V. The Thermal Beginning of the Universe

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Jan 2011

CMB discovery time-line

-1947-1948 Gamow, Alpher and Hermans model of nucleosynthesis predicts relic millimeter radiation, but the models have difficulties to produce elements heavier than Li, leads to its neglect.

-1965 Arno Penzias & Robert Wilson's serendipitous discovery of a constant excess isotropic noise with their antenna at Bell Labs (Nobel 1978).

-late 60's many groups made measurements of the intensity of the radiation and a temperature collectively showing the spectrum is that of a BB (to 10% accuracy)

-1969 Tentative detection of a dipole anisotropy by Conklin, confirmed in 1971-1977 by Henry, Wilkinson, and Smooth et al.

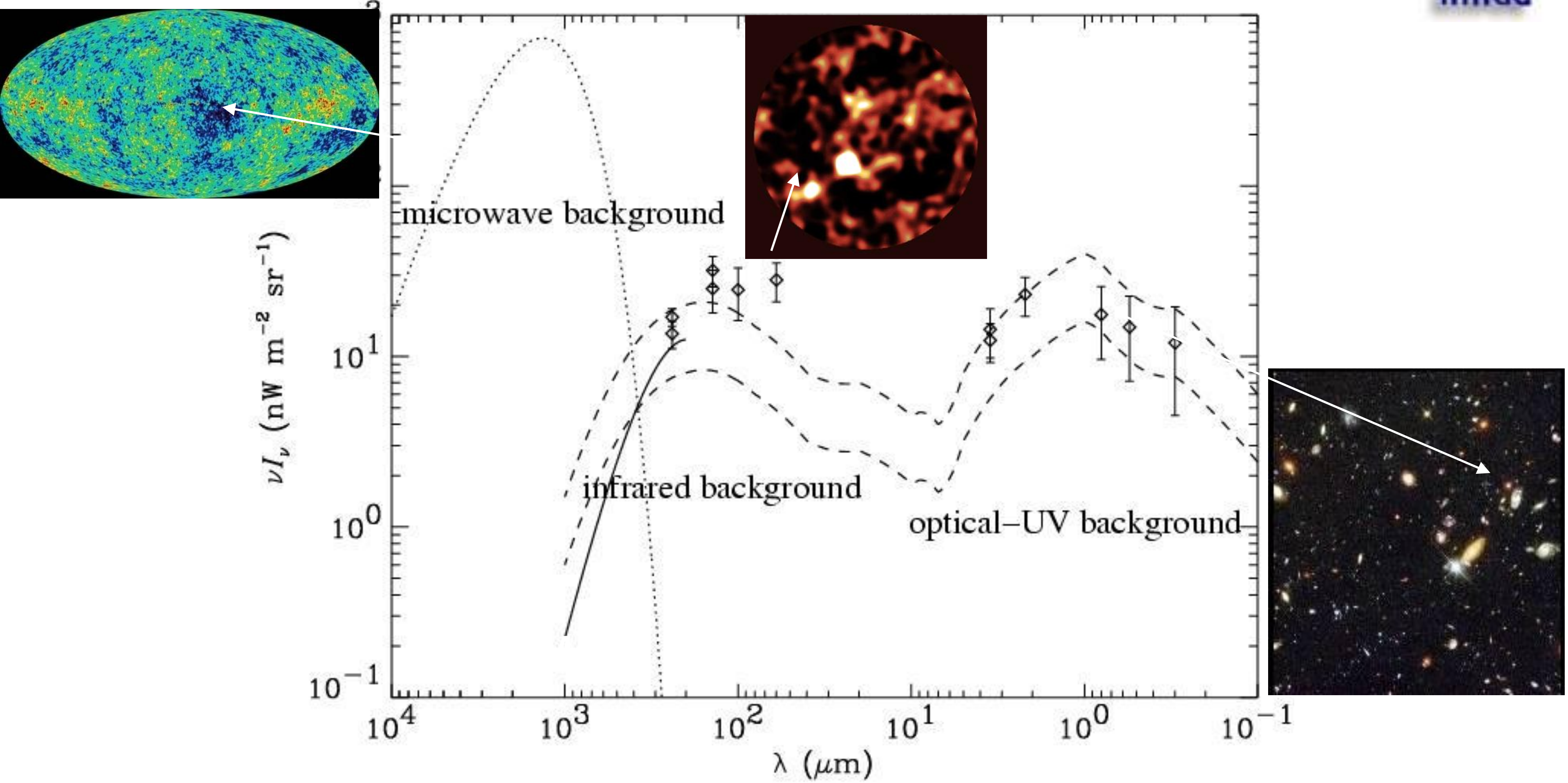
-1989 COBE (COsmic Background Explorer) launched, in 1990 first results confirming BB spectrum.

-1992 COBE's detection of non-dipole anisotropies (nobel prize 2006 for PI's Smoot & Mather.

-2003 WMAP results on precission cosmology through CMB anisotropies, end mission 2010

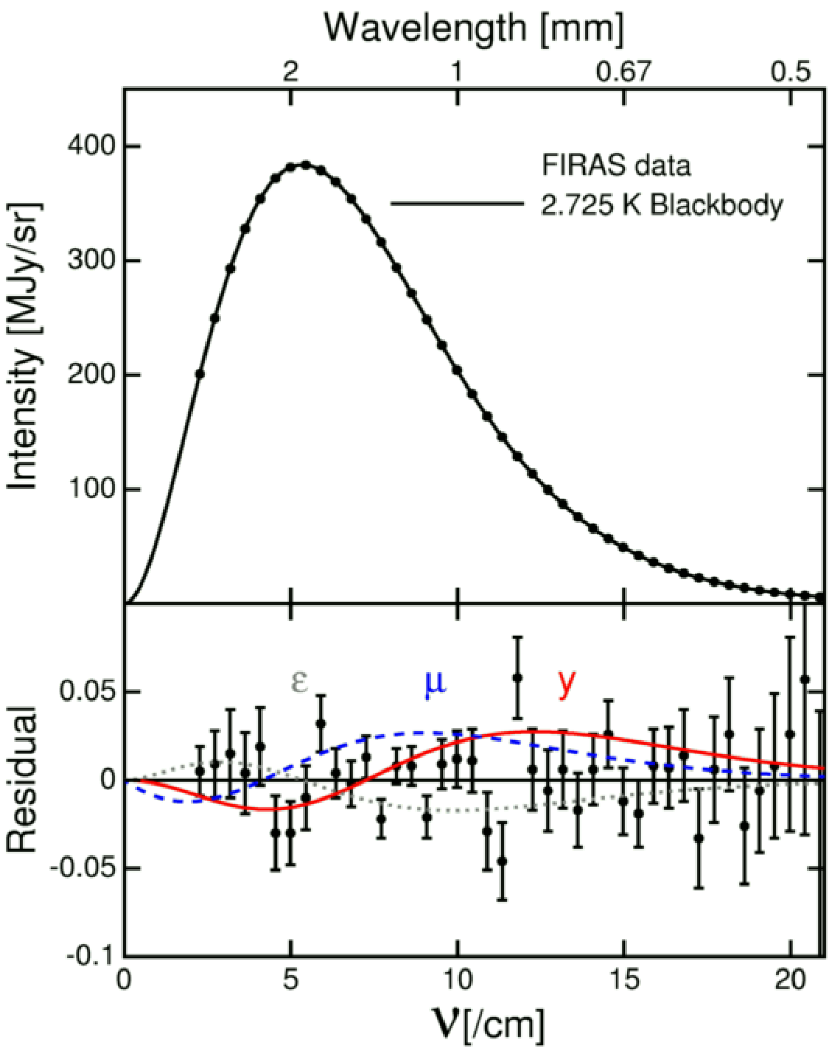
(Following E. Wright's CMB review paper)

Dominant background radiation

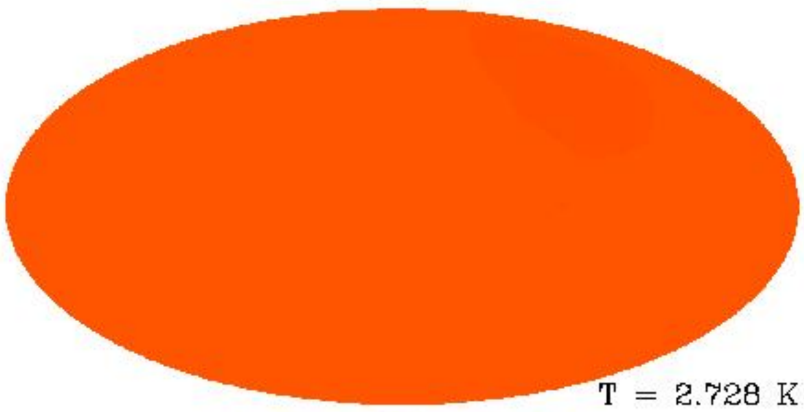


The photons of the CMB are still the largest contributors to the radiation energy in the Universe.

CMB spectrum



(E. Wright)



An almost perfect BB was measured by the FIRAS instrument aboard COBE when it was compared to a very good BB calibrator.

T=2.725±0.002 K

(Following E. Wright's CMB review paper)

CMB spectrum

The energy density of this radiation is

$$\varepsilon_{rad} = \rho_{rad}c^2 = aT^4 = 4.17\times10^{-14}\,Jm^{-3} \Rightarrow \rho_{rad} = 4.63\times10^{-31}\,Kgr\,m^{-3}$$

$$a \equiv 4\sigma/c = 7.565\times10^{-16}\ \mathrm{Jm}^{-3}K^{-4}$$

Dividing this by the critical density $\rho_c = \dfrac{3H^2}{8\pi G} = 1.88\,h^2\times10^{-26}Kgr\,m^{-3}$

we obtain the CMB density parameter $\boxed{\Omega_{rad} = 2.47\times10^{-5}\,h^{-2}}$

Because $\rho \propto T^4$, and $\rho \propto a^{-4} \Rightarrow$ $\boxed{T \propto 1/a}$ a scale factor

The radiation was hotter and had a higher energy density in the past
But was it a blackbody as well?

(From M. Georganopoulos' lecture lib)

CMB spectrum: *z* evolution

$$\varepsilon(f)df = \frac{8\pi h}{c^3}\frac{f^3 df}{\exp(hf/kT)-1}$$

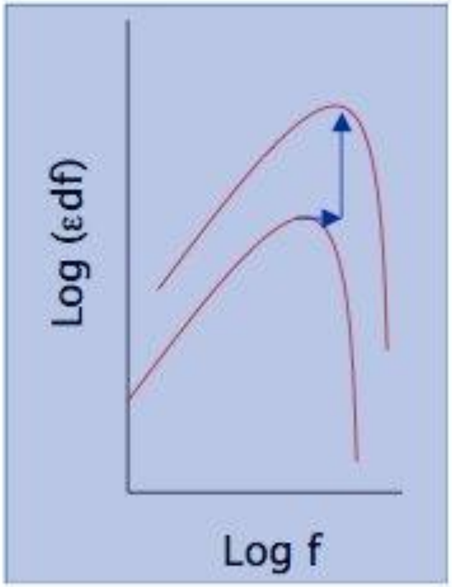
Going to earlier times, as a decreases, T increases. The denominator is invariant for f/T=constant.

If a_1 is the scale factor at given instance in the past, the peak of the emission will be at $f_{1,peak} = f_{peak}/a_1$, since $T_1 = T/a_1$. Also,

$$f_{1,peak}^3\, df_{1,peak}^3 = f_{peak}^3\, df_{peak}^3 / a_1^4$$

The peak of the curve will shift up in frequency by a factor $1/a_1$ and up in energy density by a factor $1/a_1^4$. This is the case not only for the peak frequency but for any frequency:

<u>the blackbody shape is preserved, shifting up in frequency by $1/a_1$ and in energy density by $1/a_1^4$.</u>



(From M. Georganopoulos' lecture lib)

CMB spectrum: *z* evolution

The almost perfect BB shape of the CMB backs up the expansion of the Universe, and the existence of a hotter earlier universe.

◆ If the CMB were just a tired relic light: $n_\gamma(z=0)=(1+z_e)^3 B_\nu(T_e)$ but FIRAS imposes that the factor in front of $B_\nu(T_e)$ is 1 with a precission better than 10^{-4}. Hence $(1+z_e)=1\pm1\times10^{-4} \Rightarrow z_e<0.000033$ and opaque from that onwards. But we have sources at z~4! So this is not a possibility.

◆ If the steady model were correct, there would be no evolution. Today we see CMB + FIR radiation from stars and galaxies. Energy added between 1 month and a few thousand yrs after BB will produce

$$I_{BE}(\nu,T)=\frac{2\pi\nu^3}{c^2}\frac{1}{\exp(h\nu/kT+\mu)-1}$$

but there are no deviations to the BB spectrum

(Following E. Wright's CMB review paper)

Photon/baryon ratio

If no particles are created or destroyed, is this ratio conserved?

Yes, both photon and baryon number densities scale as $1/a^3$

What is the current number density of CMB photons?

Their energy density is :

$$\boxed{\varepsilon_{rad} = aT^4 = 2.6 \times 10^5\, eV\, m^{-3}}$$

Dividing this by their mean energy E_{mean}=3kT=7.05 10^{-4} eV gives

$$\boxed{n_{rad} = \frac{\varepsilon_{rad}}{E_{mean}} = 3.7 \times 10^8\, m^{-3}}$$

(From M. Georganopoulos' lecture lib)

Photon/baryon ratio

What is the current number density of baryons?

Their density parameter, derived from nucleosynthesis, is :

$\Omega_B \approx 0.02\,h^{-2}$ This can be converted into an energy density:

$$\varepsilon_B = \rho_B c^2 = \Omega_B \rho_c c^2 \approx 3.4\times 10^{-11}\,J\,m^{-3} = 2.1\times 10^{8}\,eV\,m^{-3}$$

Divide this by the proton rest mass, E_B=939 MeV: $\boxed{n_B = 0.22\,m^{-3}}$

Recall : $\varepsilon_{rad} = 2.6\times 10^{5}\,eV\,m^{-3},\, n_{rad} = 3.7\times 10^{8}\,m^{-3}$

Currently:
- the baryon energy density dominates over that of CMB photons by ~1000.
- the CMB number density dominates over that of baryons by ~10^9. (valid for all times).

$$\boxed{\Omega_B/\Omega_\gamma \approx 1000 \quad , \quad n_\gamma/n_B \approx 10^9}$$

(From M. Georganopoulos' lecture lib)

Radiation era

We have that $\rho_M \propto R^{-3}$

$\rho_{rad} \propto R^{-4}$

There must be a z at which $\rho_M = \rho_{rad}$

Taking into account that nucleosynthesis predicts $n_\nu = 0.68\, n_\gamma$, then $\Omega_{rad} = 4.2 \times 10^{-5}\, h^{-2}$

$$1 + z_{eq} = 23900\, \Omega_m h^2 \quad \Rightarrow \; z_{eq} \approx 3100$$

Therefore the thermal history of the Universe can be divided in two main eras: a *radiation dominated era* ($z \gg z_{\rm eq}$) and a *matter dominated era* ($z \ll z_{\rm eq}$). In the radiation dominated era, in which we can neglect the curvature and Λ terms in Friedmann's equation, we have:

$$R \propto t^{1/2}\,.$$

By differentiating this relation with respect to time and using (12) we have:

$$t = \left(\frac{3}{32\pi G \rho_\gamma} \right)^{1/2} . \tag{52}$$

Using $\rho_\gamma = \pi^2 k_{\rm b} T^4 / 15 h^3 c^5$ we finally obtain the important relation between cosmic time and the temperature of the Universe in the radiation dominated era:

$$T_{\rm Kelvin} \simeq 1.3 \times 10^{10} t_{\rm sec}^{-1/2} \tag{53}$$

ie., at $t = 1$ sec the Universe had $T \sim 10^{10}$ K! It is evident that the Universe at early times was hot enough for nucleosynthesis to occur, as it had been supposed originally by Gamow. The era of nucleosynthesis takes place around $\sim 10^9$ K.

(From M. Plionis' notes or Peacock 1999)

CMB origin

$T \propto 1/a$ ->

In very early times, the energy of the CMB photons was much greater than the 13.6 eV required to ionize H. just a soup of photons, e and p (for simplicity, ignore the few He nuclei produced in nucleosynthesis).

All the mater in the Universe was ionized. If a p manages to capture an e and form an H atom, the H atom was immediately ionized by one of the abundant photons with E>13.6 eV.

In this soup of e,p, and photons, the interaction that insures thermodynamic equilibrium (a single temperature for all three species) is electron Thomson scattering:

$$\gamma + e^- \rightarrow \gamma + e^-, \quad \text{Thomson cross section } \sigma_T = 6.65 \times 10^{-29}\, m^2$$

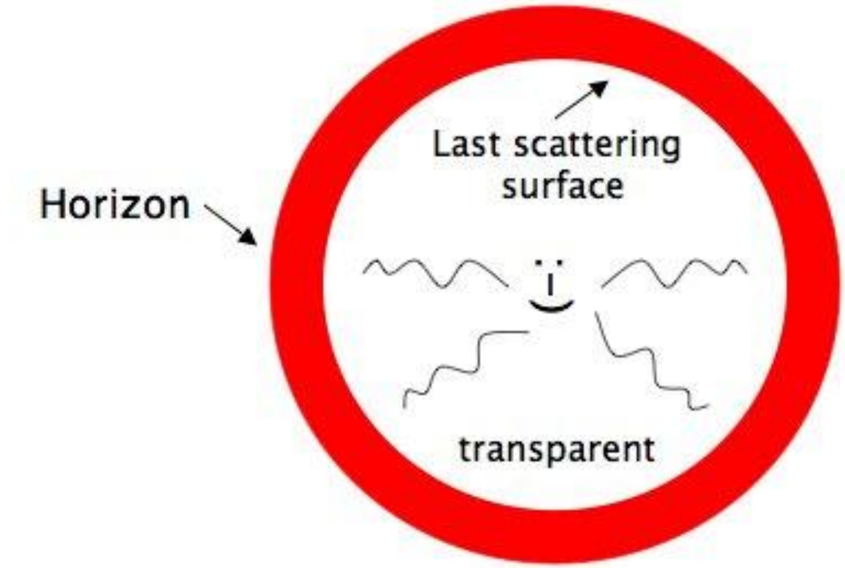
As time was passing, the CMB photons cooled down due to the expansion of the Universe, and eventually they were not able to ionize H, the Universe became neutral.

Since there were no free electrons left, the CMB photons stopped getting scattered and, after a last scattering, kept propagating unobstructed.

(From M. Georganopoulos' lecture lib

CMB origin

The last scattering surface.



Every observed is surrounded by a spherical last scattering surface. The CMB photons emerge from the last scattering surface and propagate in a straight line all the way to the observed with no further scatterings.

(From M. Georganopoulos’ lecture lib

CMB origin

3 important epochs:

1. **Recombination**, the time when the baryonic component of the Universe became neutral (number of ions=number of neutral atoms)

2. **Photon decoupling**, the time when the rate of photon scattering becomes smaller than H.
In other words, this is the epoch when the time between scatterings for a photon, becomes larger than the Hubble time.
When photons decouple, they cease to interact with electrons and the Universe becomes transparent.

3. **Last scattering.** This is the time when a typical CMB photon underwent its last scattering from an electron.

The last scattering time is very close to the photon decoupling time

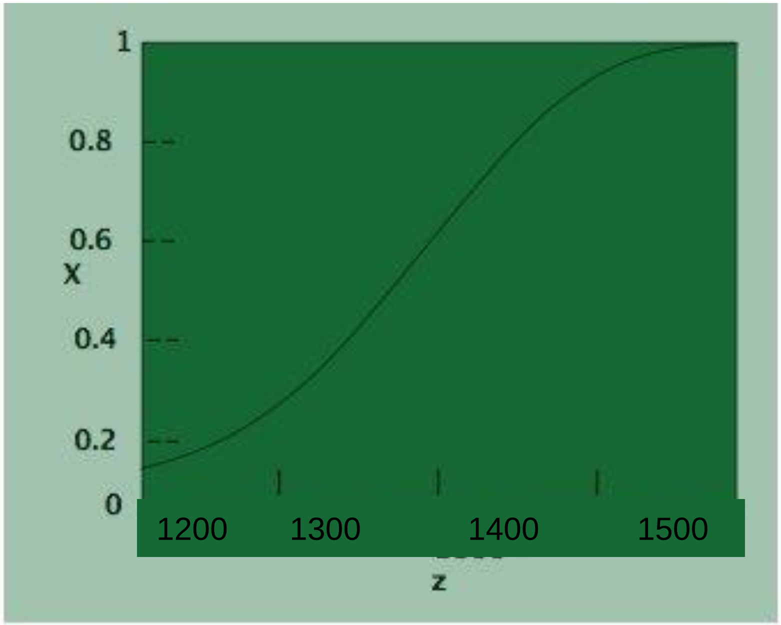
(From M. Georganopoulos' lecture lib)

CMB origin: recombination

the Saha equation gives us the ionization fraction X as a function of the ionization potential Q and the baryon to photon ratio η:

$$\frac{1-X}{X^2} = 3.84\eta\left(\frac{kT}{m_e c^2}\right)^{3/2} \exp\left(\frac{Q}{kT}\right)$$

$$X \equiv n_e / n_B = n_e / (n_H + n_p)$$



Recombination is a gradual process. Defining the moment of recombination at X=1/2 we obtain:

$$kT_{rec} = 0.323\,eV \Rightarrow T_{rec} = 3740\,K, \quad 1+z_{rec} = \frac{1}{a_{rec}} = \frac{T_{rec}}{T_{CMB,0}} = \frac{3740\,K}{2.73\,K} \approx 1371$$

(From M. Georganopoulos' lecture lib)

CMB origin: decoupling

The photon scattering rate is:

$$\Gamma = \frac{c}{\lambda} = n_e(z)\sigma_e c = X(z) n_{B,0}(1+z)^3 \sigma_e c = 4.4 \times 10^{-21} X(z)(1+z)^3 s^{-1}$$

Recombination and decoupling take place during the matter dominated era, so Friedmann's eq. is:

$$\frac{H^2}{H_0^2} = \frac{\Omega_{m,0}}{a^3} = \Omega_{m,0}(1+z)^3 \Rightarrow H = 1.24 \times 10^{-18}(1+z)^{3/2}$$

Setting Γ=H, we obtain

$$1 + z_{dec} = \frac{43.0}{X(z_{dec})^{2/3}} \Rightarrow z_{dec} = 1130$$

(From M. Georganopoulos' lecture lib)

CMB origin: decoupling

In our calculation we used the Saha equation, which assumes that photoionization is always in equilibrium. This is not true when Γ becomes comparable to H. A detailed calculation gives:

$$z_{dec} \approx 1100, T_{dec} \approx 3000K.$$

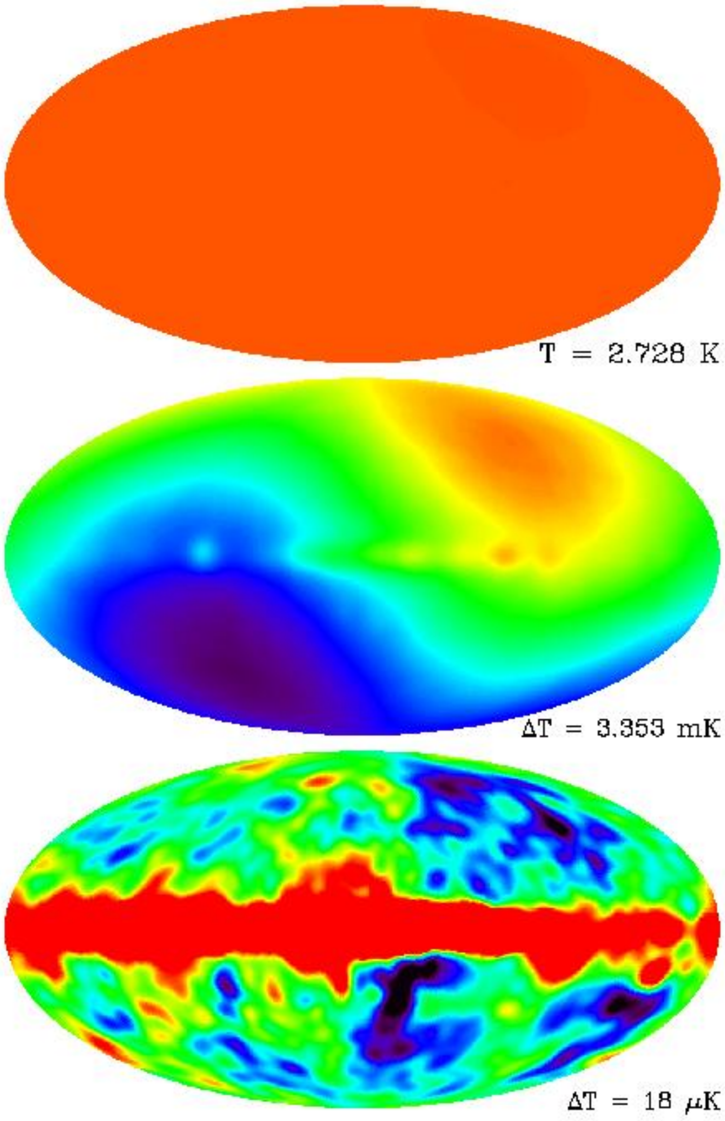
Event	Redshift	Temperature	**Time (yr)**
Radiation-mattter equality	3570	9730	**47000**
Recombination	1370	3740	**256000**
Photon decoupling	1100	3000	**372000**
Last scattering	1100	3000	**372000**

›Before decoupling, photon pressure on the matter smoothed out density fluctuations in the photon baryon field at distances smaller than the horizon distance back then.

›After, the hydrogen gas was free to collapse under its self gravity (and that of the dark matter) to form structure in the Universe

(From M. Georganopoulos' lecture lib)

CMB spectrum: dipole anisotropy



Dipole anisotropy in COBE data can be explained as a Doppler effect between the frame of reference of the solar system and that at rest with the observable CMB.

$$\nu' = \gamma(1-\beta\cos\theta)\nu, \quad \text{with}\, \beta \equiv v/c$$

$$\text{and } \gamma \equiv 1/\sqrt{1-\beta^2}$$

$$T(\theta) = T_0/\gamma(1-\beta\cos\theta) \approx T_0 + T_0\beta\cos\theta$$

A fit to the image $T_0\beta$=3353±24μK
And with T_0=2.735K

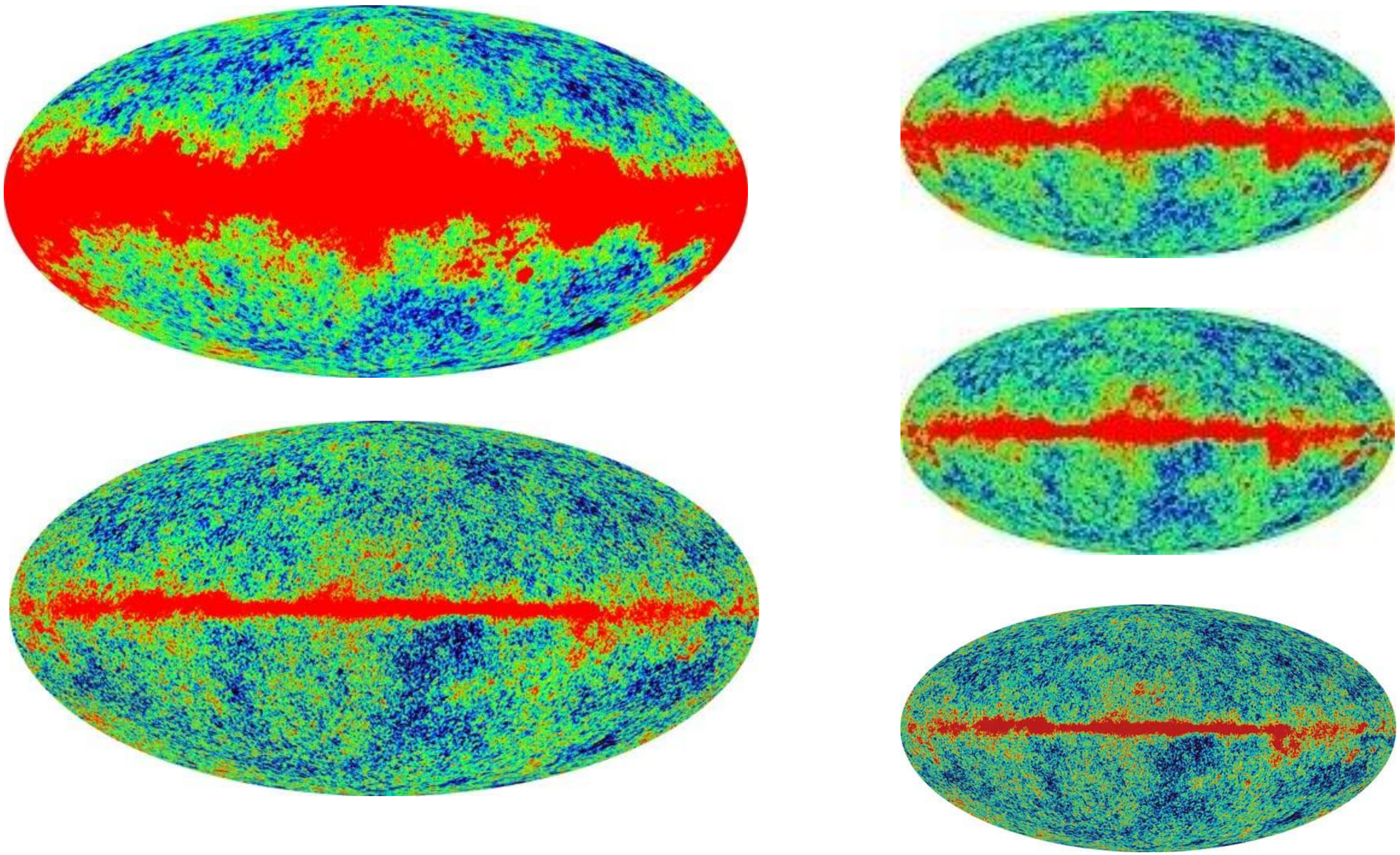
$$\vec{v}_{sun} - \vec{v}_{CMB} = 369 \pm 3 \text{ km s}^{-1}$$

Taking into account the movement around the MW, and the movement of the LG
towards *(l,b)*≈(277°, 30°)
Signature of local attractors.

$$\vec{v}_{LG} - \vec{v}_{CMB} \approx 620 \pm 45 \text{ km s}^{-1}$$

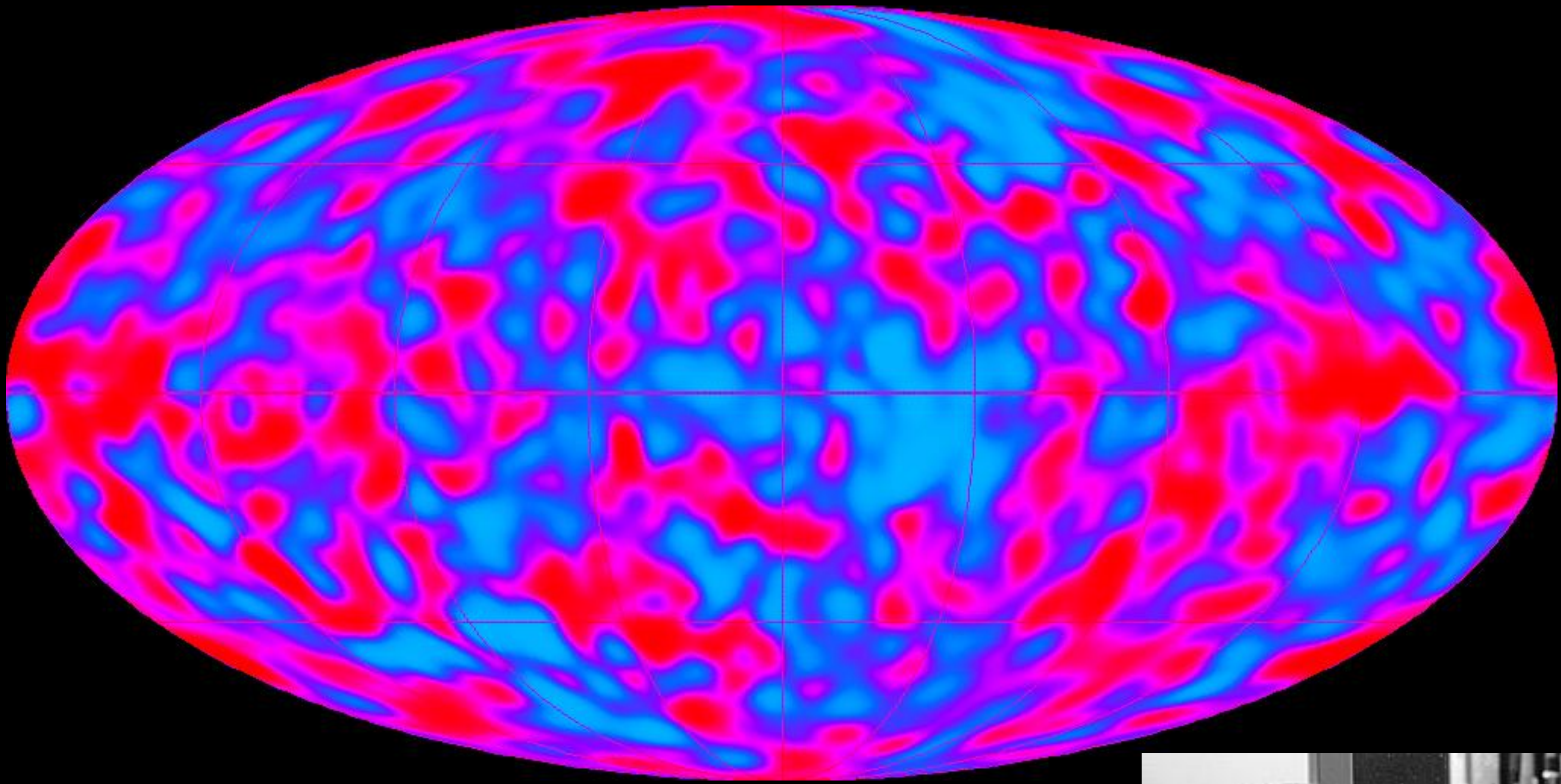
(Following E. Wright's CMB review paper)

CMB spectrum: removing the galaxy



WMAP: 23 to 90 GHz image of the CMB after dipole subtraction. The galaxy emission is dominated by dust

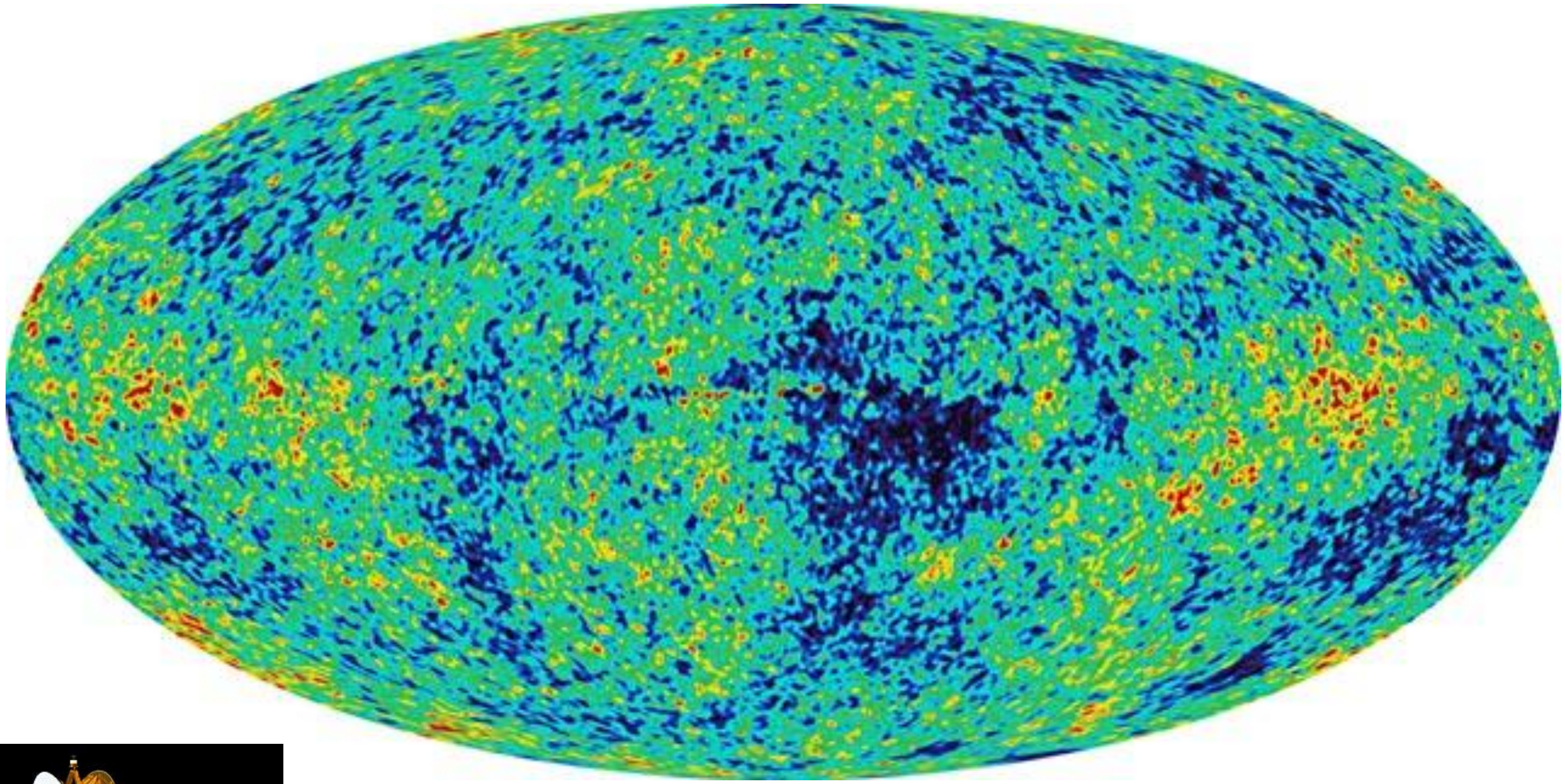
CMB spectrum: removing the galaxy

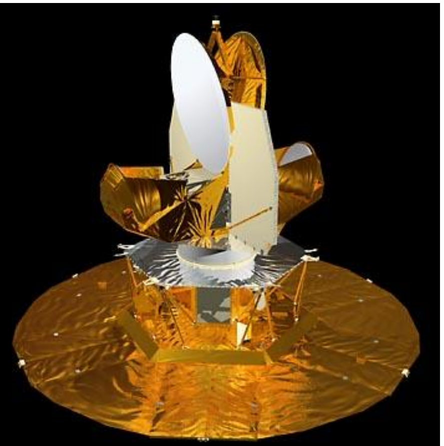


Cosmic Background Explorer COBE (1992):

$\Delta T/T = 10^{-5}$

CMB spectrum: removing the galaxy

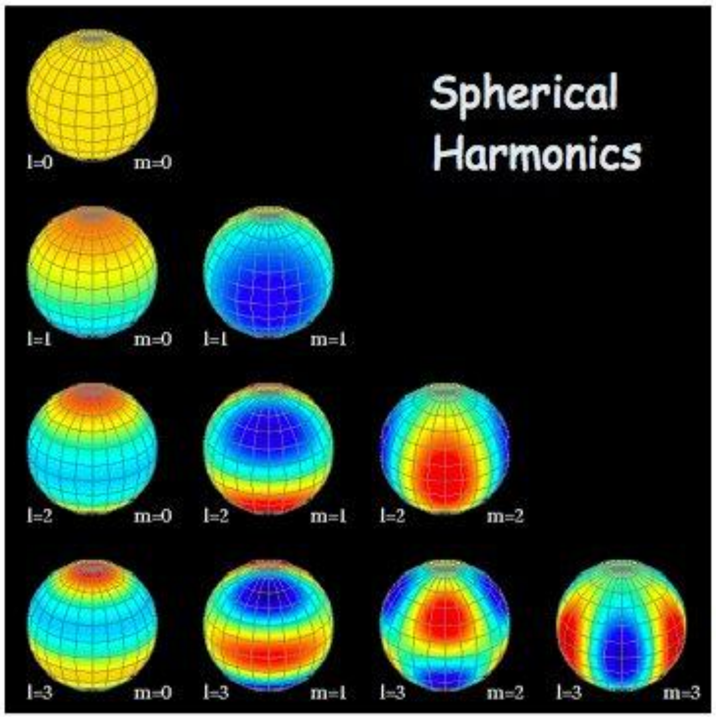


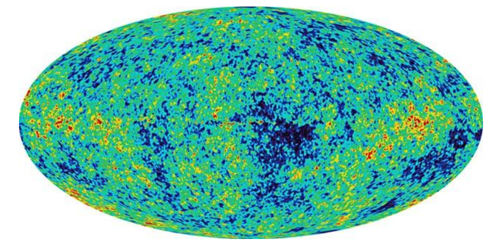


WMAP full sky view

CMB spectrum: statistical properties

T(l,b) can be fully specified by either the angular correlation function $C(\theta)$ or its Legendre transformation, the angular power spectrum C_{l}.





Write the temperature fluctuations as a series of spherical harmonics:

$$\frac{\delta T(\theta,\phi)}{T} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta,\phi)$$

$$a_{\ell m} = \int \frac{\Delta T(\theta,\phi)}{\mathrm{T_{CMB}}} Y_{\ell m}(\theta,\phi)\, \mathrm{d}\Omega$$

$$C_\ell = \langle |a_{\ell m}|^2 \rangle$$

The correlation function, a CMB map derived function:

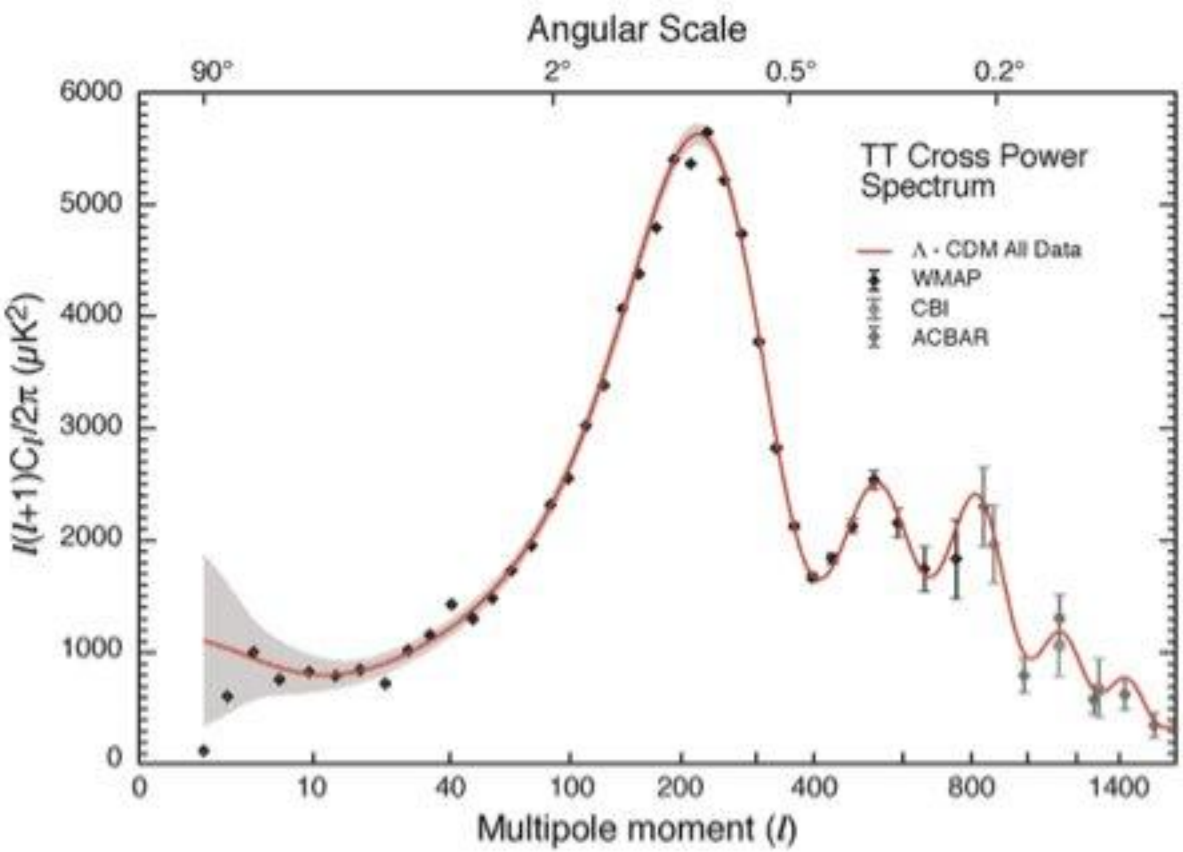
$$C(\theta) = \left\langle \frac{\delta T(\hat{n})}{T} \frac{\delta T(\hat{n}')}{T} \right\rangle_{\hat{n}\hat{n}'=\theta}$$

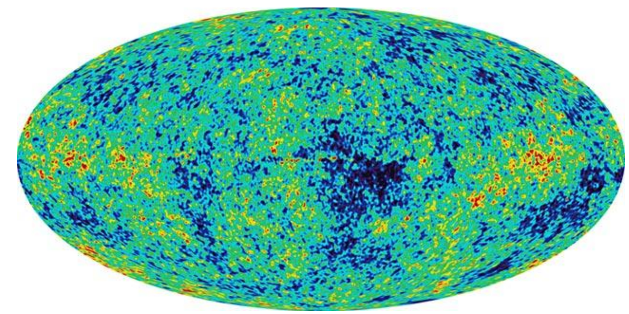
Which are related by $C(\theta) = \frac{1}{4\pi} \sum_{l} (2l+1) C_l P_l(\cos\theta)$ where P_l are Legendre polynomials of order *l.*

A term C_l is a measure of angular fluctuations on the angular scale $\theta \sim 180^o/l$.

In order to have equal power for all scales the spherical harmonics impose C_l = cte/*l(l+1)*. If the sky had equal power on all scales $lC_l(l+1)$ should be a constant.

CMB spectrum: power spectrum

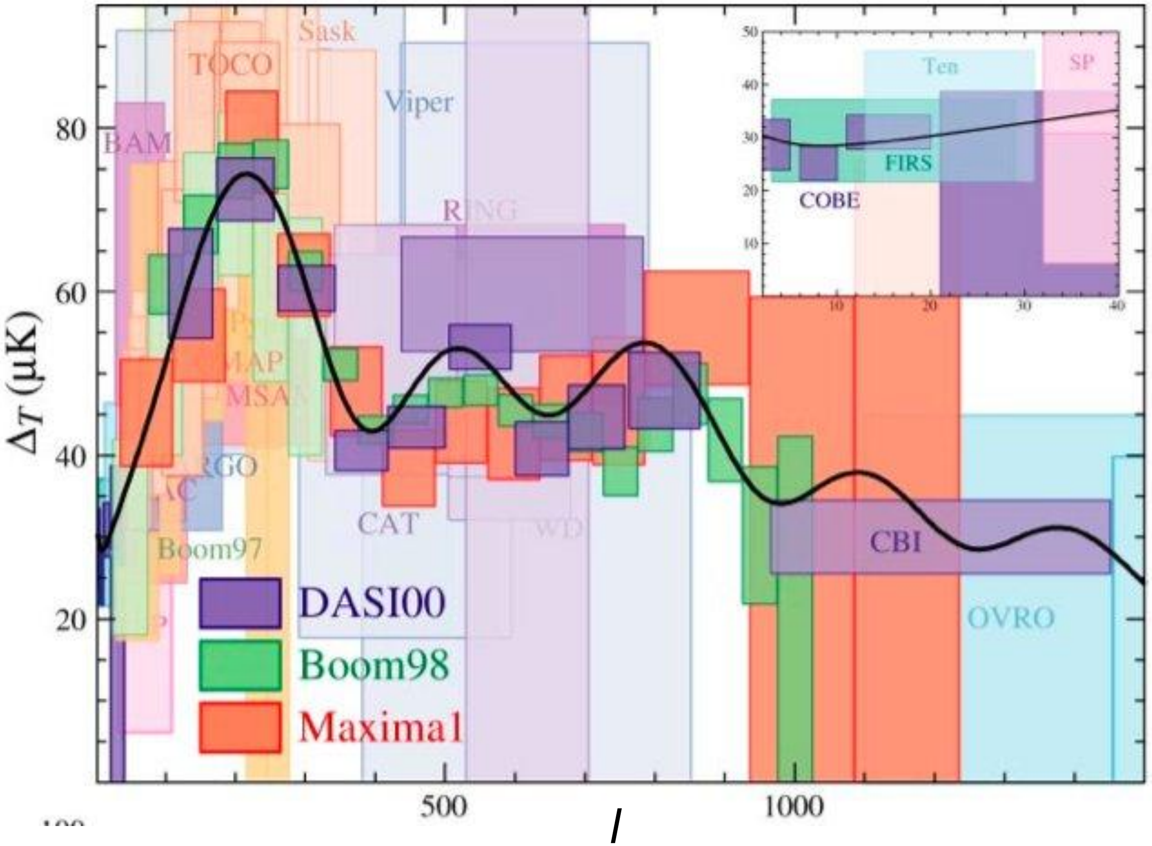




Why is there a major peak at l~200 (θ~1°)?
And what are the secondary peaks?

(From M. Georganopoulos' lecture lib)

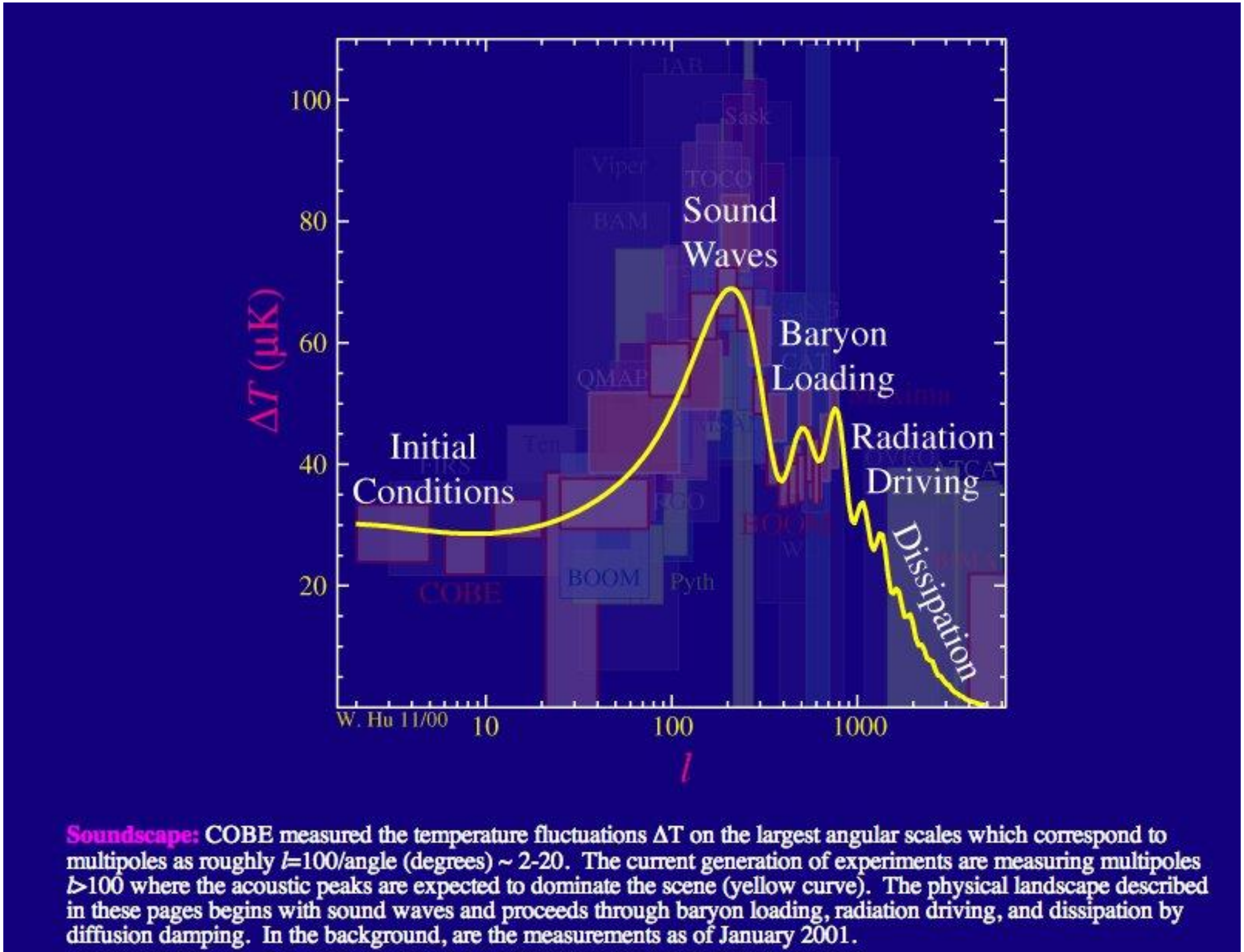
CMB spectrum: power spectrum



First peak had already been constrained by an array of 1992-2000 missions, and sampled in its full amplitude by Boomerang (1998) & Maxima (2000)

(Hu & Dodelson 2002)

CMB spectrum: power spectrum



(From Hu’s webpage)

CMB spectrum: power spectrum

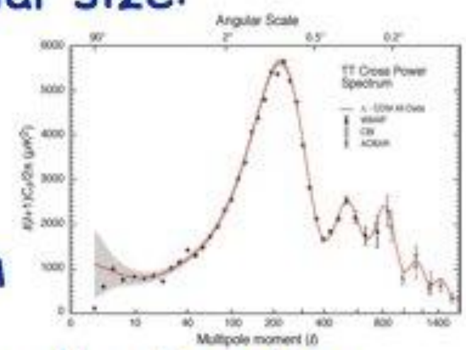
At the time of last scattering the Universe was matter dominated and the Hubble distance was:

$$\frac{c}{H(z_{ls})} = \frac{c}{H_0(z+1)^{3/2}} = \frac{3\times10^{8}\ m\,s^{-1}}{1.24\times10^{-18}\,s^{-1}(1101)^{3/2}} \approx 6.6\times10^{21}\ m \approx 0.2\ Mpc$$

Seen from Earth this has an angular size:

$$\theta_H = \frac{c/H(z_{ls})}{d_A} \approx \frac{0.2\,Mpc}{13\,Mpc} \approx 0.015\ rad \approx 1^{o}$$

This is the first peak of the power spectrum



The Hubble distance at the time of decoupling is ~ the maximum length information (photons, matter) could have traveled. The fluctuations at $\theta > \theta_H$ and $\theta < \theta_H$ must reveal different things.

(From M. Georganopoulos' lecture lib)

CMB spectrum: power spectrum

At the time of last scattering the Universe, the non-baryonic dark matter dominates:

$$\Omega_{dm} : \Omega_{\gamma} : \Omega_{m} = 6.4 : 1.4 : 1$$

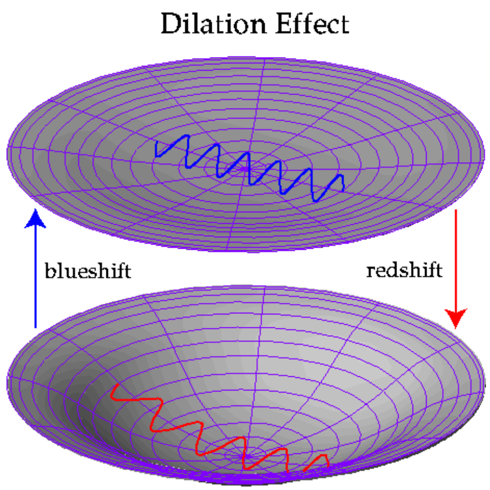
If the dark matter density distribution at the time of last scattering has a spatially varying component $\delta\rho$, then there is a spatially varying gravitational potential $\delta\Phi$ given by Poison's equation:

$$\nabla^2(\delta\Phi) = 4\pi G \delta\rho$$

(From M. Georganopoulos' lecture lib)

CMB spectrum: power spectrum $\theta > \theta_H$

If at the time of last scattering a CMB photon is at a local minimum (in a "potential well"), it will spend energy to climb out of the well and it will be redshifted.



If it is at a local maximum, it will roll down the maximum and gain energy (it will be blueshifted). In general, Sachs and Wolfe (1967) showed that:

$$\frac{\delta T}{T} = \frac{1}{3}\frac{\delta \Phi}{c^2}$$

All these are valid for angular scales $\theta > \theta_H$, because the photon-baryon fluid has no time to move further than θ_H. At smaller scales, we need to consider the fact that the photon-baryon fluid has time to move substantially under the influence of the gravity of dark matter.

(From M. Georganopoulos' lecture lib)

CMB spectrum: power spectrum $\theta<\theta_H$

The photon-baryon fluid moves under the influence of the non baryonic matter that dominates. When the fluid finds itself in a potential of dark matter, it flows toward the bottom of the well.

As it falls its pressure rises. As the pressure rises, it slows down, stops and inverts the fluid fall. The fluid expands, and a series of oscillations takes place, called "acoustic oscillations".

The oscillations stop at recombination, when photons decouple from baryons. How many oscillations take place? The larger the well the fewer the oscillations. For $\theta\sim\theta_H$ there is time for ~1 oscillation.

If the photon-baryon fluid is compressed in a well at the time of recombination, the decoupled photons will be hotter than average.

If the photon-baryon fluid is expanded at the time of recombination, the decoupled photons will be cooler than average.

Graphic by Wayne Hu, http://background.uchicago.edu/~whu/beginners/introduction.html

The highest peak at $\theta\sim\theta_H$ (l~200) represents the wells where the photon-baryon fluid had just reach maximum compression at the recombination time.

(From M. Georganopoulos' lecture lib)

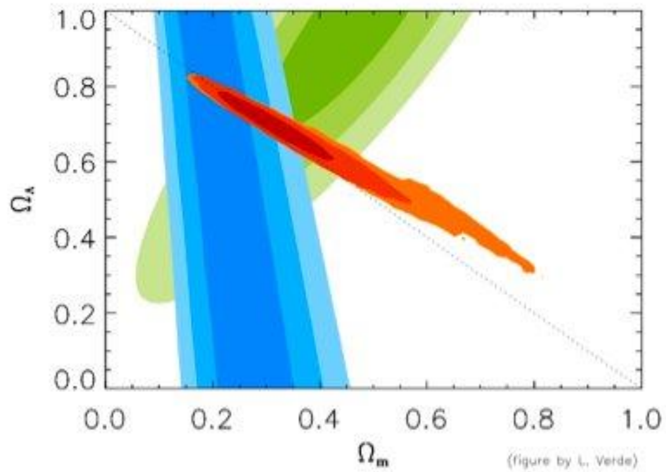
CMB spectrum: 1st peak of power spectrum

In a negatively curved Universe, the angular size of an object of known intrinsic size at a given redshift is smaller than it is in a positively curved Universe.

If the Universe were negatively curved, the first peak would be seen at l>180 or θ<1°.

If the Universe were positively curved ,the first peak would be seen at l<180 or θ>1°.

The observe peak at l~200 is consistent with k=0 or Ω_0=1



Supernovae, large scale structure, CMB

The observe peak at l~200 is consistent with k=0 or Ω_m+Ω_Λ=1

(From M. Georganopoulos' lecture lib)

CMB spectrum: 1st peak of power spectrum

The amplitude of the first peak depends on the sound speed of the photon baryon fluid:

$$c_s = c\sqrt{w_{pb}}$$

The equation of state, $p_{pw}=w_{pb}\rho$ depends of the baryon to photon ratio. To reproduce the power spectrum we need

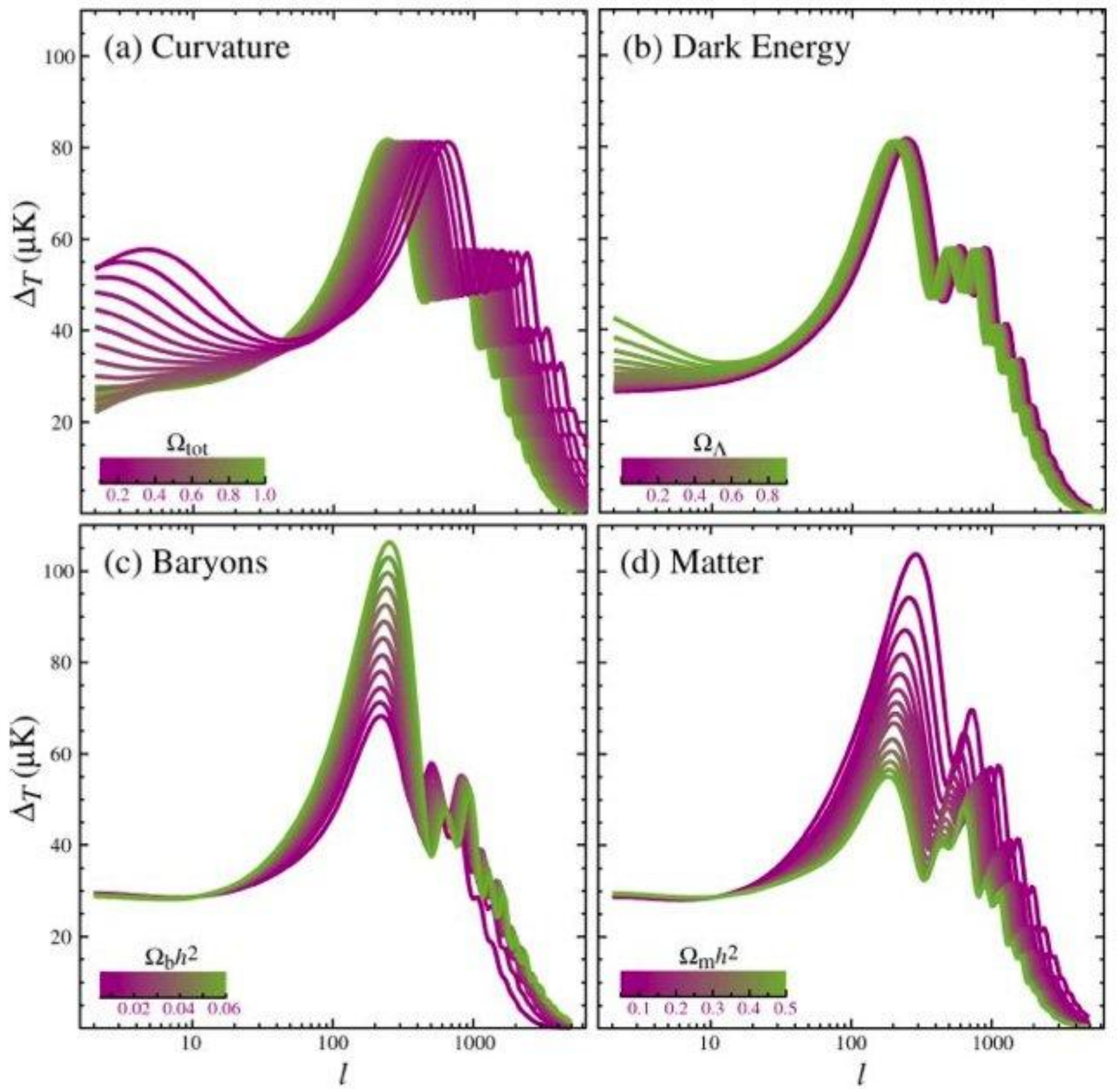
$$\Omega_{bary,0} = 0.04 \pm 0.02.$$

How nice! This is in good agreement with the nucleosynthesis result!

There is much more to the CMB power spectrum.

(From M. Georganopoulos' lecture lib)

CMB spectrum: cosmic dependences



Varied around a fidutial model Ω_{tot}=1, Ω_Λ=0.65, Ω_B=0.02h^{-2}, Ω_m=0.147h^{-2}, n=1, z_{ri}=0, E=0

(Hu & Dodelson 2002)

WMAP spectrum: precission cosmology

Table 2: Power Law ΛCDM Model Parameters and 68% Confidence Intervals. The Three Year fits in this Table assume no SZ contribution, $A_{SZ} = 0$, to allow direct comparision with the First Year results. Fits that include SZ marginalization are given in Table 5 (first column) and represent our best estimate of these parameters.

Parameter	First Year Mean	WMAPext Mean	Three Year Mean	First Year ML	WMAPext ML	Three Year ML
$100\Omega_b h^2$	$2.38^{+0.13}_{-0.12}$	$2.32^{+0.12}_{-0.11}$	2.23 ± 0.08	2.30	2.21	$10^2\Omega_b h^2 = 2.233^{+0.072}_{-0.091}$
$\Omega_m h^2$	$0.144^{+0.016}_{-0.016}$	$0.134^{+0.006}_{-0.006}$	0.126 ± 0.009	0.145	0.138	0.128
H_0	72^{+5}_{-5}	73^{+3}_{-3}	74^{+3}_{-3}	68	71	73
τ	$0.17^{+0.08}_{-0.07}$	$0.15^{+0.07}_{-0.07}$	0.093 ± 0.029	0.10	0.10	0.092
n_s	$0.99^{+0.04}_{-0.04}$	$0.98^{+0.03}_{-0.03}$	0.961 ± 0.017	0.97	0.96	0.958
Ω_m	$0.29^{+0.07}_{-0.07}$	$0.25^{+0.03}_{-0.03}$	0.234 ± 0.035	0.32	0.27	0.24
σ_8	$0.92^{+0.1}_{-0.1}$	$0.84^{+0.06}_{-0.06}$	0.76 ± 0.05	0.88	0.82	0.77

Table 6: ΛCDM Model

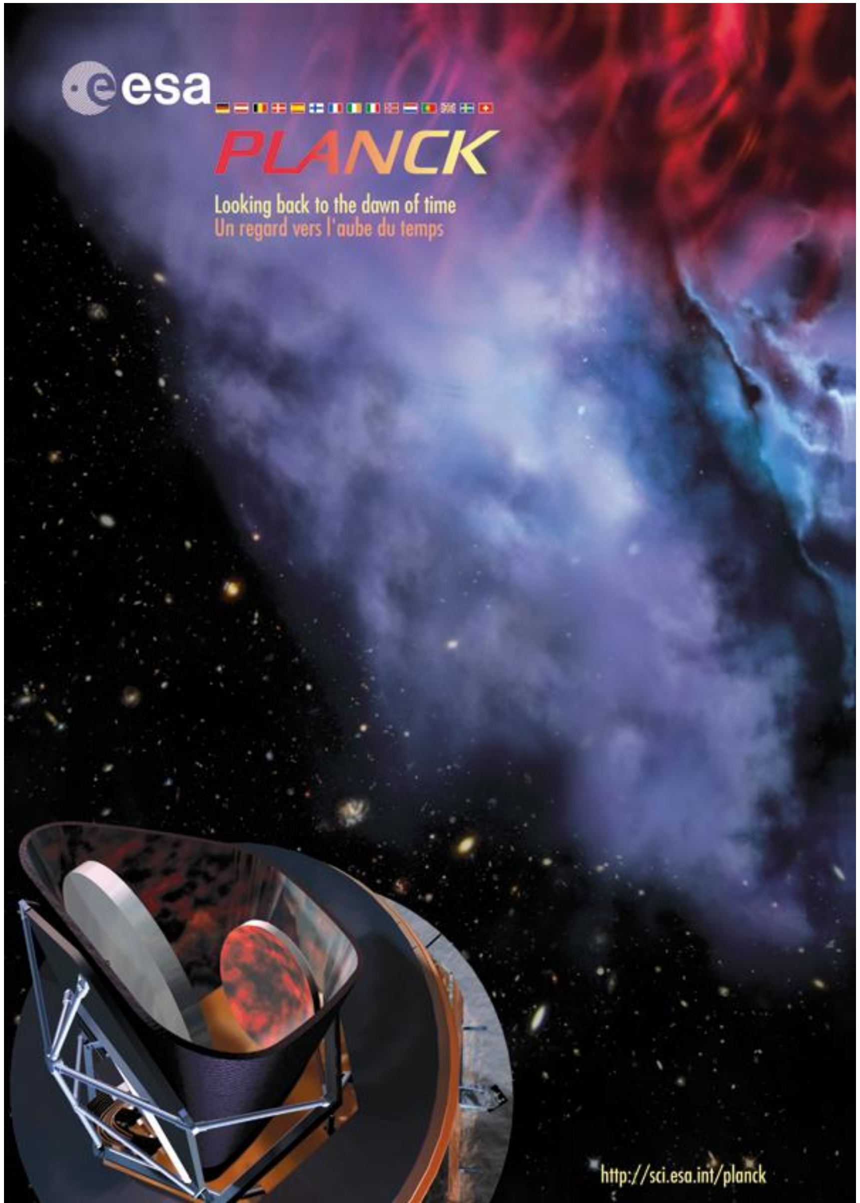
Parameter	WMAP+ SDSS	WMAP+ LRG	WMAP+ SNLS	WMAP + SN Gold	WMAP+ CFHTLS
$100\Omega_b h^2$	$2.233^{+0.062}_{-0.086}$	$2.242^{+0.062}_{-0.084}$	$2.233^{+0.069}_{-0.088}$	$2.227^{+0.065}_{-0.082}$	$2.255^{+0.062}_{-0.083}$
$\Omega_m h^2$	$0.1329^{+0.0056}_{-0.0075}$	$0.1337^{+0.0044}_{-0.0061}$	$0.1295^{+0.0056}_{-0.0072}$	$0.1349^{+0.0056}_{-0.0071}$	$0.1408^{+0.0034}_{-0.0050}$
h	$0.709^{+0.024}_{-0.032}$	$0.709^{+0.016}_{-0.023}$	$0.723^{+0.021}_{-0.030}$	$0.701^{+0.020}_{-0.026}$	$0.687^{+0.016}_{-0.024}$
A	$0.813^{+0.042}_{-0.052}$	$0.816^{+0.042}_{-0.049}$	$0.808^{+0.044}_{-0.051}$	$0.827^{+0.045}_{-0.053}$	$0.846^{+0.037}_{-0.047}$
τ	$0.079^{+0.029}_{-0.032}$	$0.082^{+0.028}_{-0.033}$	$0.085^{+0.028}_{-0.032}$	$0.079^{+0.028}_{-0.034}$	$0.088^{+0.026}_{-0.032}$
n_s	$0.948^{+0.015}_{-0.018}$	$0.951^{+0.014}_{-0.018}$	$0.950^{+0.015}_{-0.019}$	$0.946^{+0.015}_{-0.019}$	$0.953^{+0.015}_{-0.019}$
σ_8	$0.772^{+0.036}_{-0.048}$	$0.781^{+0.032}_{-0.045}$	$0.758^{+0.038}_{-0.052}$	$0.784^{+0.035}_{-0.049}$	$0.826^{+0.022}_{-0.035}$
Ω_m	$0.266^{+0.026}_{-0.036}$	$0.267^{+0.018}_{-0.025}$	$0.249^{+0.024}_{-0.031}$	$0.276^{+0.023}_{-0.031}$	$0.299^{+0.019}_{-0.025}$

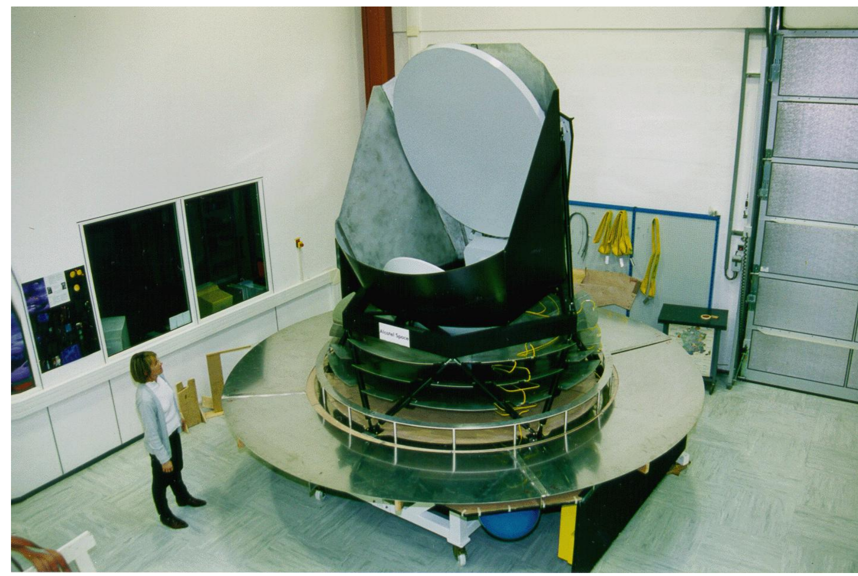
Data Set Constraints on Geometry and Vacuum Energy

Data Set	Ω_K	Ω_Λ
WMAP + $h = 0.72 \pm 0.08$	$-0.003^{+0.013}_{-0.017}$	$0.758^{+0.035}_{-0.058}$
WMAP + SDSS	$-0.037^{+0.021}_{-0.015}$	$0.650^{+0.055}_{-0.048}$
WMAP + 2dFGRS	$-0.0057^{+0.0061}_{-0.0088}$	$0.739^{+0.026}_{-0.029}$
WMAP + SDSS LRG	$-0.010^{+0.011}_{-0.015}$	$0.728^{+0.020}_{-0.028}$
WMAP + SNLS	$-0.015^{+0.020}_{-0.016}$	$0.719^{+0.021}_{-0.029}$
WMAP + SNGold	$-0.017^{+0.022}_{-0.017}$	$0.703^{+0.030}_{-0.038}$

(Spergel et al. 2006)

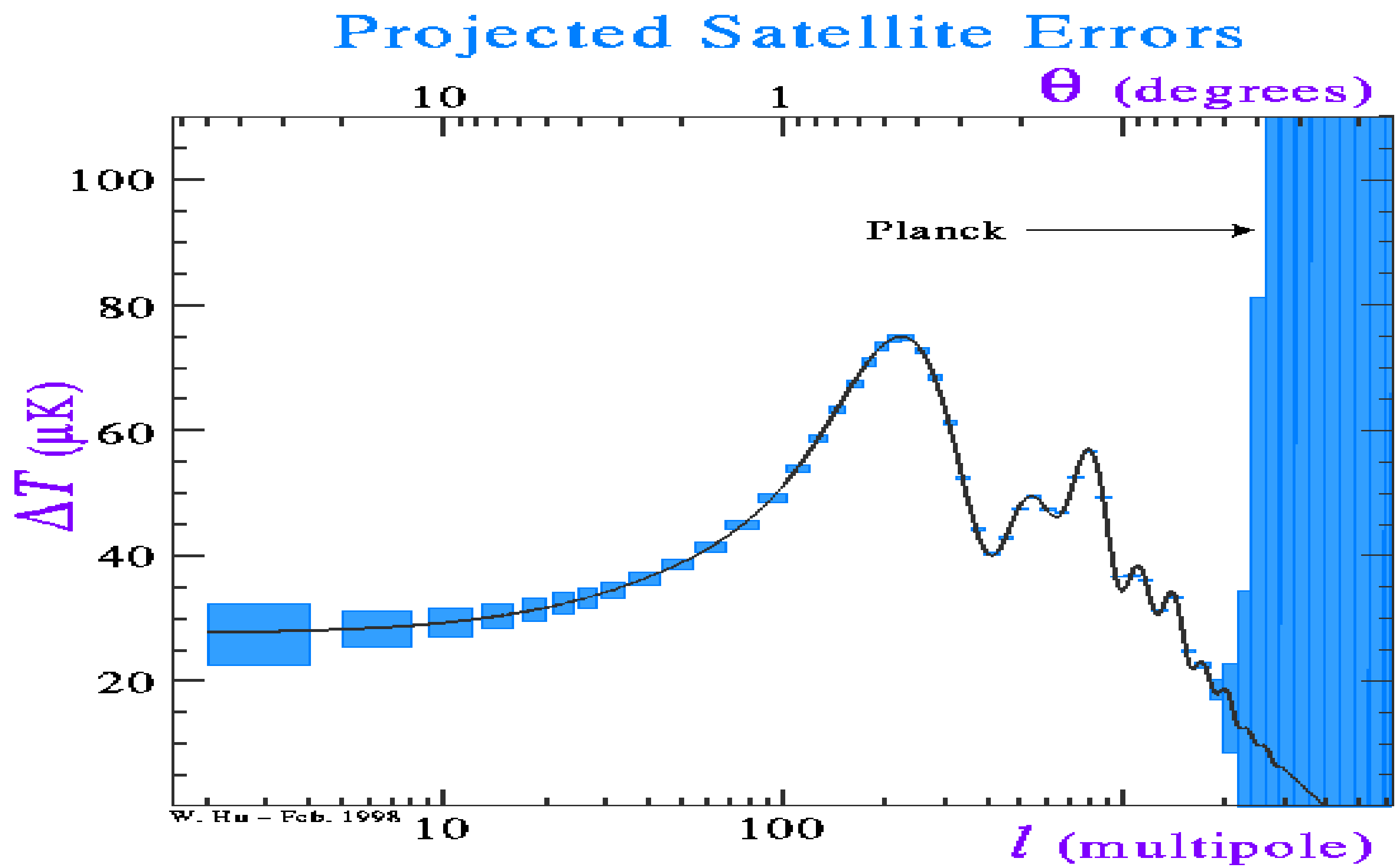
CMB spectrum: the future





(From Planck website)

CMB spectrum: the future



(From Planck website)