological Principle

"The Universe is homogeneous and isotropic on large-scales"

As can be seen by the 2-point correlation function of galaxies, which are clustered in scales of few x h^{-1} Mpc.

Other LSS scales: supercluster associations ~ 100 h^{-1} Mpc
filaments ~ 100 –250 h^{-1} Mpc
voids ~ 60 h^{-1} Mpc

There is a characteristic scale 300 h^{-1} Mpc $\leq l \leq cH_0^{-1}$ averaged over which the Universe can be considered homogeneous.



The original Hubble diagram

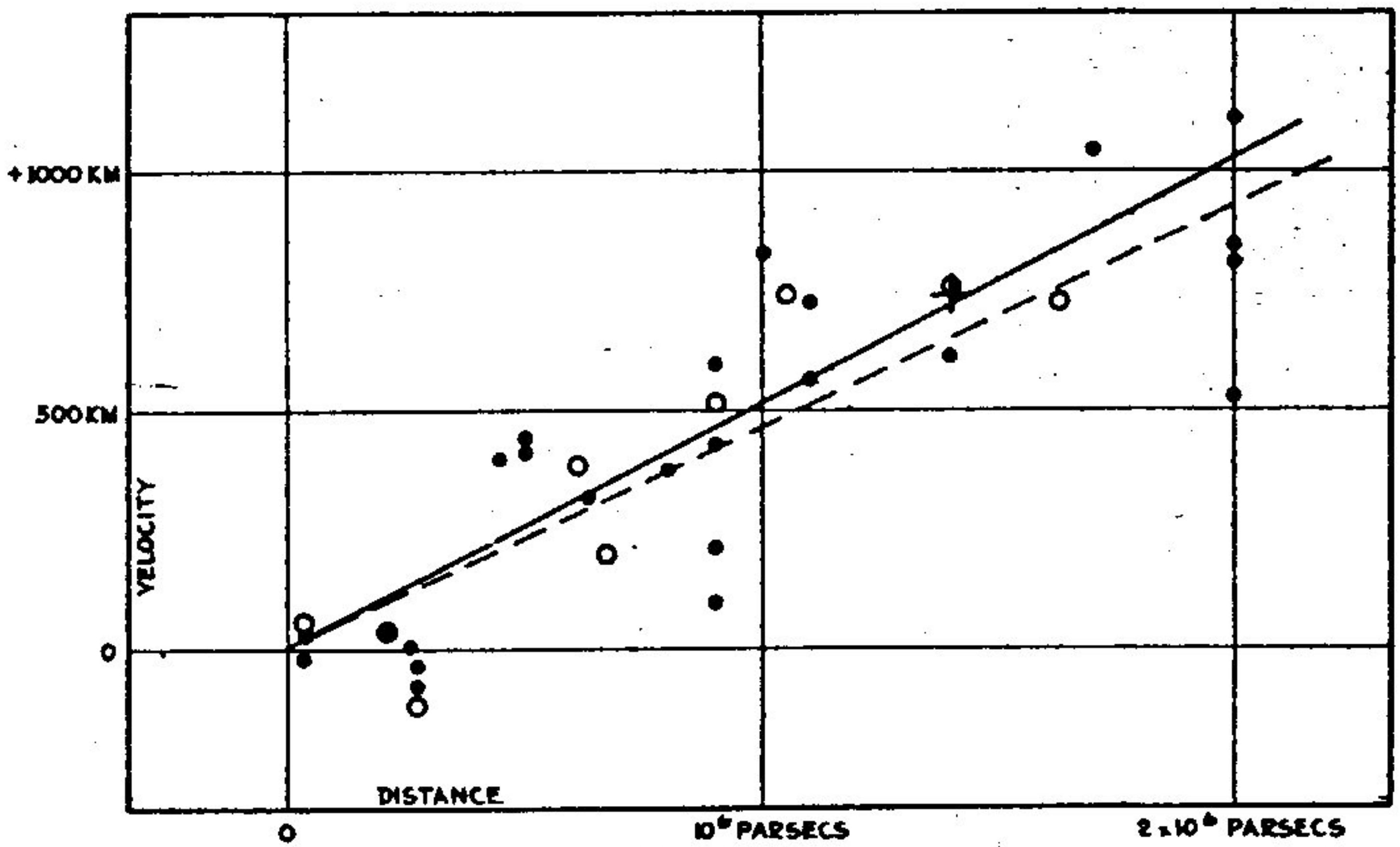
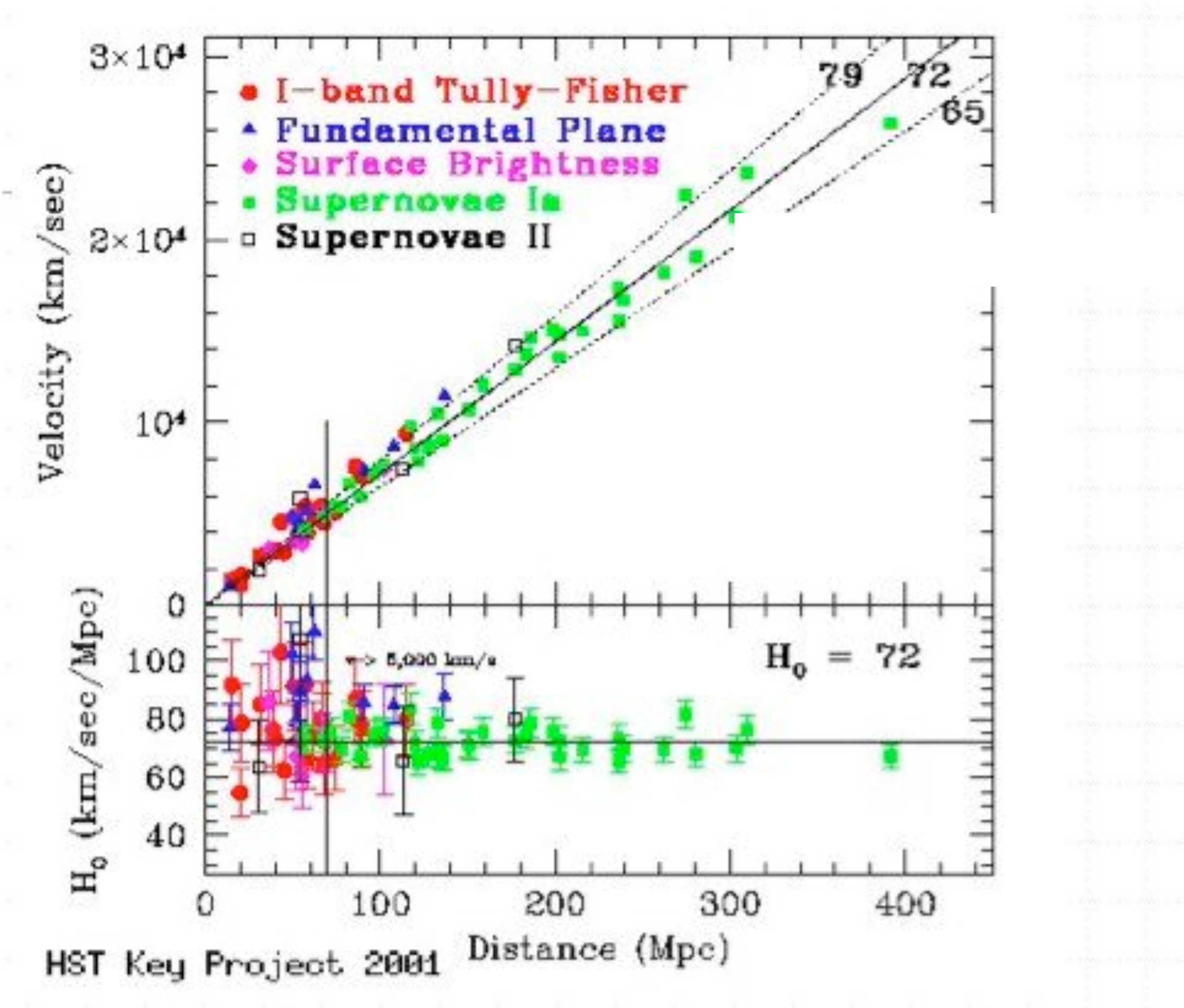


FIGURE 1

Hubble (1936)

The value of H_0:

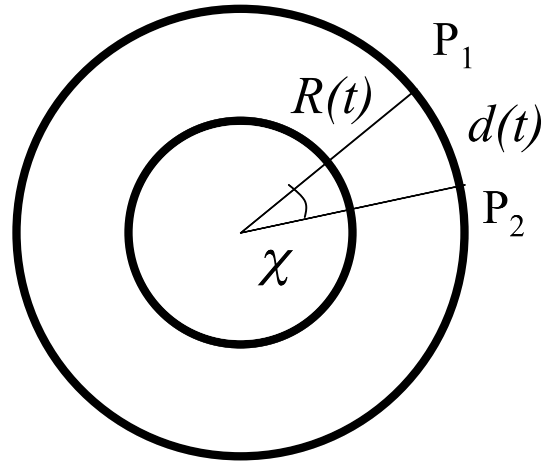
H_0=72±8 km/s/Mpc (Freedman et al. 2001)



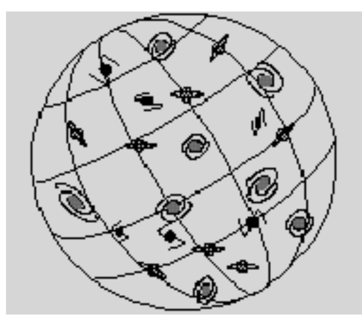
The origin of the Hubble "constant"

Can be deduced from an expanding homogeneous universe.

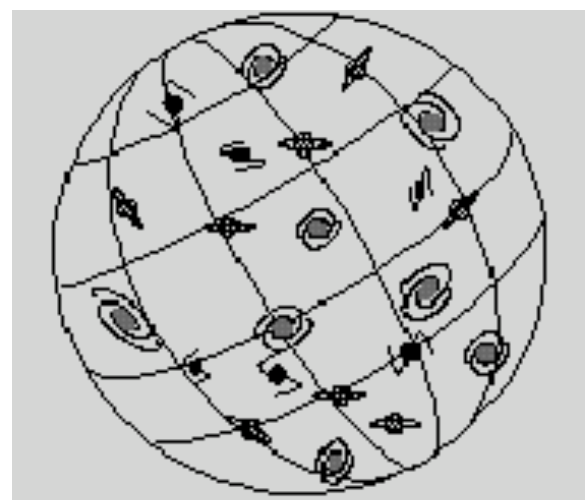
Let's imagine a 1D Universe, on an expanding circle:
d(t) is the proper distance between two points P_1P_2
R(t) is the scale factor or growth
χ is the comoving distance between P_1P_2 (comoving with the expanding universe)



$$\left.\begin{array}{l} d = R\chi \Rightarrow v = \dot{d} = \dot{R}\chi = \dfrac{\dot{R}}{R}d \\ v = H_0 d \text{ in general } v = Hd \end{array}\right\} \Rightarrow \boxed{H = \frac{\dot{R}}{R}}$$



$\Rightarrow$



The Friedmann-Lemaître-Robertson-Walker metric (1922-1936)

$$ds^2 = c^2dt^2 - R^2\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

where *(r,θ,φ)* are spherical comoving coordinates, *R* is the scale factor, and *k* is a constant related to the curvature.

It can be deduced purely from symmetry alone for a homogeneous universe

For a 2D universe on the surface of a sphere, the proper distance P_1P_2

$$(dl)^2 = (Rd\theta)^2 + (bd\phi)^2 = \left(\frac{db}{\sqrt{1-b^2/R^2}}\right)^2 + (bd\phi)^2$$

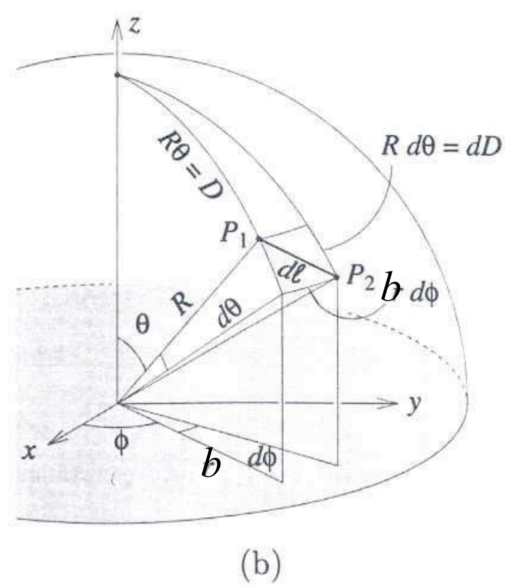
where $K=1/R^2$ is the curvature at t

For a 3D universe

$$(dl)^2 = \left(\frac{db}{\sqrt{1-Kb^2}}\right)^2 + (bd\theta)^2 + (b\sin\theta d\phi)^2$$

For space-time, introducing a time-independent curvature $K \equiv k/R^2$, and the comoving coordinate r, such that $b=Rr$, the geodesic is given by

$$(ds)^2 = (cdt)^2 - (dl)^2 = (cdt)^2 - R^2\left[\left(\frac{dr}{\sqrt{1-kr^2}}\right)^2 + (rd\theta)^2 + (r\sin\theta d\phi)^2\right]$$



(following Carroll & Ostlie's "Modern Astrophysics")

The cosmological origin of redshift

$z \equiv (\lambda - \lambda_0)/\lambda_0$

It can be deduced from the Robertson-Walker metric

Light travels along null geodesics $ds=0$. If we follow the path of light from r_1 to $r=0$, the null geodesic follows constant (θ,ϕ), and $d\theta=d\phi=0$.

Hence, the RW metric $\Rightarrow$ $$\frac{cdt}{R} = \pm \frac{dr}{\sqrt{1-kr^2}}$$

Two consecutive crests leave at t_1 and $t_1+\Delta t_1$ and are received at t_0 and $t_0+\Delta t_0$

$$\int_{t_1}^{t_0} \frac{cdt}{R(t)} = -\int_{r_1}^{0} \frac{dr}{\sqrt{1-kr^2}} = \int_{0}^{r_1} \frac{dr}{\sqrt{1-kr^2}} \tag{4}$$

A successive crest, leaving $\mathcal{L}_1$ at $t_1 + \delta t_1$ will arrive at $r = 0$ at $t_0 + \delta t_0$, but since we have constant radial coordinates:

$$\int_{t_1+\delta t_1}^{t_0+\delta t_0} \frac{cdt}{R(t)} = \int_{0}^{r_1} \frac{dr}{\sqrt{1-kr^2}} \Longrightarrow \int_{t_1}^{t_0} \frac{cdt}{R(t)} = \int_{t_1+\delta t_1}^{t_0+\delta t_0} \frac{cdt}{R(t)} \tag{5}$$

and rearranging the limits of integration:

$$\int_{t_1}^{t_1+\delta t_1} + \int_{t_1+\delta t_1}^{t_0} = \int_{t_1+\delta t_1}^{t_0} + \int_{t_0}^{t_0+\delta t_0}$$

we get that:

$$\int_{t_1}^{t_1+\delta t_1} \frac{cdt}{R(t)} = \int_{t_0}^{t_0+\delta t_0} \frac{cdt}{R(t)}$$

Now if $\delta t \ll t$ we can consider $R(t)$ constant over the integration time and therefore $\delta t_1/R(t_1) = \delta t_0/R(t_0)$ and since $\delta t_{1,0}$ is the time between successive wave crests of the emitted (detected) light, it is also the wavelength of the emitted (detected) light:

$$\frac{\lambda_1}{\lambda_0} = \frac{R(t_1)}{R(t_0)}$$

and the *Cosmological* redshift, z is defined as the ratio of the detected wavelength to that emitted:

$$\boxed{1 + z = \frac{\lambda_0}{\lambda_1} = \frac{R(t_0)}{R(t_1)}} \tag{6}$$

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Hence, the RW metric $\Rightarrow$ $\frac{cdt}{R} = \pm \frac{dr}{\sqrt{1-kr^2}}$ $\Rightarrow$ $1+z = \frac{\lambda_0}{\lambda_1} = \frac{R(t_0)}{R(t_1)} \Rightarrow R \propto (1+z)^{-1}$

It is not a Doppler effect, but rather a property of the expanding non-Euclidean space-time.

The wavelength of light shifts to the red $\lambda \propto R(t)$

The energy carried by the wave decreases as the Universe expands $E=hc/\lambda \propto 1/R(t)$

In general, every single quantity has to be converted

You might find $z \geq \sim 1$ interpreted as recession velocity, using the relativistic Doppler effect formula:

$$1+z = \sqrt{\frac{1+v/c}{1-v/c}}$$