OBSERVATIONAL METHODS IN RADIO ASTRONOMY II : INTERFEROMETRY

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(based on ALMA community days
and NRAO Synthesis workshop)

INTRODUCTION

WHY INTERFEROMETRY?

The problem:

- Single-dish telescope resolution is ~ λ / D
- For a given λ, to increase resolution one would need to increase D, the antenna size…
- But: keeping/increasing high surface accuracy and pointing precision

  → difficult to achieve technically !
- The largest single-dish antenna (fully directional) are the 100m Effelsberg radiotelescope (Germany) and the Green Bank Telescope (USA)

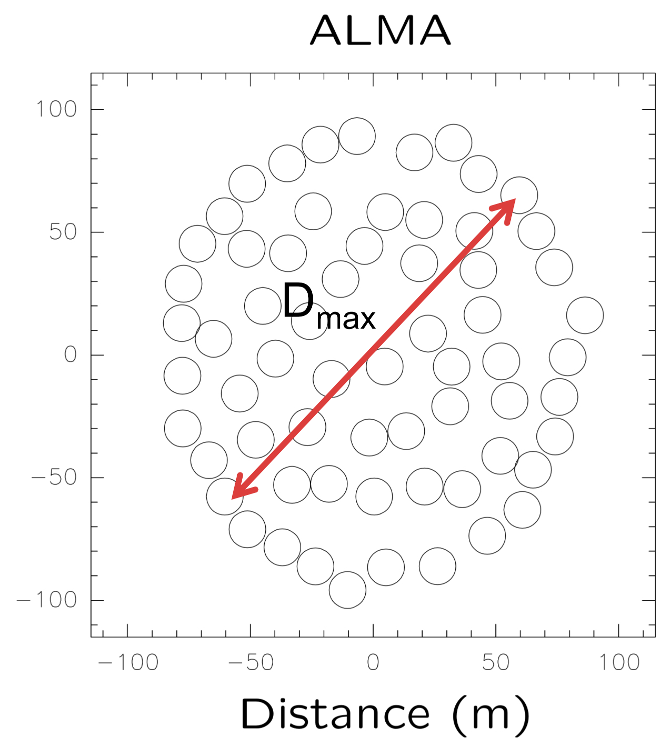
WHY INTERFEROMETRY?

The solution: Aperture synthesis

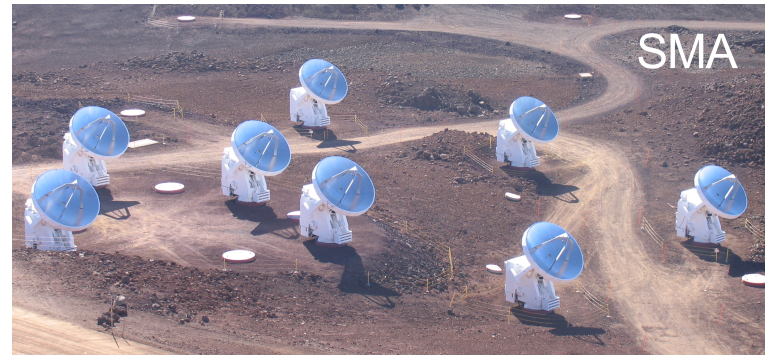
- Use a group of smaller antennas to "simulate" a very large one
- Interferometer resolution is ~ λ / D_{max}
- → difficult technically, but feasible !

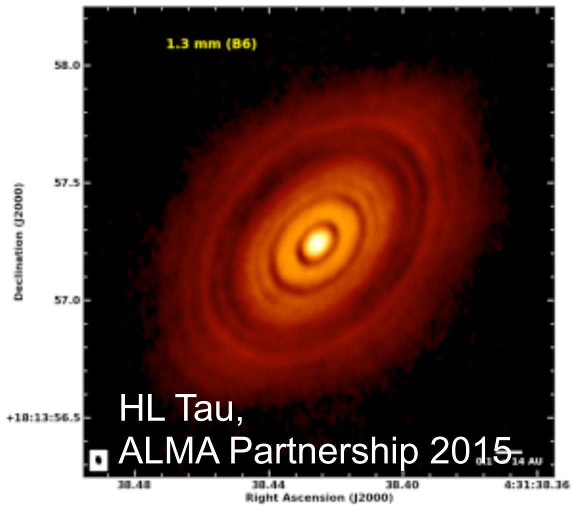
Some vocabulary…

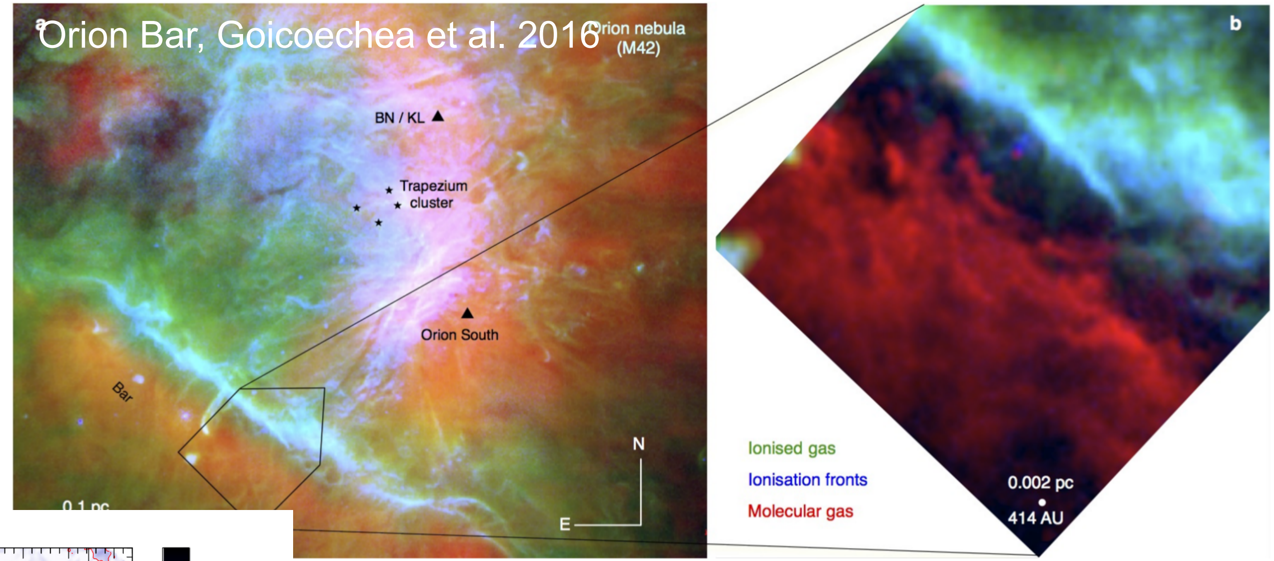
- Baseline: line between two antennas
- Configuration: antenna distribution
- Field of view: primary beam on one antenna
- Synthesized beam: resolution of the array

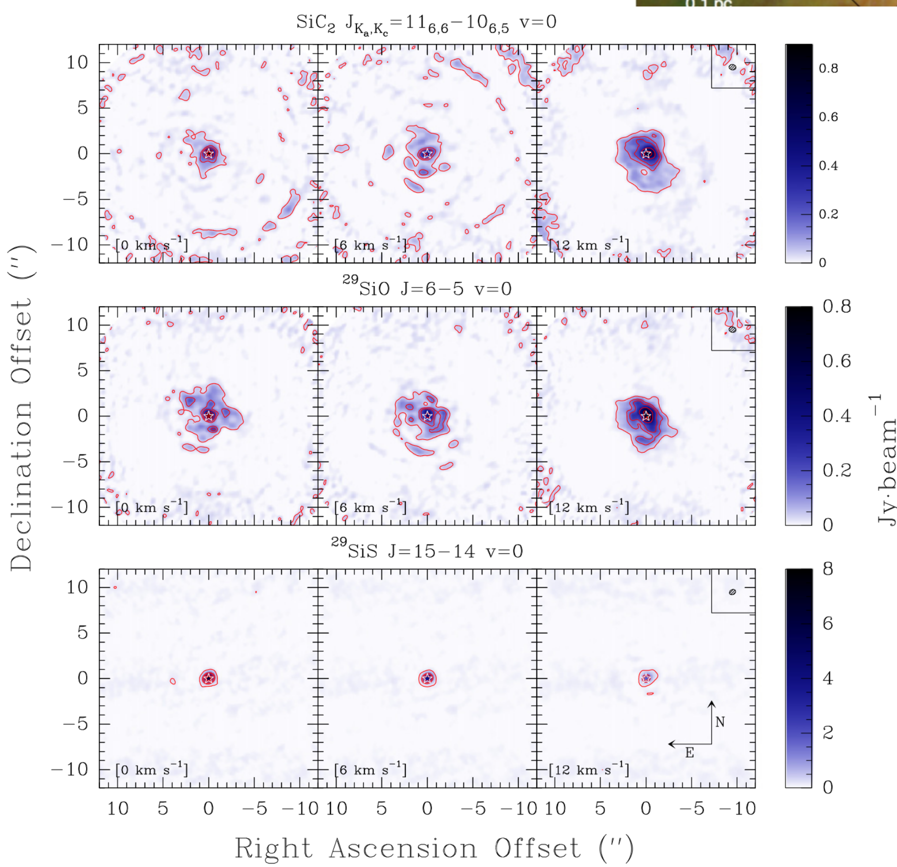


MM-SUBMM INTERFEROMETERS



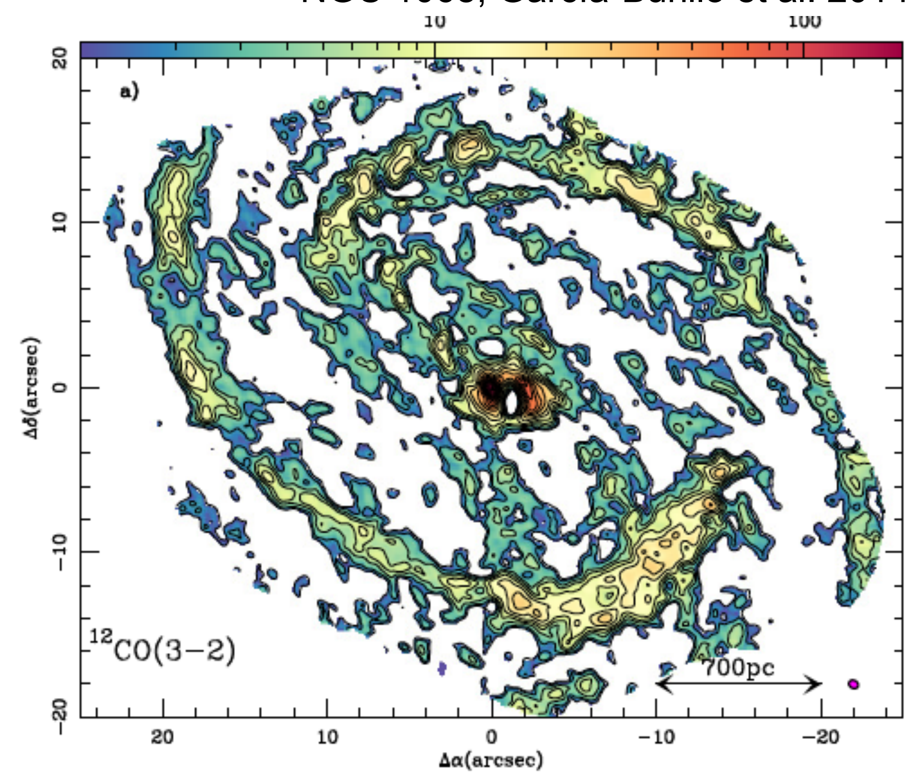






IRC+10216, Velilla-Prieto et al. 2015

NGC 1068, García-Burillo et al. 2014



GOALS / QUESTIONS

Goals

- Measure the signal emitted from a particular region in the sky
- Obtain high spatial images of the source (cont and/or lines)
- Determine chemical and/or physical properties

Questions

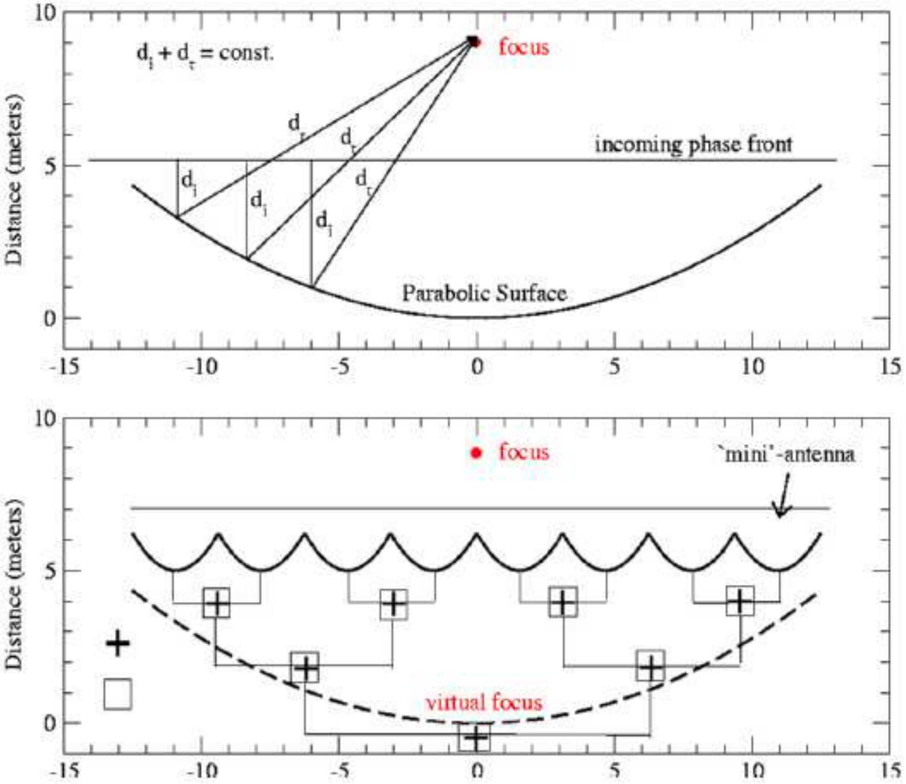
- How does interferometry work?
- Calibration of interferometers
- Image fidelity

INTERFEROMETRY BASICS

INTERFEROMETRY: THE BASICS

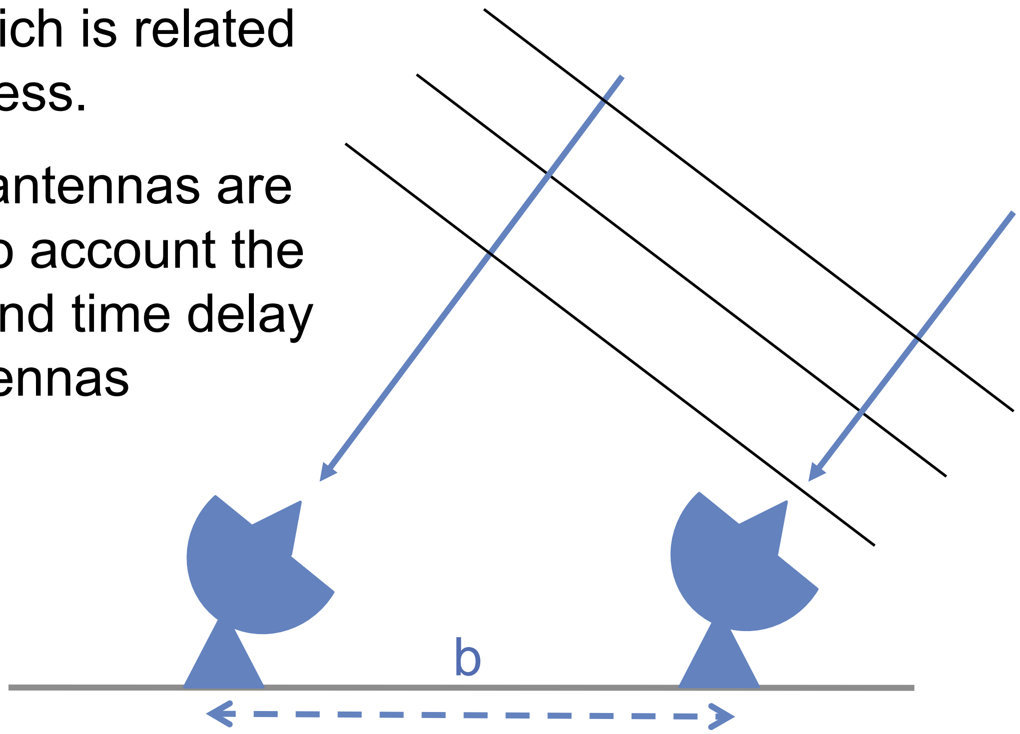
- Antenna's response is the result of coherent vector summation of the electric field at the focus
- We can consider this as a series of small antennas whose signals are summed

→ We don't need a single parabolic antenna !

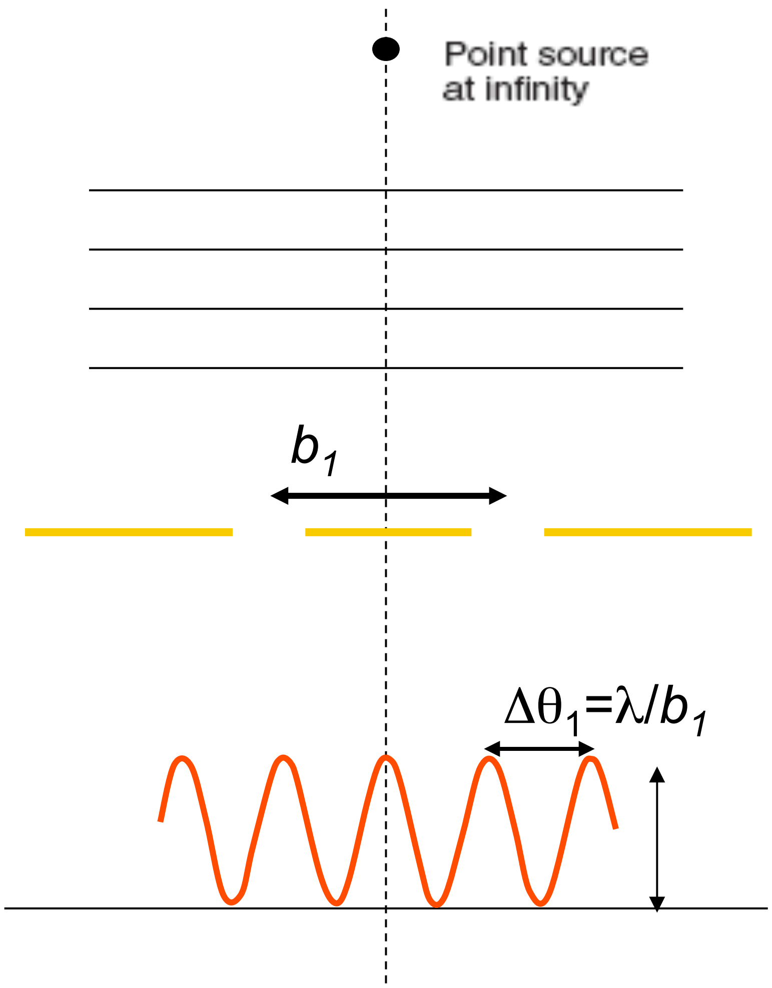


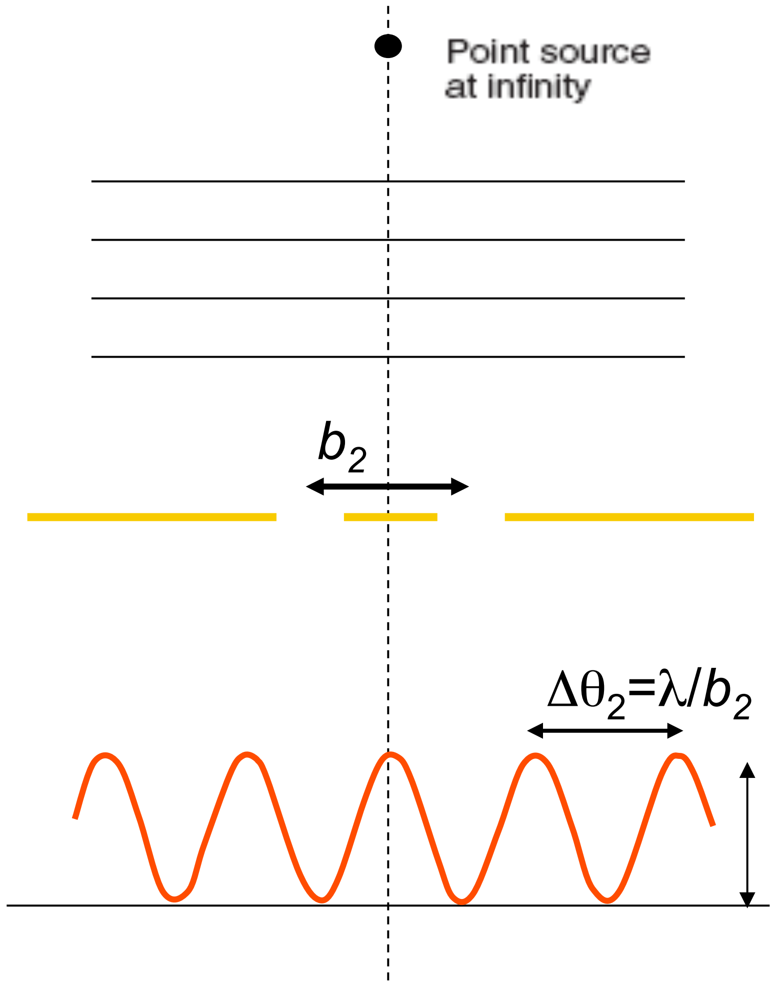
INTERFEROMETRY: THE BASICS

- Interferometry: a method to "synthesize" a large aperture by combining signals collected by separated small apertures
- An Interferometer measures the interference pattern produced by two apertures, which is related to the source brightness.
- The signals from all antennas are correlated, taking into account the distance (baseline) and time delay between pairs of antennas

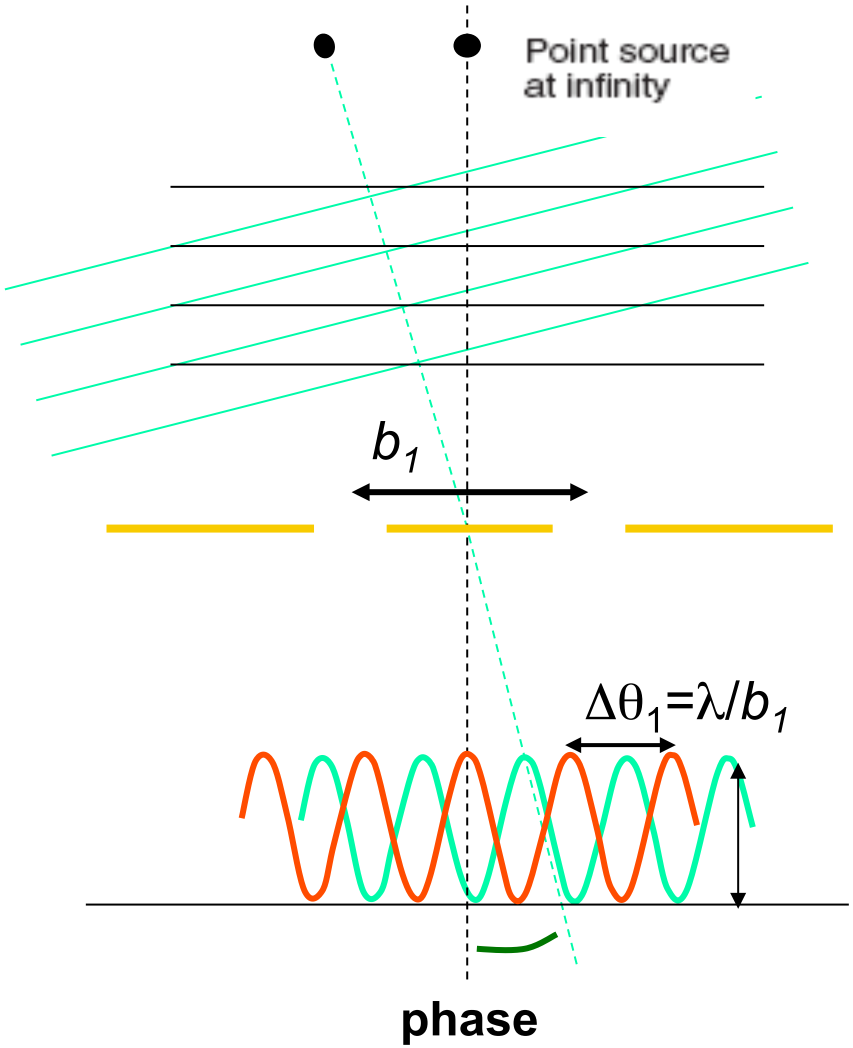


INTERFEROMETRY: THE BASICS

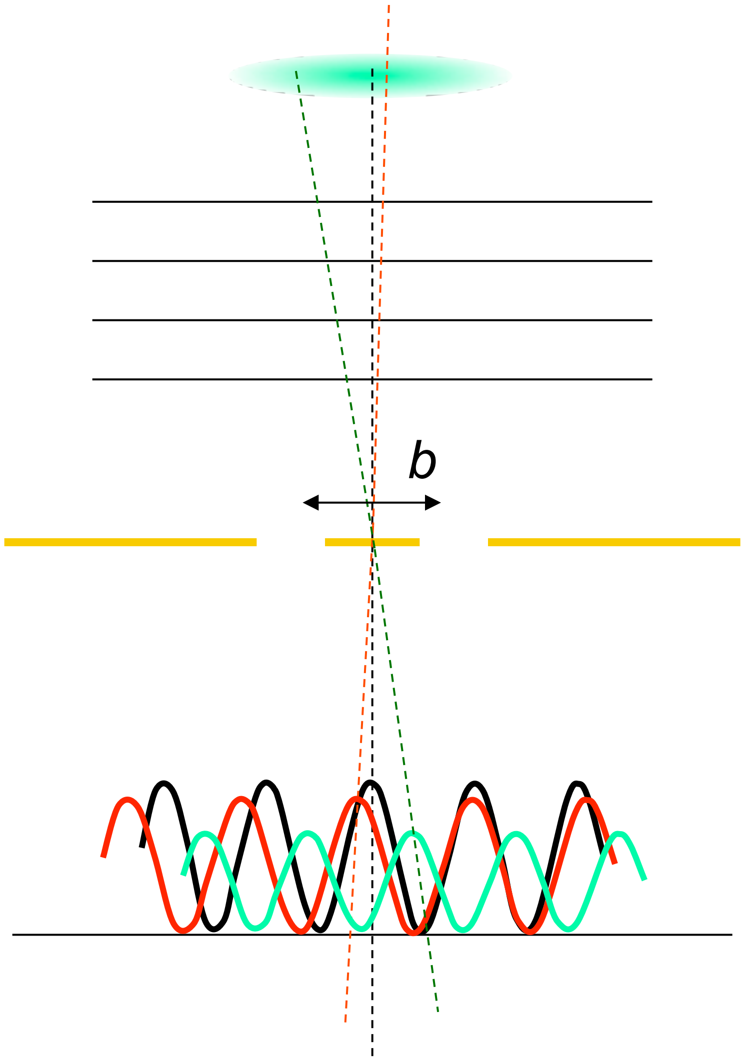




INTERFEROMETRY: THE BASICS



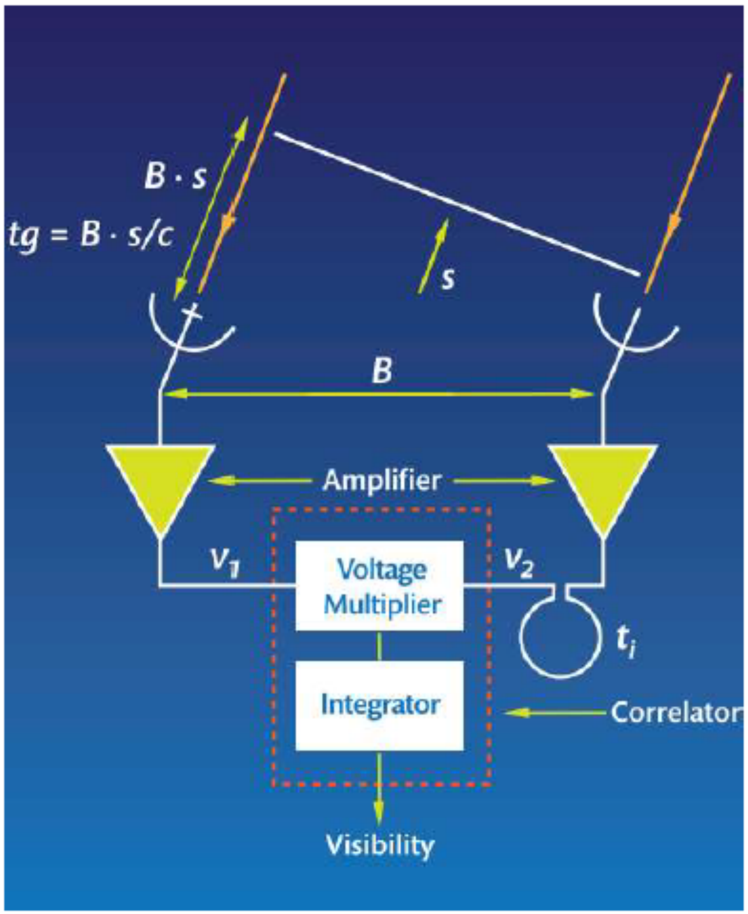
INTERFEROMETRY: THE BASICS



- Amplitude tells “how much” of a certain frequency component
- Phase tells “where” this component is located

Visibility

THE TWO ELEMENT INTERFEROMETER



- plane waves from a distant point source travel an extra distance to reach antenna 1

$$\vec{b} \cdot \hat{s} = b \cos\theta$$

- output of antenna 1 is delayed by geometric delay
- $$\tau_g = \vec{b} \cdot \hat{s} / c$$
- output voltages:

$$V_1 = E \cos[\omega(t - \tau_g)]$$

$$V_2 = E \cos(\omega t)$$

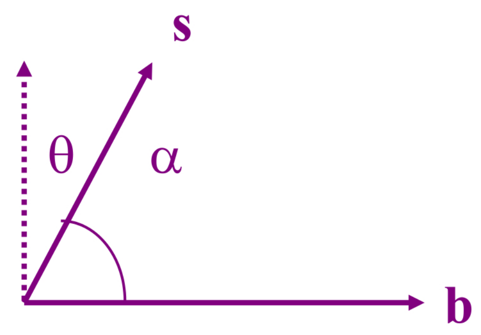
- correlator first multiplies:

$$V_1 V_2 = \frac{E^2}{2}[\cos(2\omega t - \omega\tau_g) + \cos(\omega\tau_g)]$$

and then time-averages:

$$R = \langle V_1 V_2 \rangle = \frac{E^2}{2} \cos(\omega\tau_g)$$

THE TWO ELEMENT INTERFEROMETER



In 1D: $$\frac{\mathbf{b} \bullet \mathbf{s}}{\lambda} = u\cos\alpha = u\sin\theta = ul$$

- u = b/λ is the baseline length in wavelengths
- θ is the angle w.r.t. the plane perpendicular to the baseline
- Thus:

$$R = \frac{E^2}{2}\cos(\omega\tau_g) = \frac{E^2}{2}\cos(2\pi\frac{b \cdot s}{\lambda}) = \frac{E^2}{2}\cos(2\pi ul)$$

$$\omega = 2\pi\nu$$

Angular frequency

RESPONSE FROM AN EXTENDED SOURCE

- Obtained by summing the responses at each antenna to all the emission over the sky, multiplying the two, and averaging:

$$R_C = \left\langle \iint V_1 d\Omega_1 \times \iint V_2 d\Omega_2 \right\rangle$$

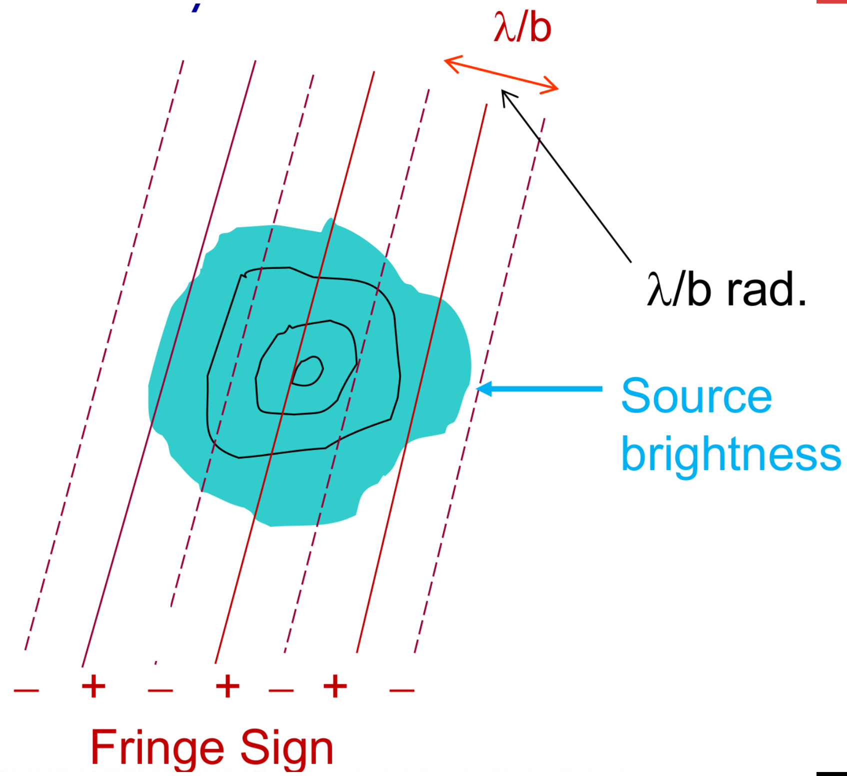
- Developing the integrals for each antenna…

$$R_C = \iint I_\nu(\mathbf{s}) \cos(2\pi\nu \mathbf{b} \cdot \mathbf{s}/c) d\Omega$$

- Which relates what we want, the source brightness $I_\nu(\mathrm{s})$, with what we measure, the response of the interferometer R_c
- Can we recover $I_\nu(\mathrm{s})$ from R_c ?

RESPONSE FROM A
EXTENDED SOURC

$$R_C = \iint I_\nu(\mathbf{s}) \cos(2\pi\nu \mathbf{b}\cdot\mathbf{s}/c)\, d\Omega$$



The correlator multiplies the source brightness by this coherence pattern (cosinusoidal function), and integrates (sums) the result over the sky

- Orientation set by baseline geometry
- Fringe separation set by (projected) baseline length and wavelength
- BUT one cosinus function is not enough (only sensitive to the "even" part of the brightness) → we need an "odd" function to recover that part of the brightness: sinus function

Extended source

- Then, complex correlator is a combination of cosine and sine correlators
- We define complex visibility: $$V \equiv R_c - iR_s = Ae^{-i\phi}$$

  where visibility amplitude and phase:

  $$A = (R_c^2 + R_s^2)^{1/2} \qquad \phi = \tan^{-1}(R_s / R_c)$$

- Response to an extended source of a 2-element interferometer with a complex correlator is the complex visibility:

Brightness temperature distribution

$$V_\nu = \int I_\nu(\hat{s}) \exp(-i2\pi b \cdot \hat{s} / \lambda) d\Omega$$

- this is a 2D Fourier transform, giving us a well established way to recover *I(s)* from *V(b)*

Visibilities

$$V(u,v) = \int I(l,m) \exp[-i2\pi(ul + vm)] dl dm$$

VISIBILITY AND SKY BRIGHTNESS

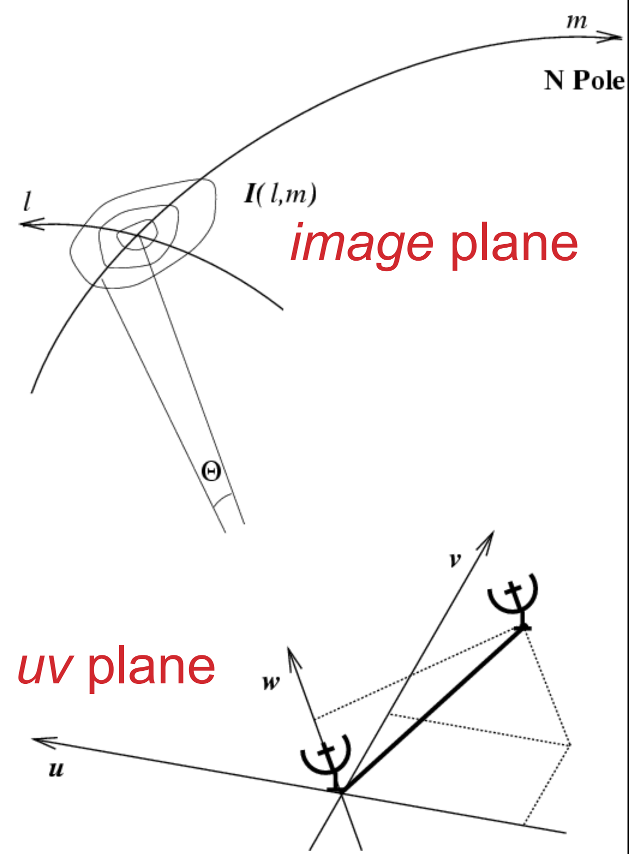
For small fields of view: the complex visibility, *V(u,v)*, is the 2D Fourier transform of the brightness on the sky, *I(l,m)*

(van Cittert-Zernike theorem)

$$V(u,v) = \iint I(l,m) e^{2\pi i (ul+vm)} \, dl dm$$

$$I(l,m) = \iint V(u,v) e^{-2\pi i (ul+vm)} \, du dv$$

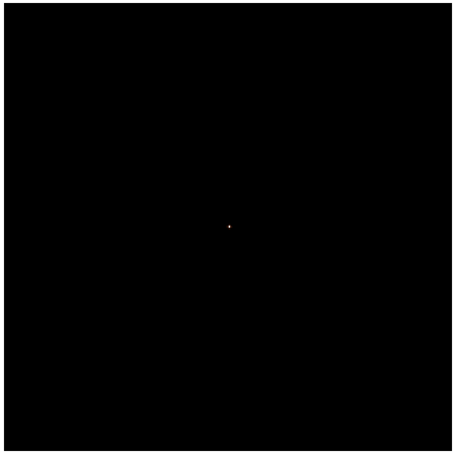
- *u,v* (wavelengths) are spatial frequencies in E-W and N-S directions, i.e. the baseline lengths
- *l,m* (rad) are angles in tangent plane relative to a reference position in the E-W and N-S directions



2D FOURIER TRANSFORMS

$I(l,m)$ Amp{$V(u,v)$}

δ Function



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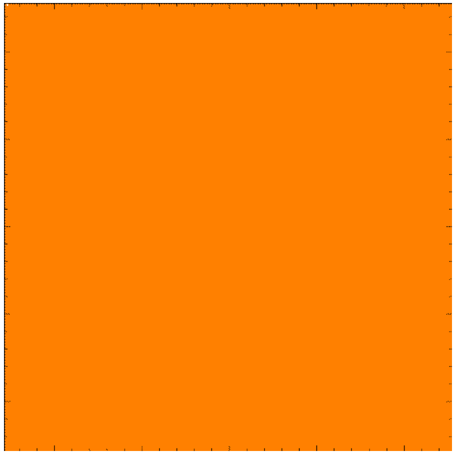
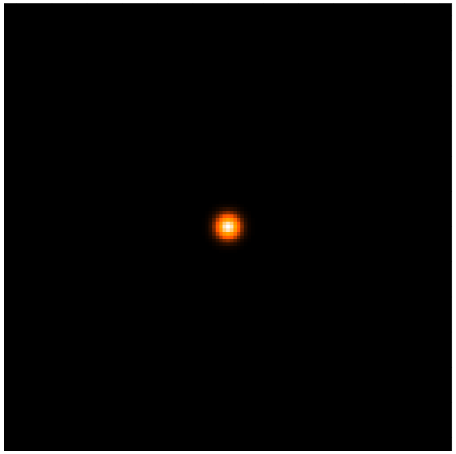


Constant

Gaussian



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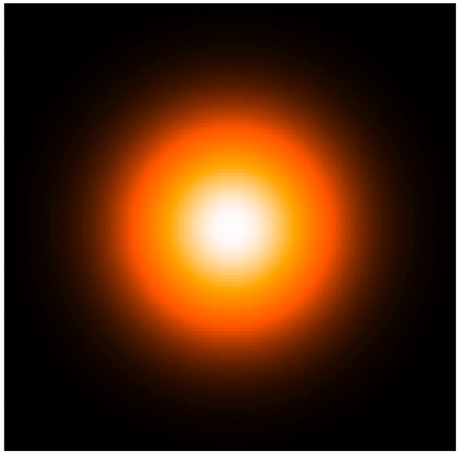


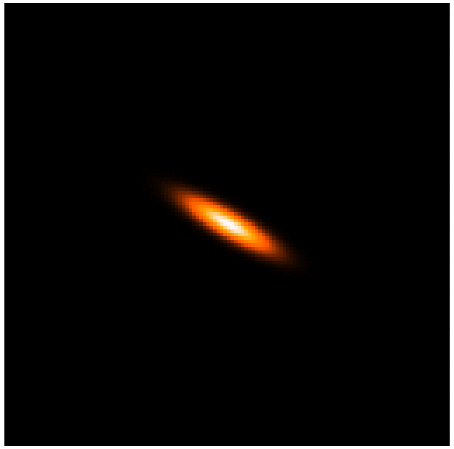
Gaussian

→ narrow features transform to wide features (and vice-versa)

2D FOURIER TRANSFORMS

$I(l,m)$ | Amp{$V(u,v)$}

elliptical Gaussian



⇌

elliptical Gaussian

Disk

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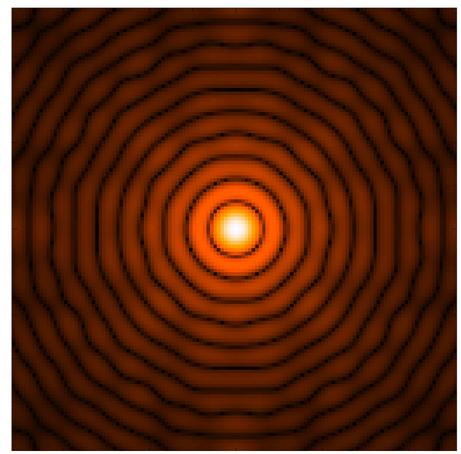
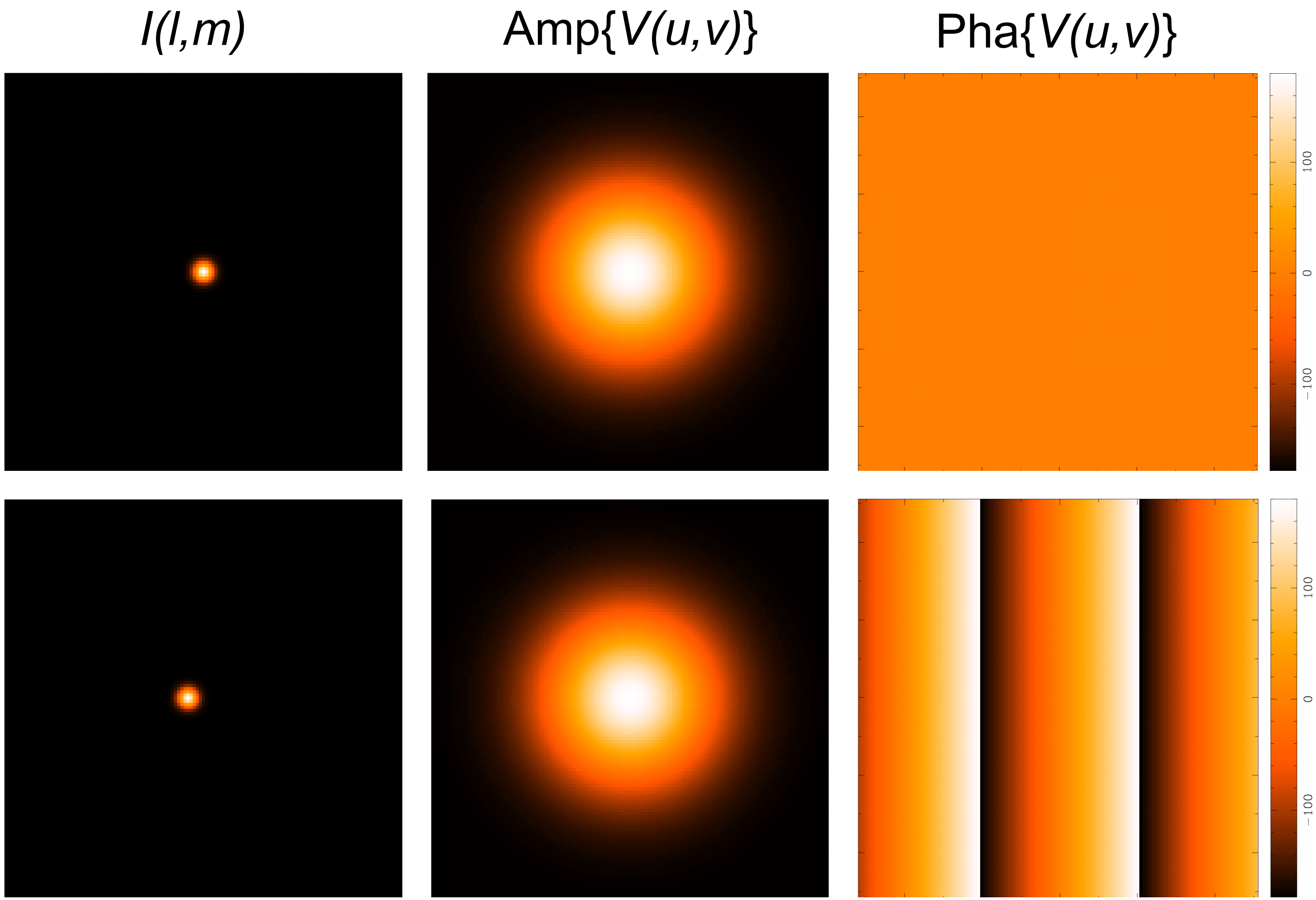


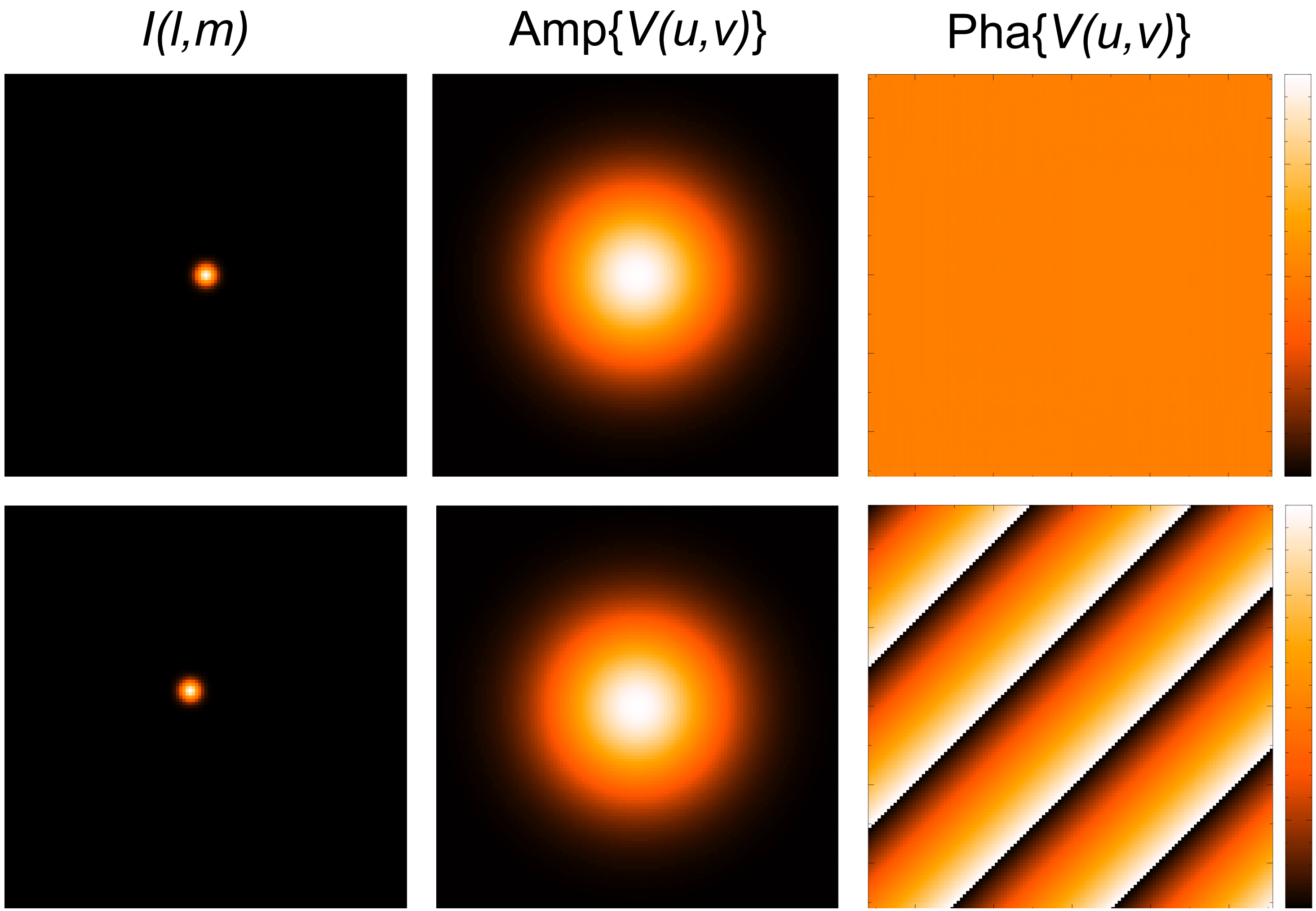
Bessel

→ sharp edges result in many high spatial frequencies

VISIBILITY: AMPLITUDE AND PHASE



VISIBILITY: AMPLITUDE AND PHASE



CALIBRATION

CALIBRATION: amplitude and phase

- Each pair of antennas: 1 visibility (amplitude and phase)
- But…

$$V_{i,j}^{obs}(\nu,t) = G_{i,j}(\nu,t)V_{i,j}(\nu,t)$$

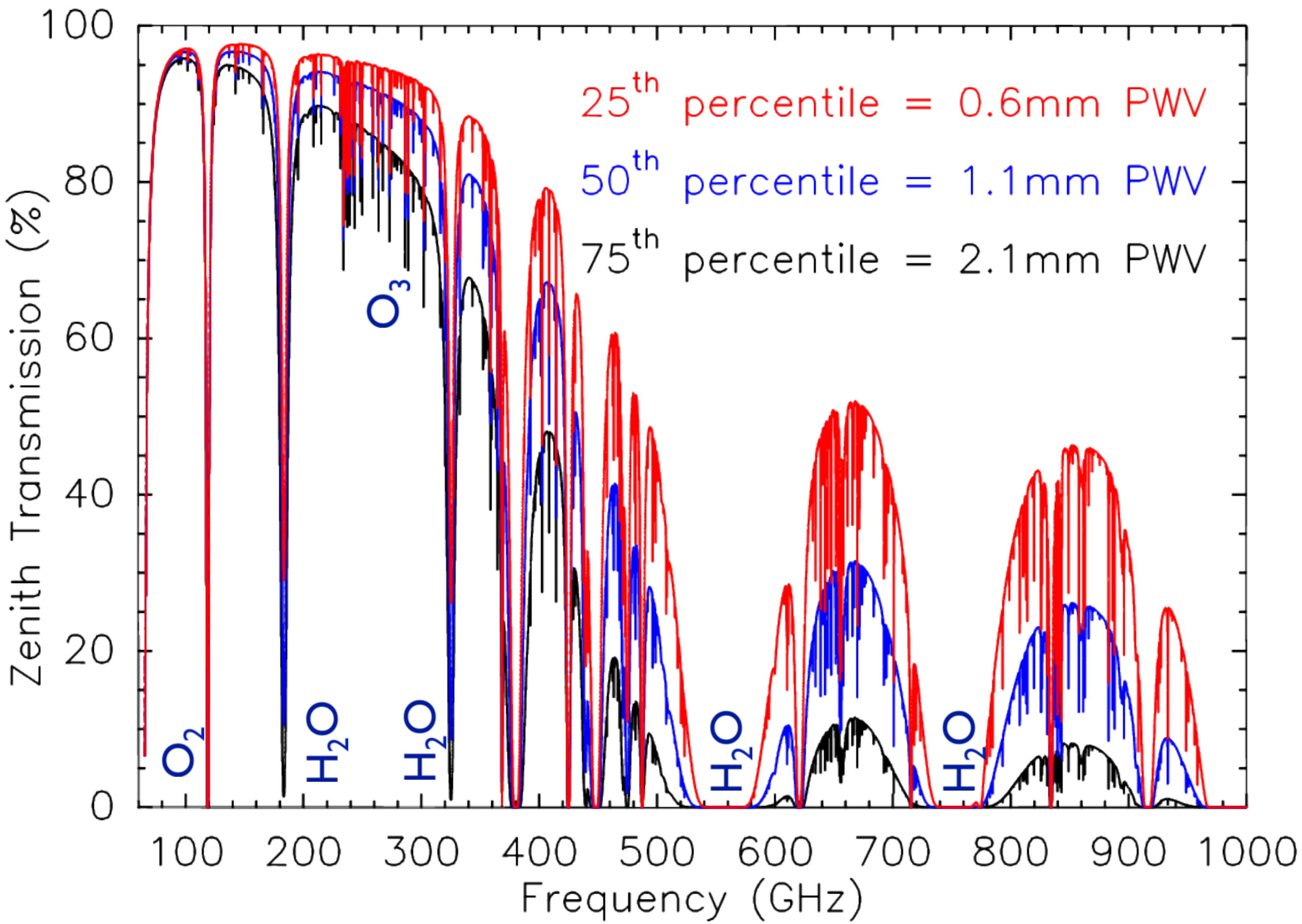
  - Weather
  - Real antennas
  - Electronics: receivers, backends, cables,…

→ Goal of calibration is to correct visibilities for atmospheric and instrumental effects: complex gains (amplitude and phase), in time and frequency

- Electronics have variable gains, both in amplitude and phase, both in frequency and time
- Atmosphere: absorption (amplitude, T_{sys}) and path length fluctuations (phase)

→ We need to observed some sources whose visibilities are known (calibrators –QSOs), then transfer $G_{i,j}$ to the target source

ATMOSPHERIC OPACITY (PWV = PRECIPITABLE WATER VAPOR)

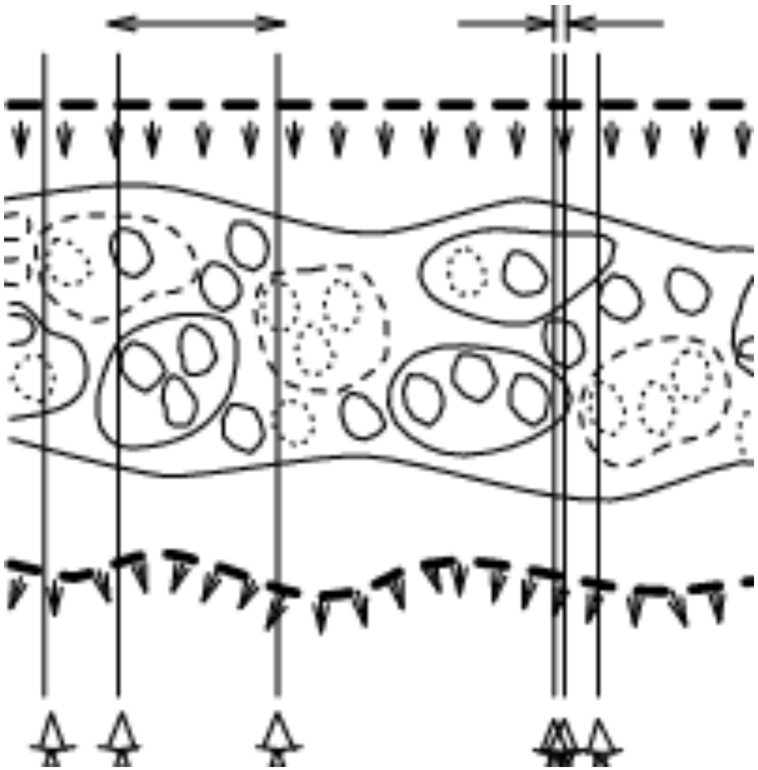


ATMOSPHERIC PHASE FLUCTUATIONS

Variations in the amount of precipitable water vapor (pwv) cause phase fluctuations, which are worse at shorter wavelengths (higher frequencies), and result in:

- Low coherence (loss of sensitivity)
- Radio "seeing"
- Anomalous pointing offsets
- Anomalous delay offsets

You can observe in apparently excellent submm weather (in terms of transparency) and still have terrible "seeing" i.e. phase stability.



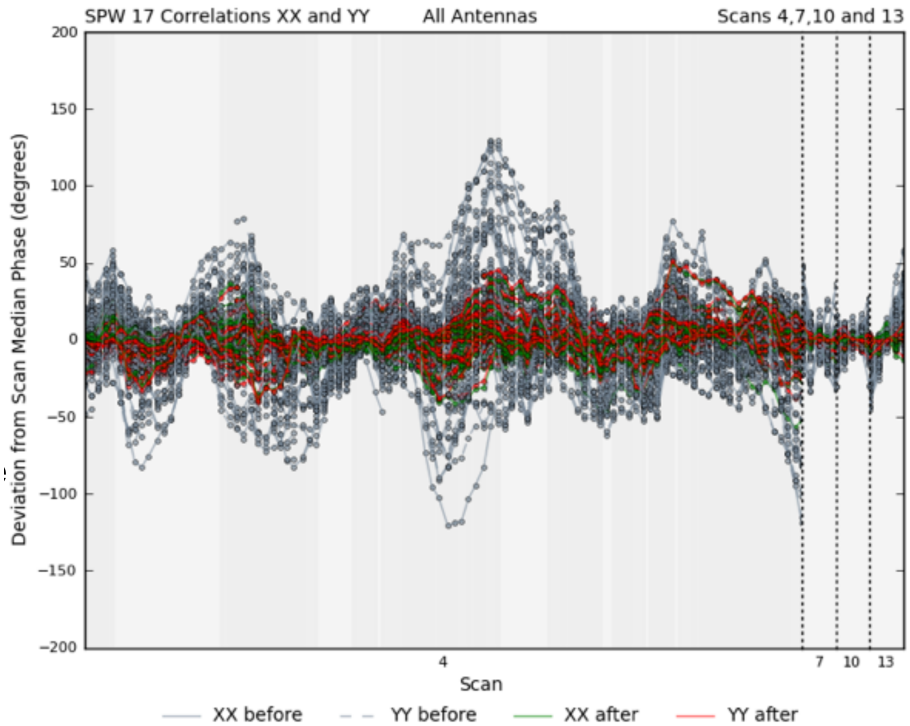
Patches of air with different water vapor content (and hence index of refraction) affect the incoming wave front differently.

ATMOSPHERIC PHASE FLUCTUATIONS

Water vapor radiometer (WVR):

- Measure rapid fluctuations in H_2O lines (183 GHz at ALMA, 325 GHz at NOEMA)
- Use these measurements to derive changes in water vapor column (*w*) and convert these into phase corrections using:

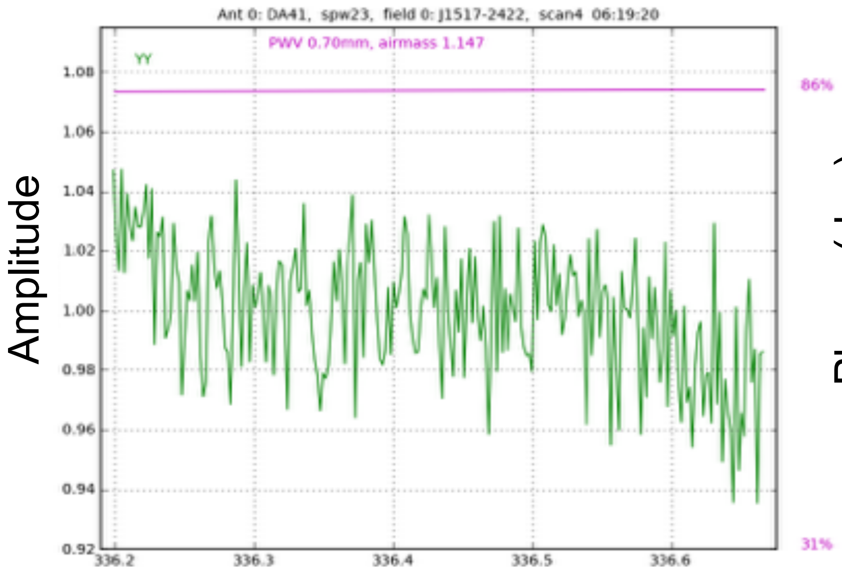
$$\Delta\phi_e \approx 12.6\,\pi\,\Delta w / \lambda$$

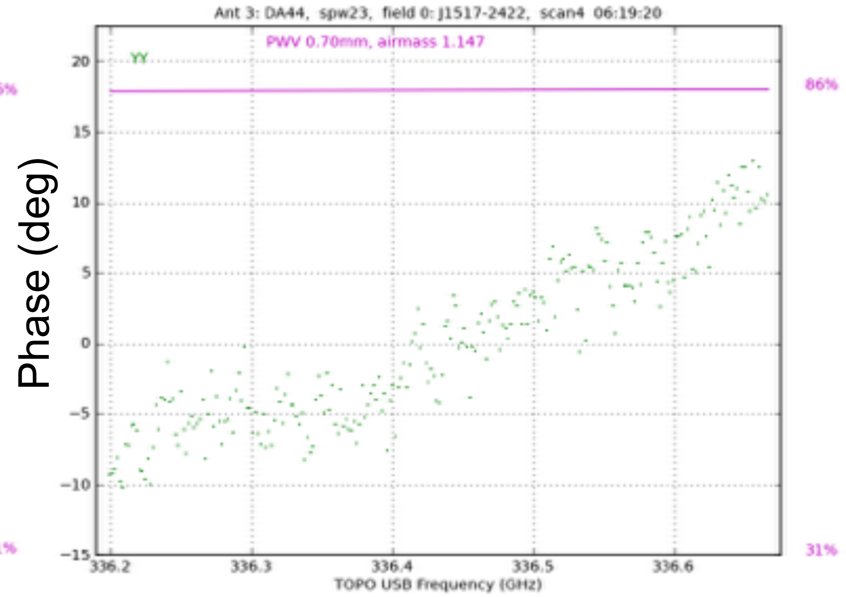


- Higher impact at high frequencies
- Higher impact at long baselines
- After corrections, phase noise should decrease

BANDPASS CALIBRATION

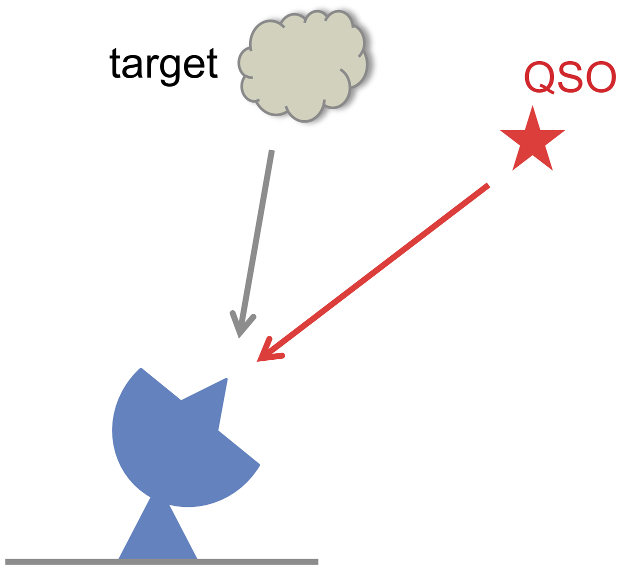
- Determine the variations of phase and amplitude with frequency → <u>mainly instrumental</u>
- Use **strong** calibrator, no need to be near science target
- Assumed to be independent of time: observe once during the observing run, typically at the beginning

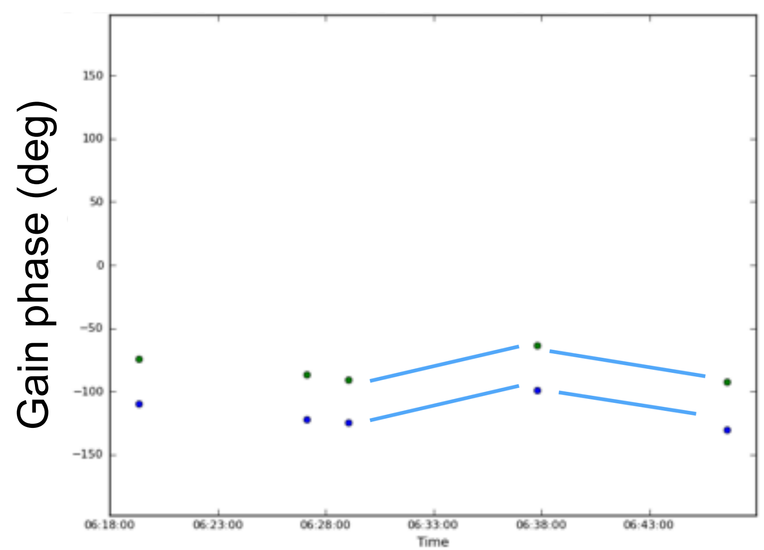




GAIN CALIBRATION

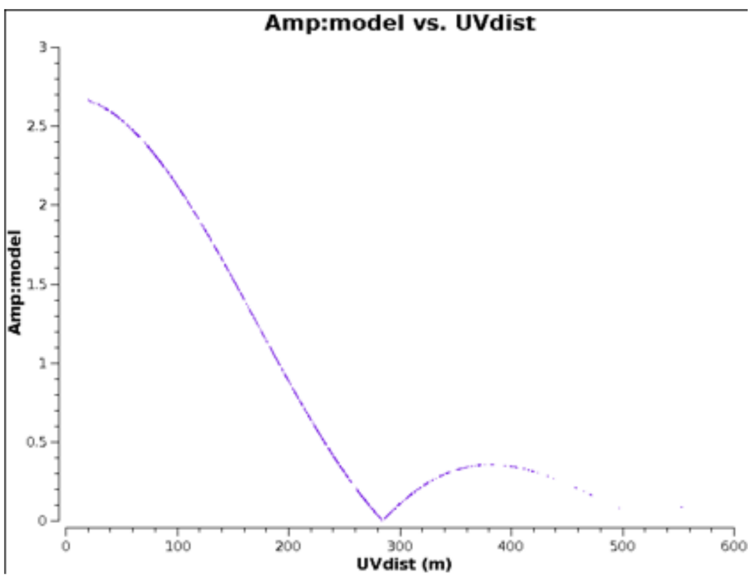
- Determine the long time variations of phase and amplitude with time → mainly atmosphere
- Assumed to be independent of frequency
- Use **point-like** calibrator **close** to science target
- Observe regularly: switching between target and calibrator

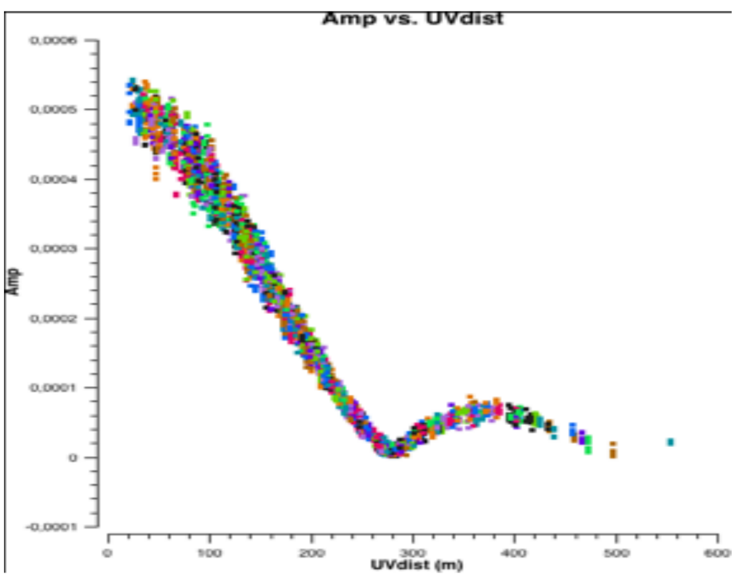




ABSOLUTE FLUX CALIBRATION

- Determine Jy/K scale
- Observe a known flux calibrator, then transferred to calibrators and science target
- Use **no variable** objects: planets, moons, QSOs only if regularly monitored; no need to be close to science target
- Observe once, typically at the beginning



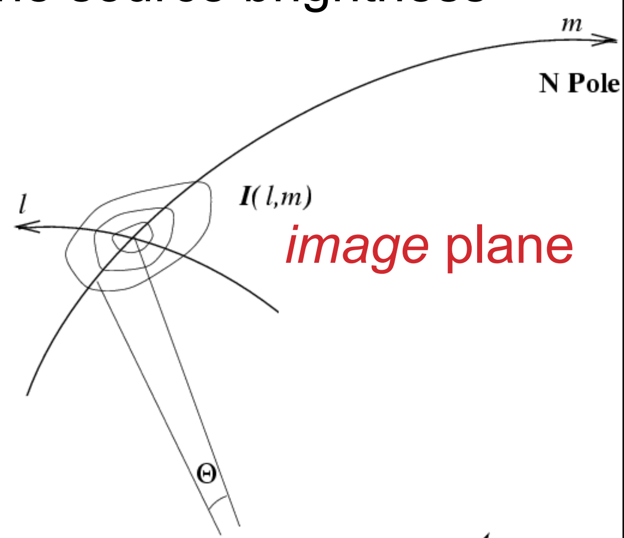


IMAGING AND DECONVOLUTION

VISIBILITY AND SKY BRIGHTNESS

- An interferometer measures the interference pattern produced by two apertures
- The interference pattern is directly related to the source brightness
- For small fields of view: the complex visibility, *V(u,v)*, is the 2D Fourier transform of the brightness on the sky,*I(l,m)*

  (van Cittert-Zernike theorem)

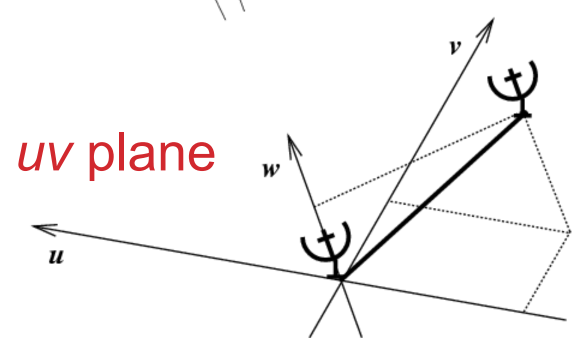


Fourier space/domain

$$V(u,v) = \iint I(l,m) e^{2\pi i(ul+vm)} \, dldm$$

Image space/domain

$$I(l,m) = \iint V(u,v) e^{-2\pi i(ul+vm)} \, dudv$$



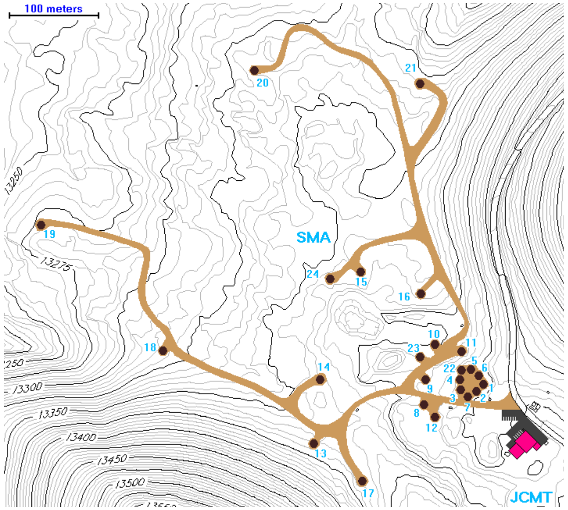
APERTURE SYNTHESIS

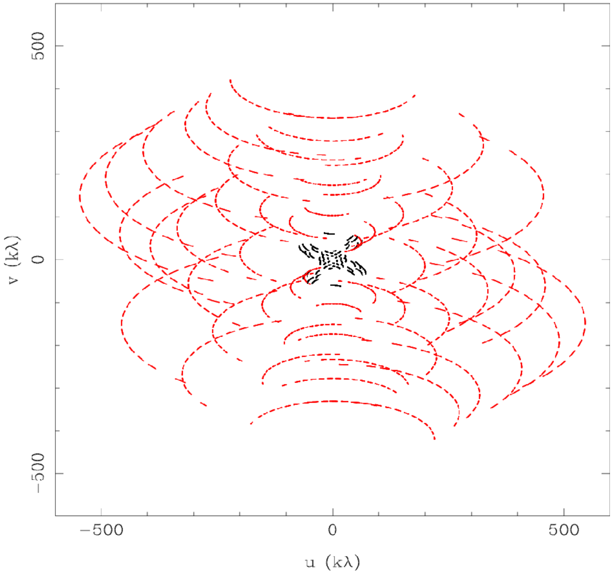
The Fourier transform of the array baseline configuration, projected onto the sky defines the spatial frequencies that the array is sensitive to

- 1 pair of telescopes → 1 (u,v) sample at a time
- N telescopes → number of samples = N(N-1)/2 ("snapshot")

A good image quality requires a good coverage of the uv plane

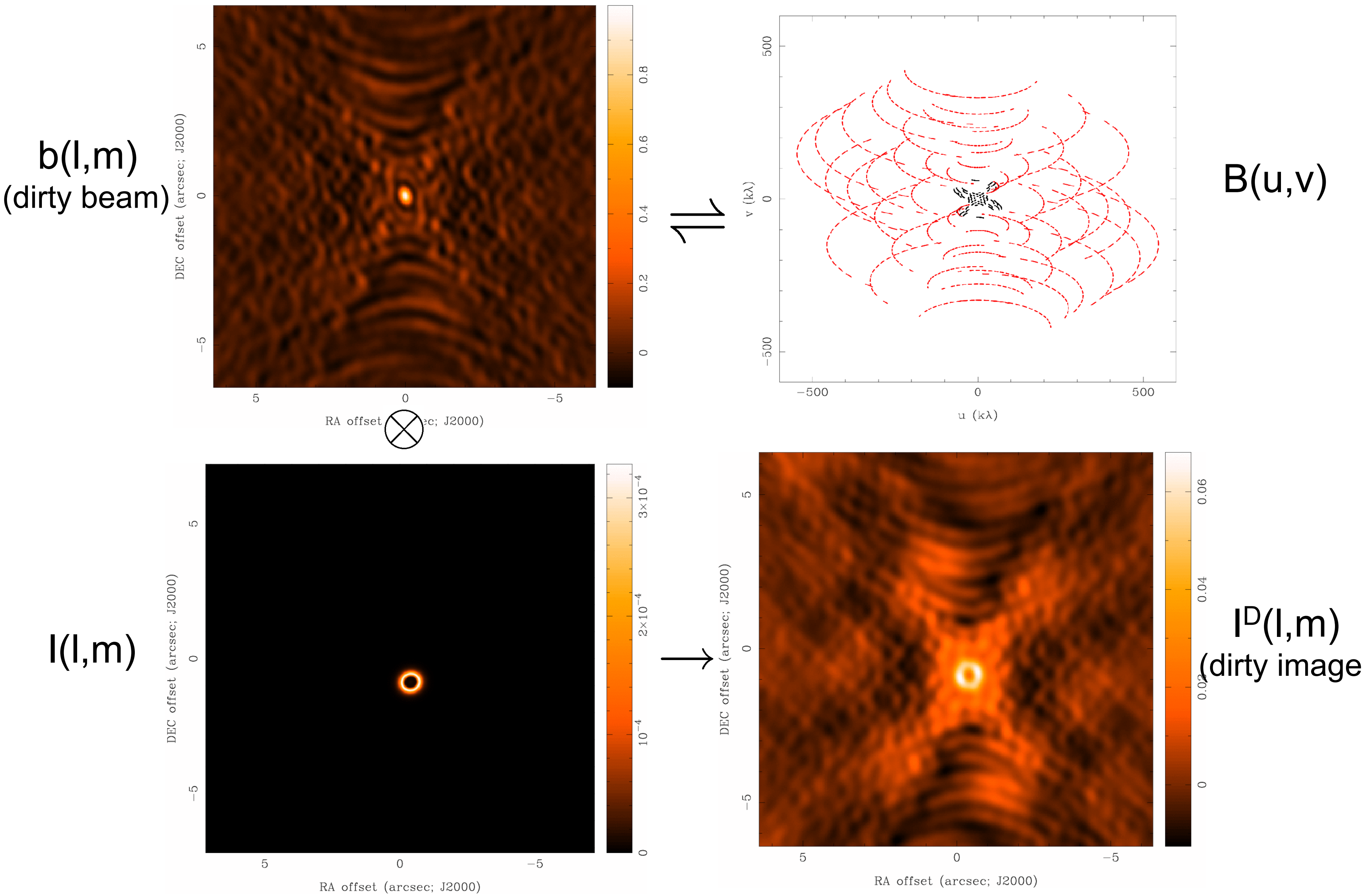
- fill in (u,v) plane by making use of Earth rotation ("track")
- reconfigure physical layout of N telescopes





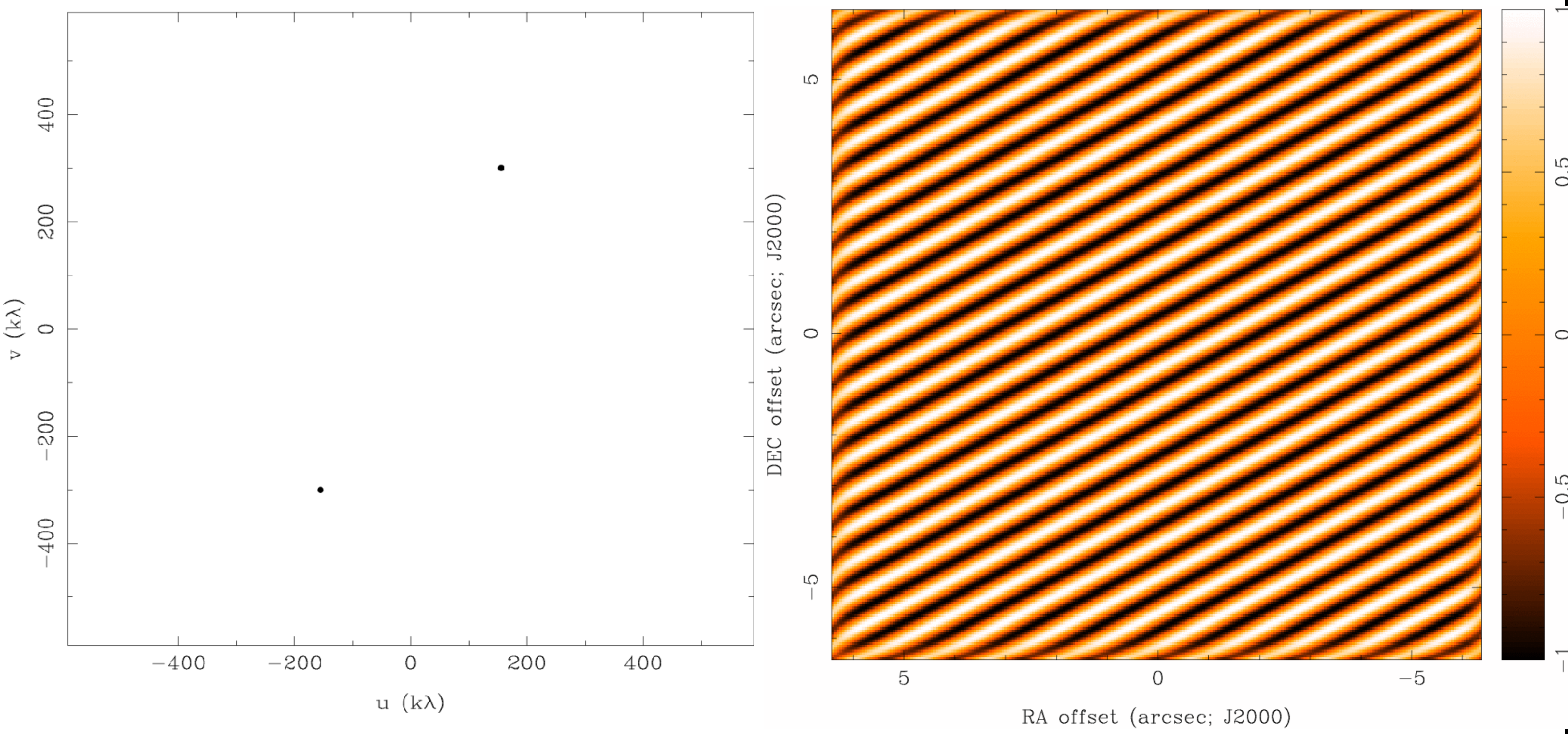
2 configurations
of 8 SMA antennas
345 GHz
Dec = -24 deg

DIRTY BEAM AND DIRTY IMAGE



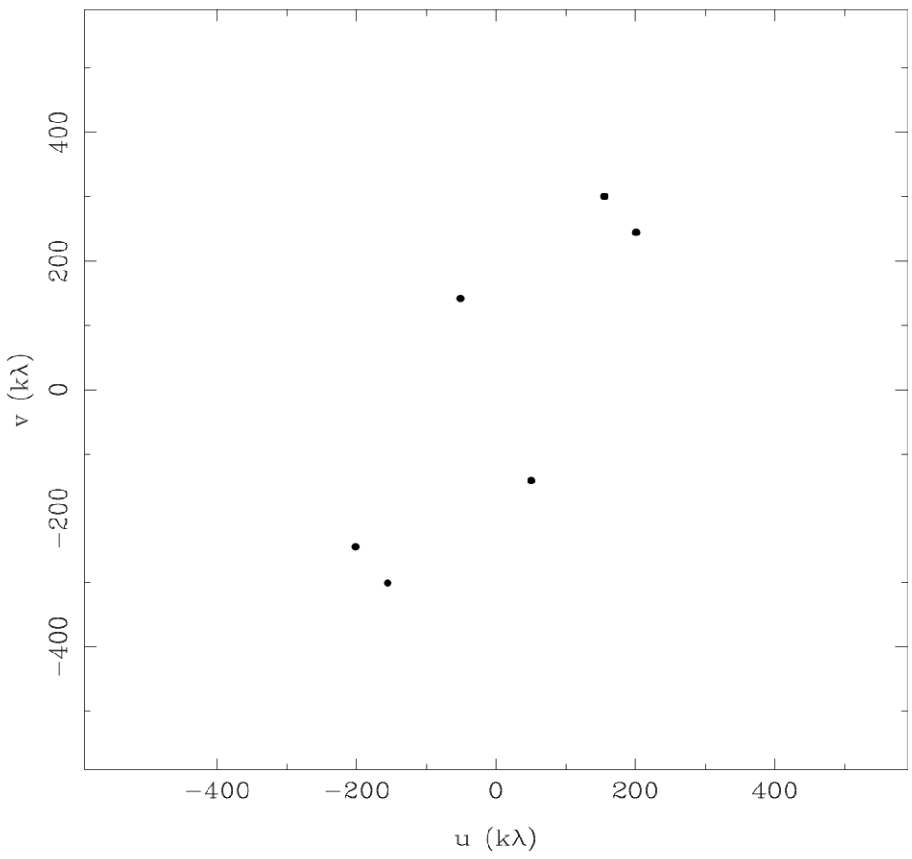
DIRTY BEAM SHAPE AND N ANTENNAS

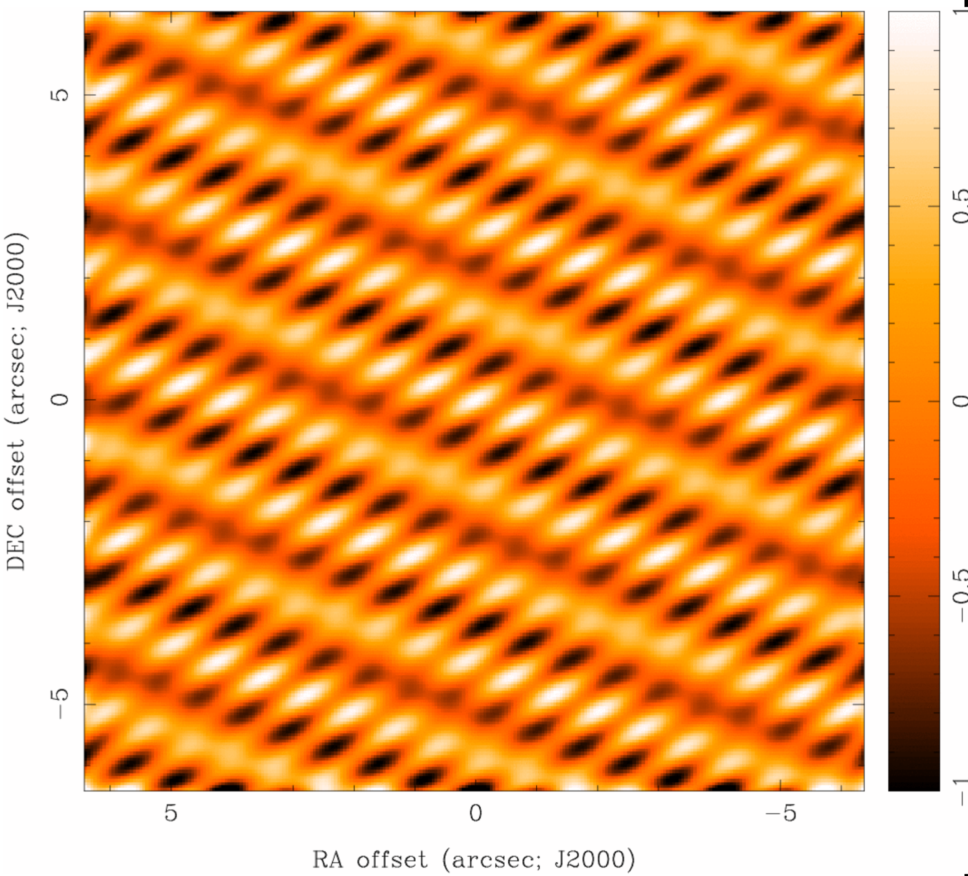
2 Antennas



DIRTY BEAM SHAPE AND N ANTENNAS

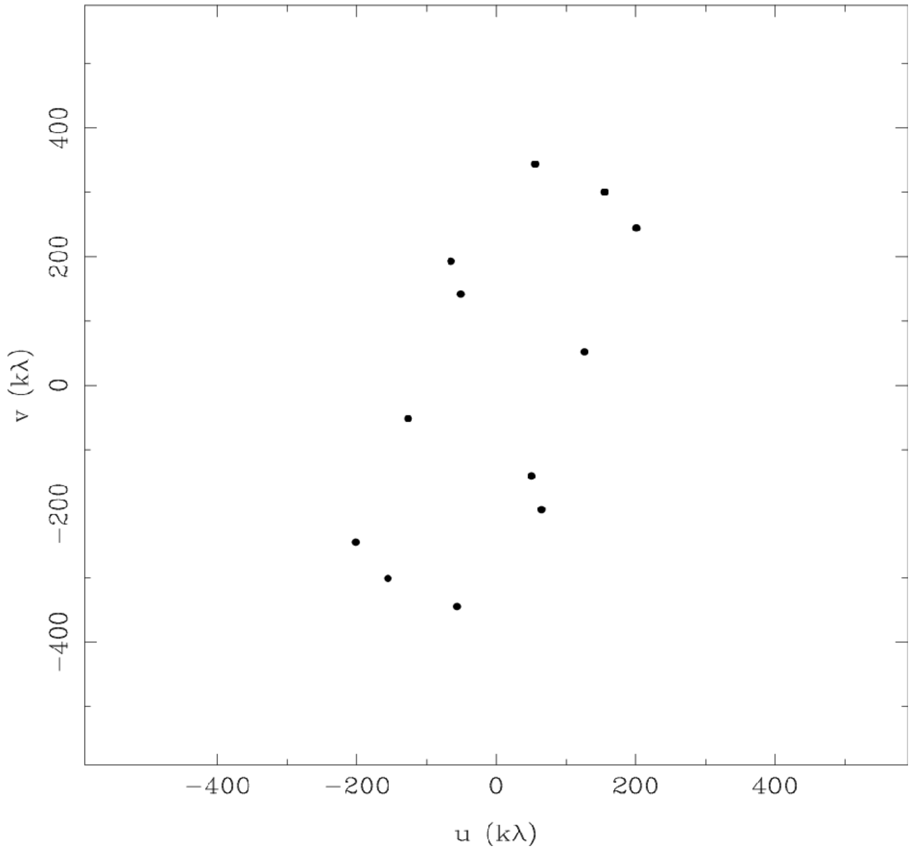
3 Antennas

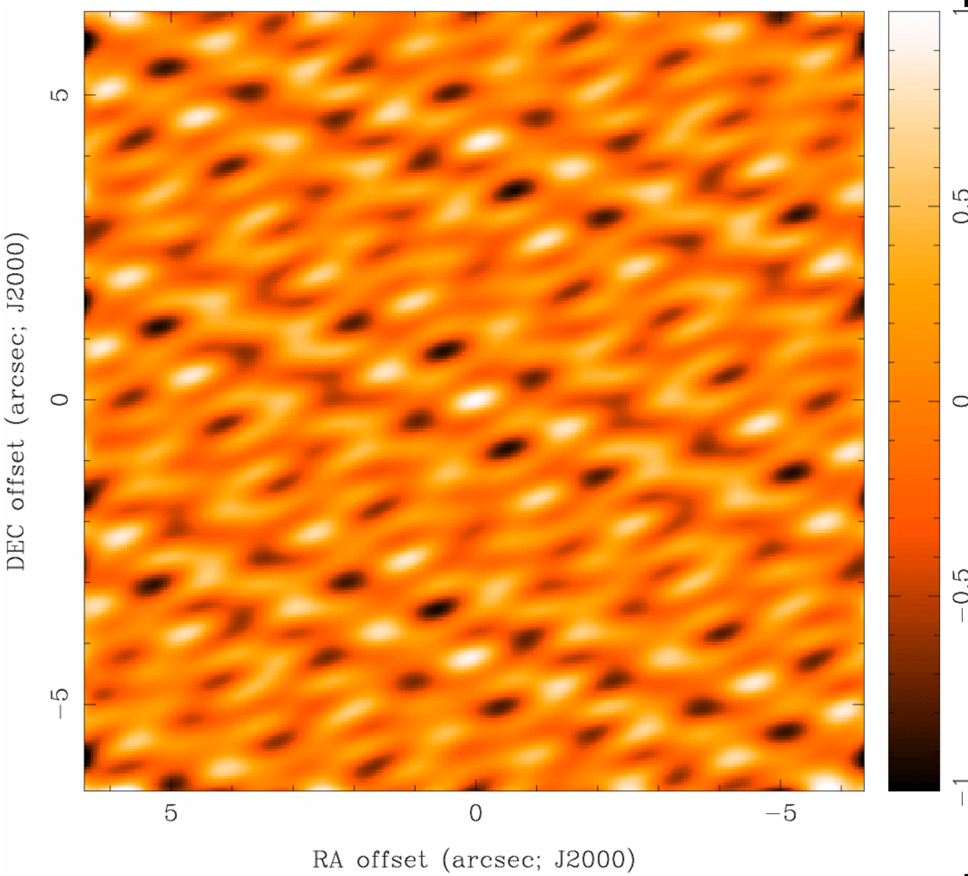




DIRTY BEAM SHAPE AND N ANTENNAS

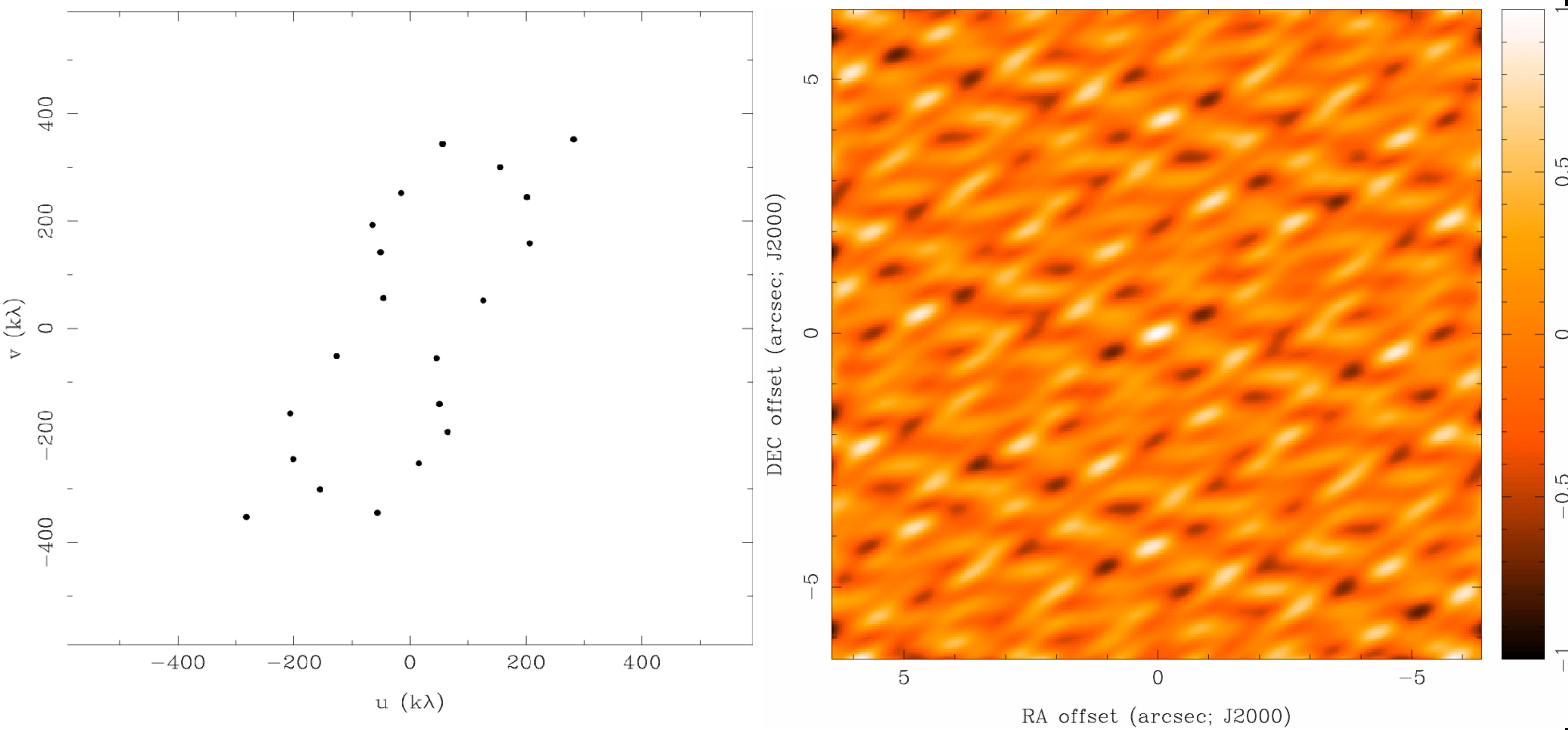
4 Antennas





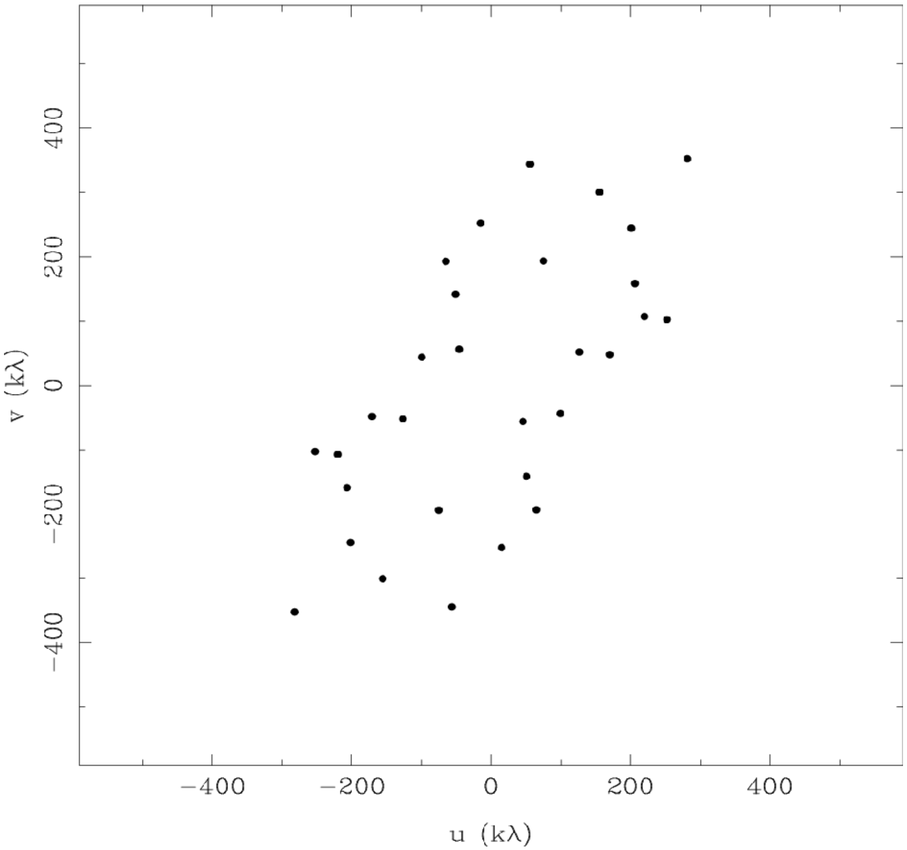
DIRTY BEAM SHAPE AND N ANTENNAS

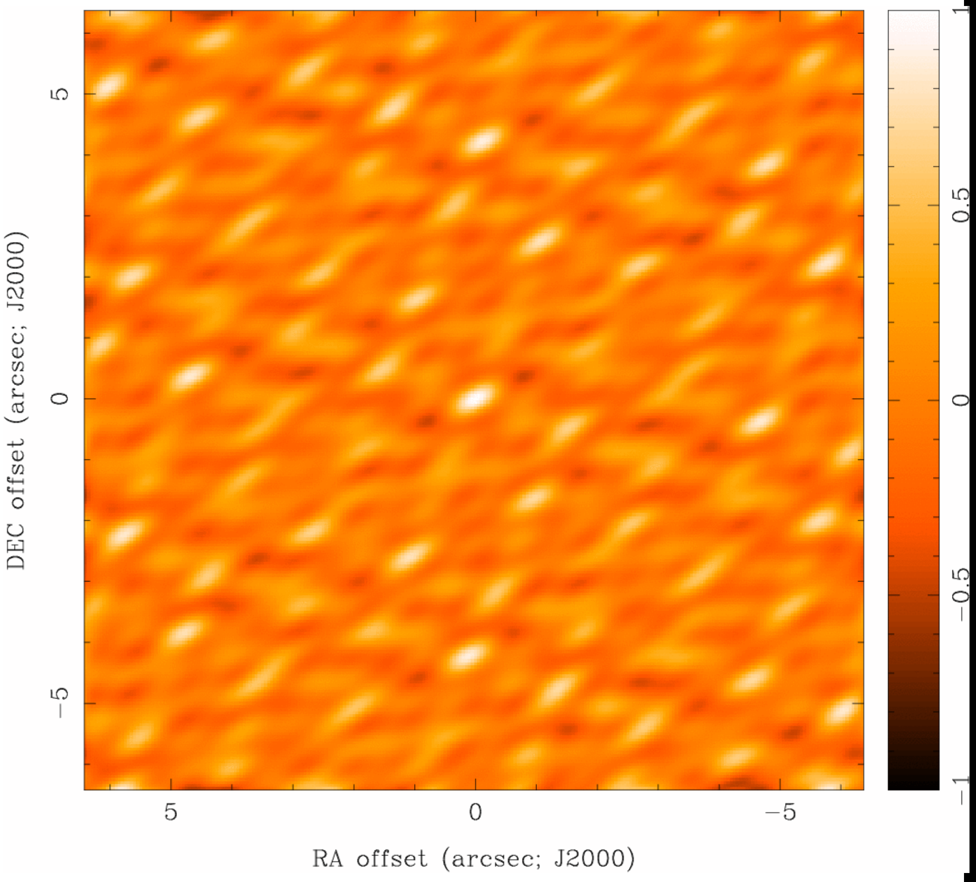
5 Antennas



DIRTY BEAM SHAPE AND N ANTENNAS

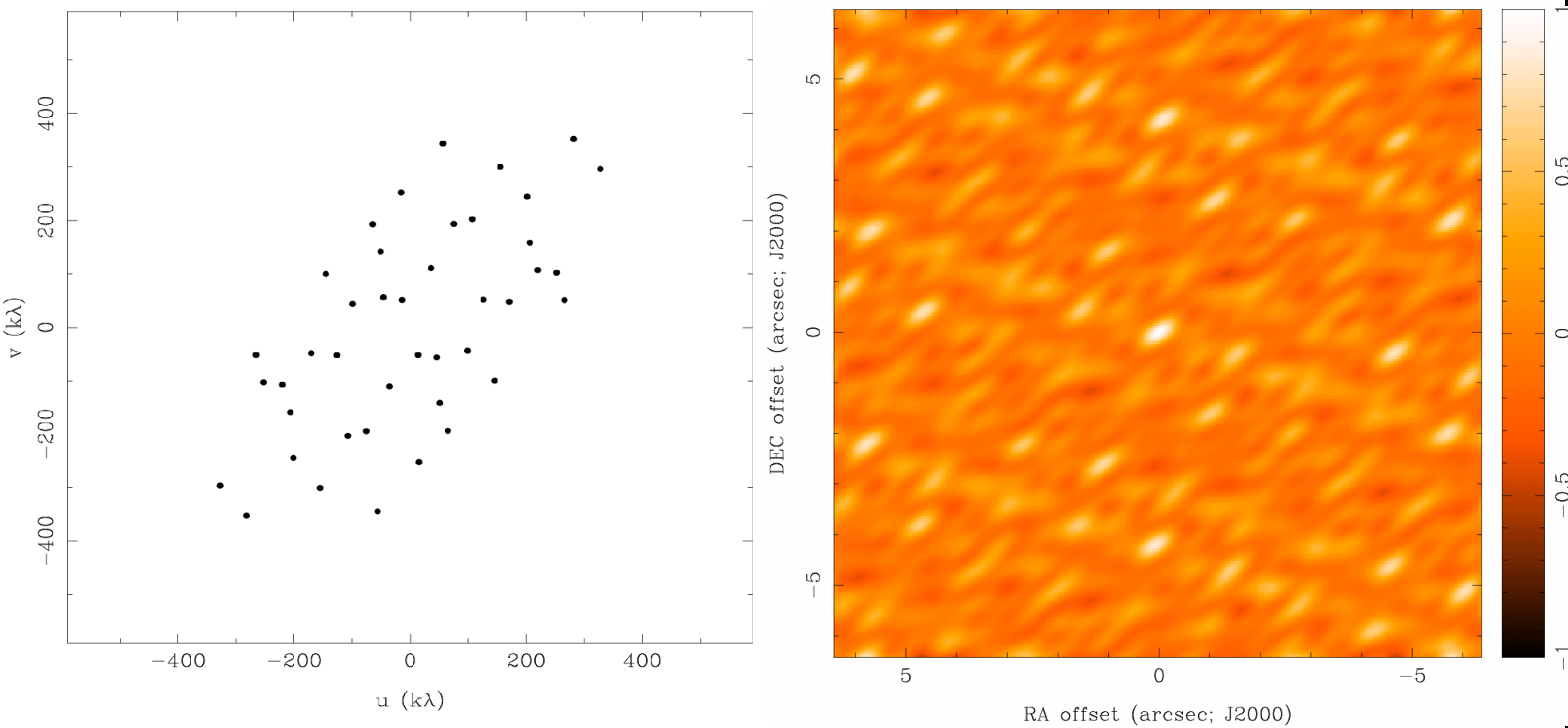
6 Antennas





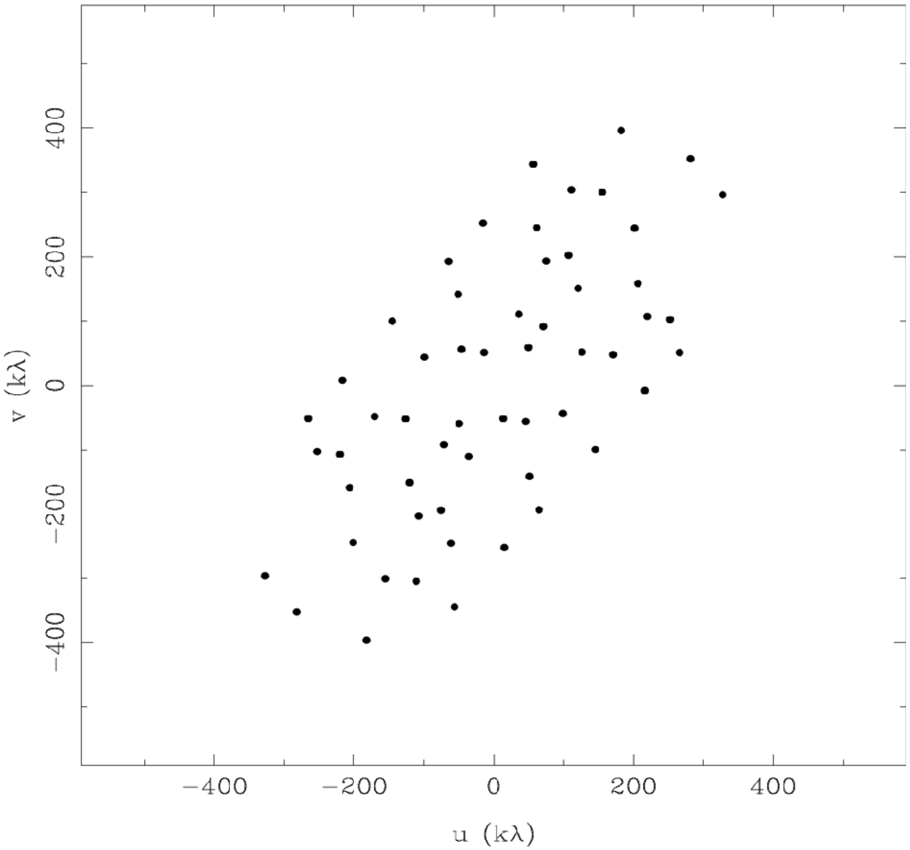
DIRTY BEAM SHAPE AND N ANTENNAS

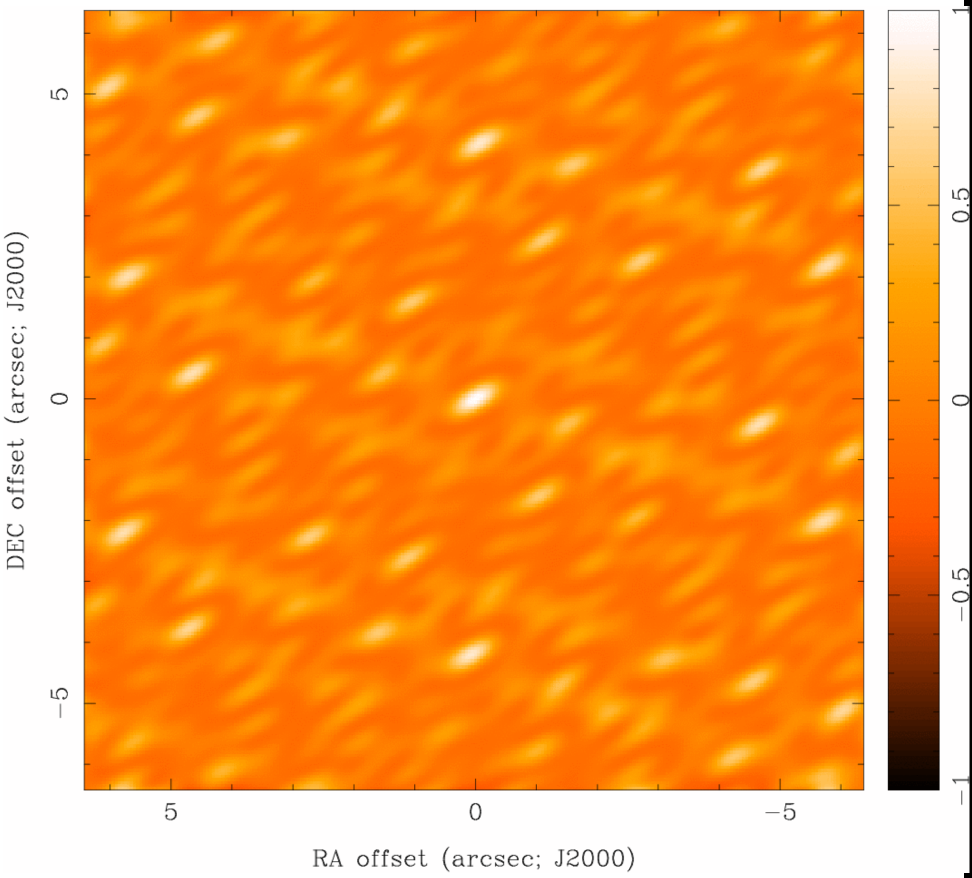
7 Antennas



DIRTY BEAM SHAPE AND N ANTENNAS

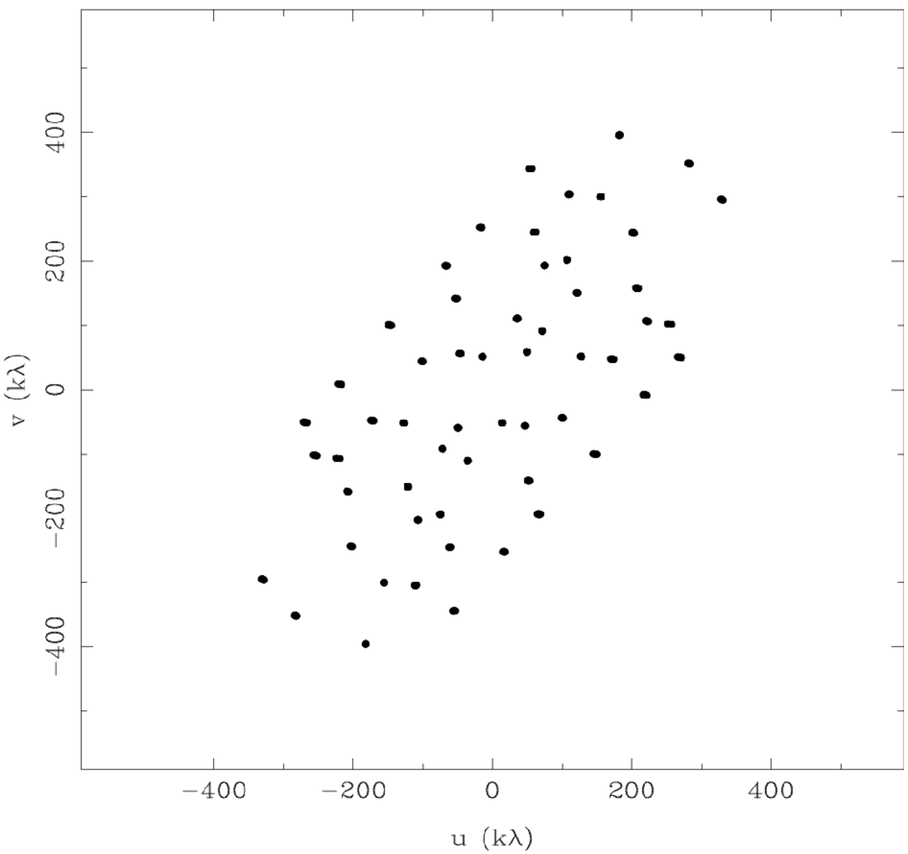
8 Antennas

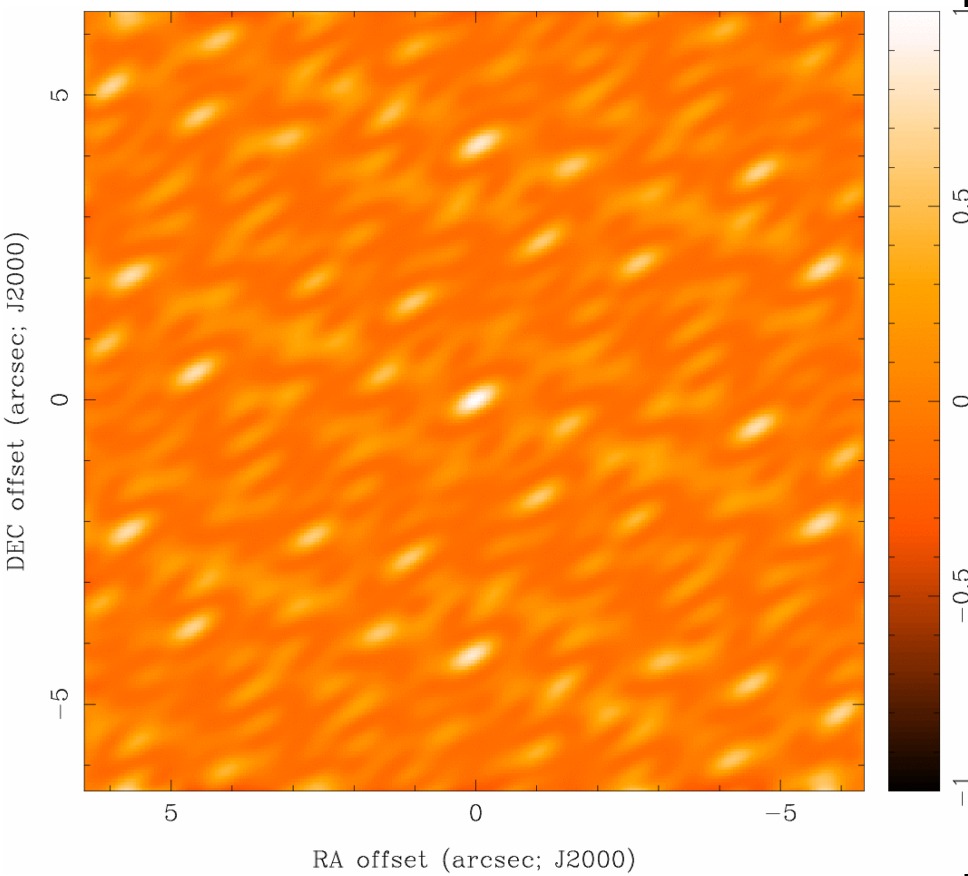




DIRTY BEAM SHAPE AND N ANTENNAS

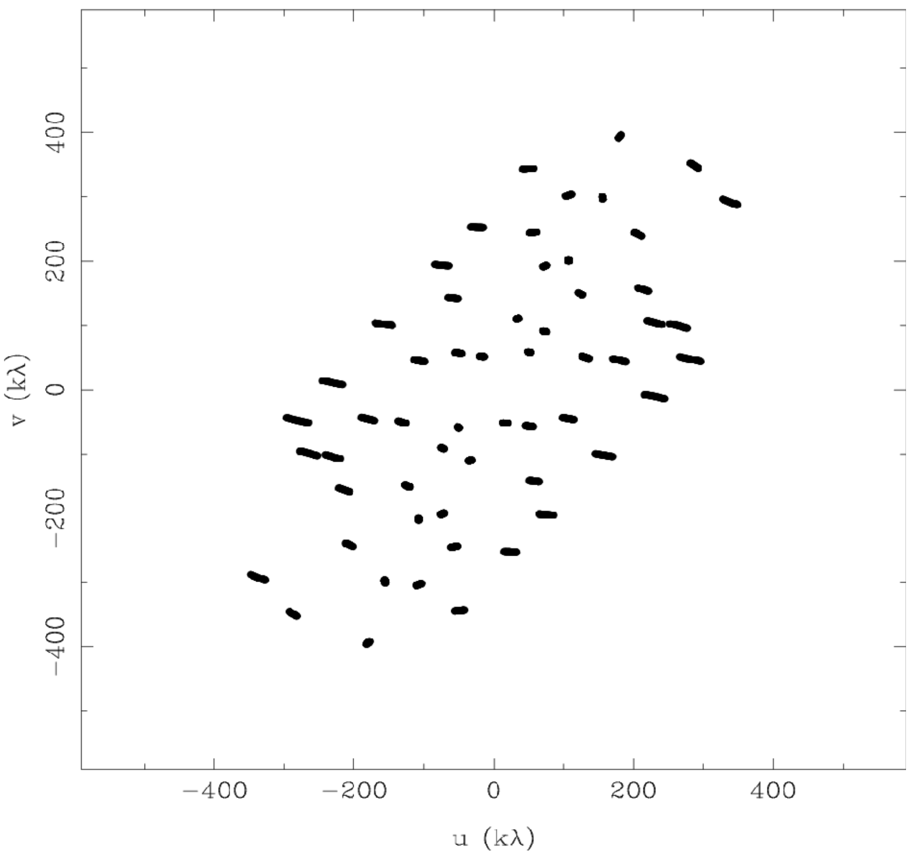
8 Antennas x 6 Samples

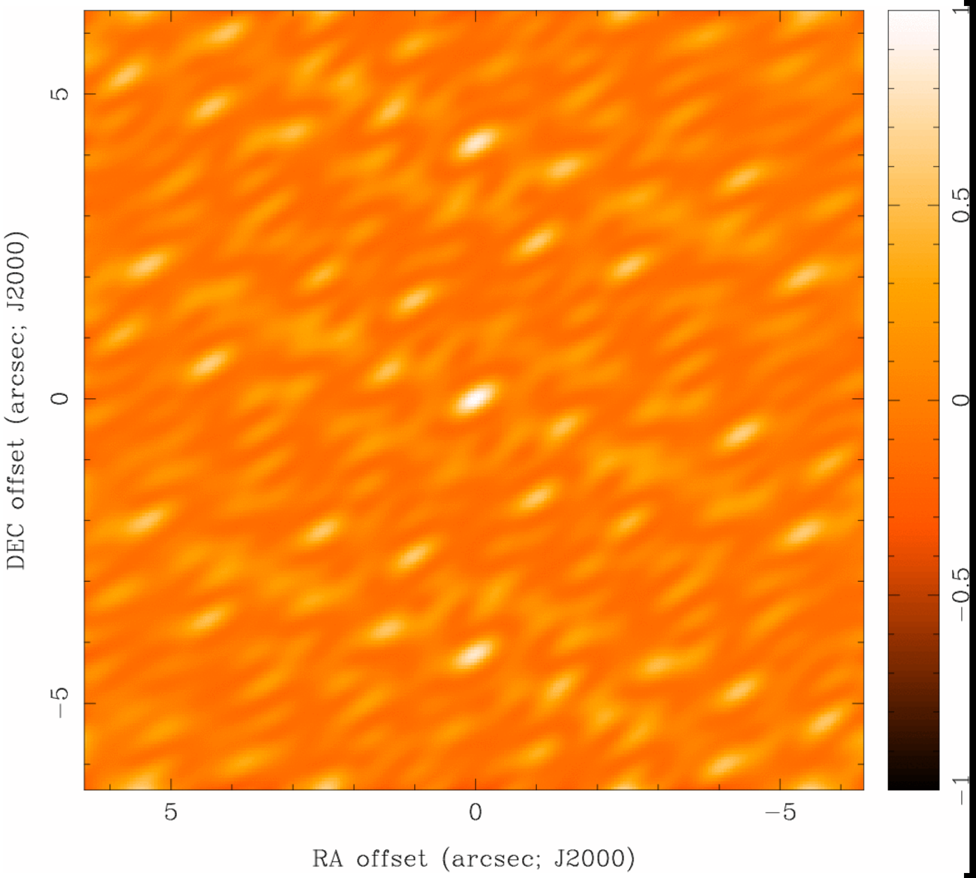




DIRTY BEAM SHAPE AND N ANTENNAS

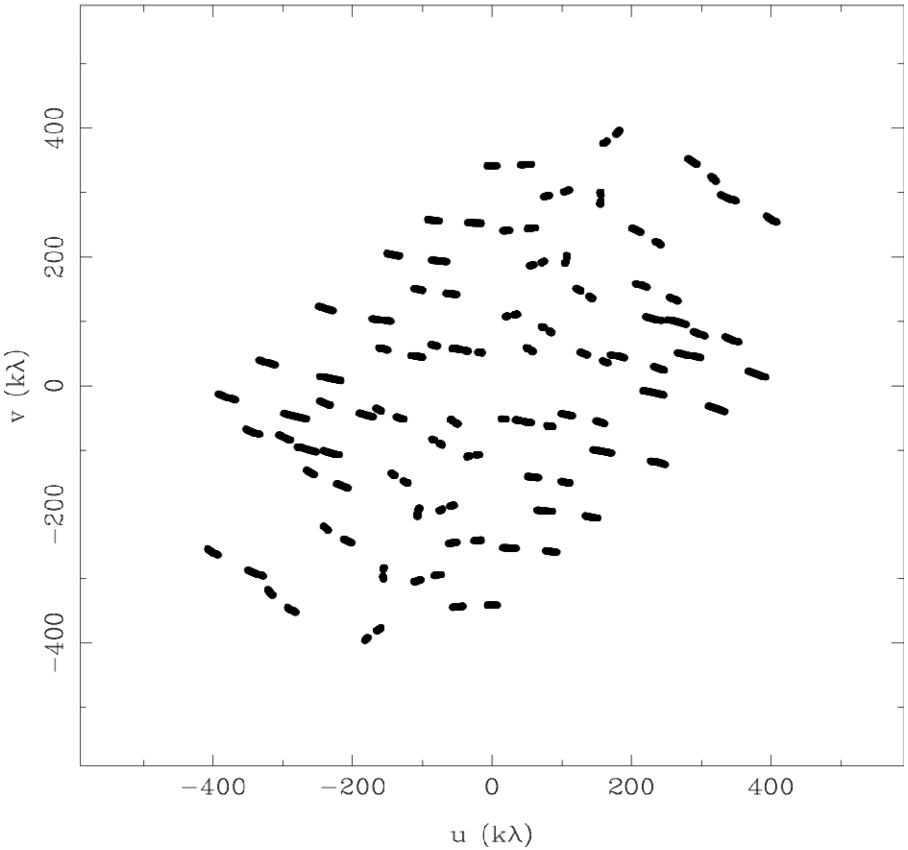
8 Antennas x 30 Samples

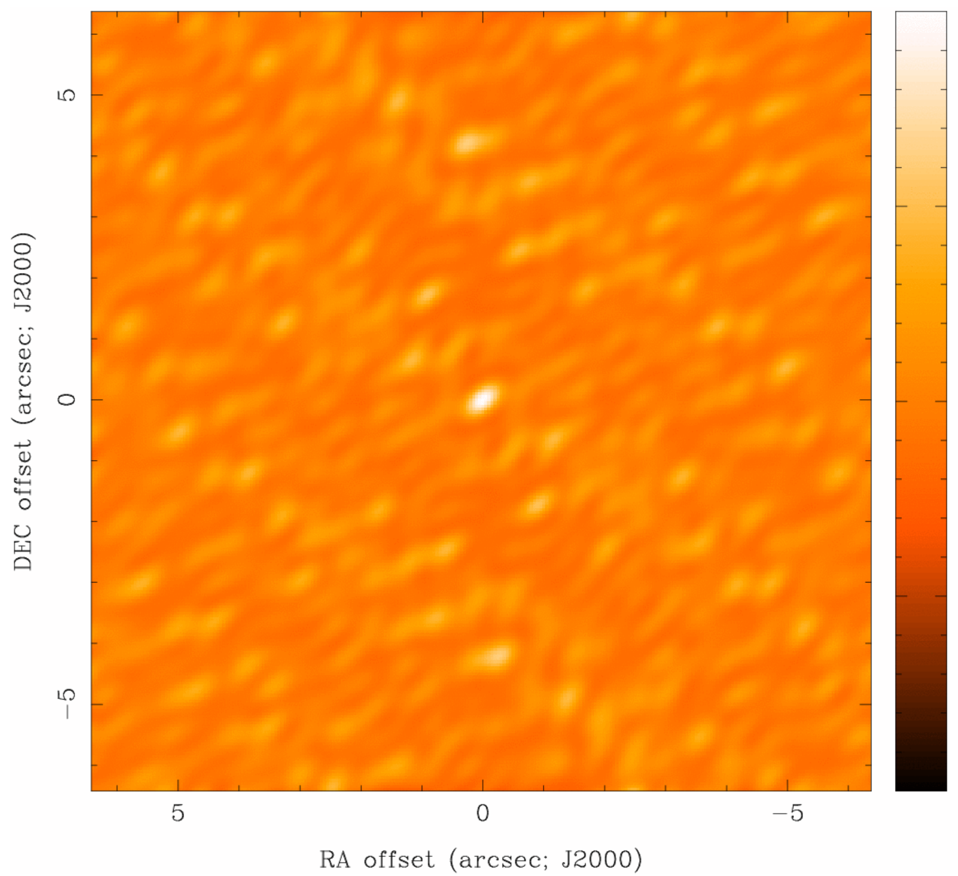




DIRTY BEAM SHAPE AND N ANTENNAS

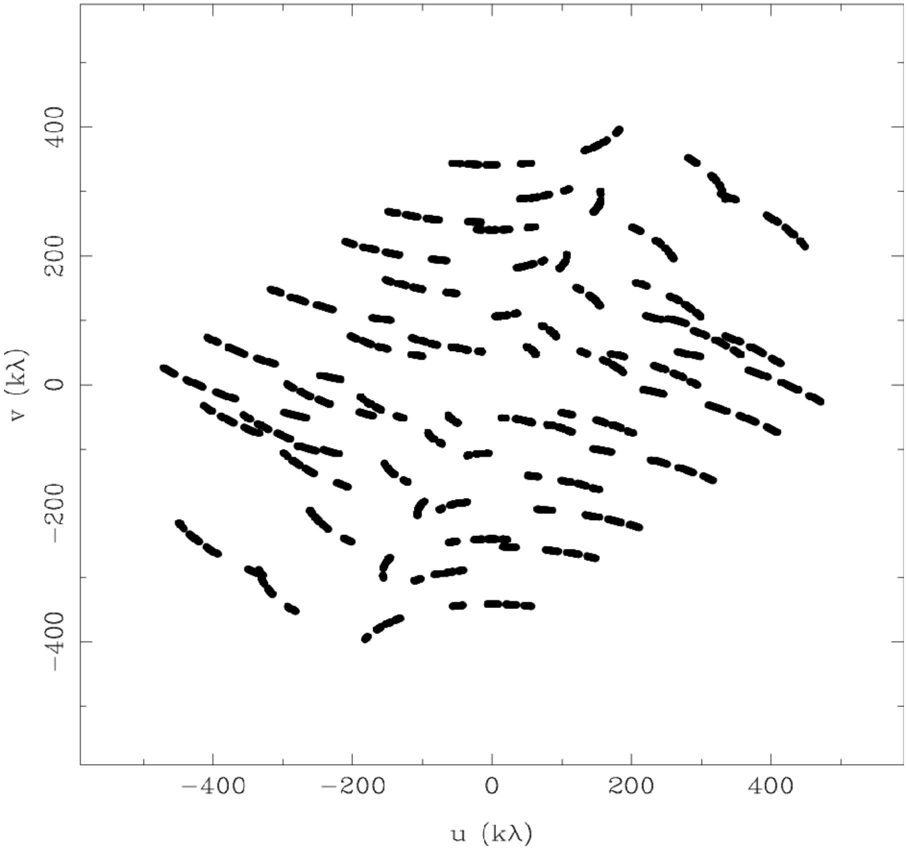
8 Antennas x 60 Samples

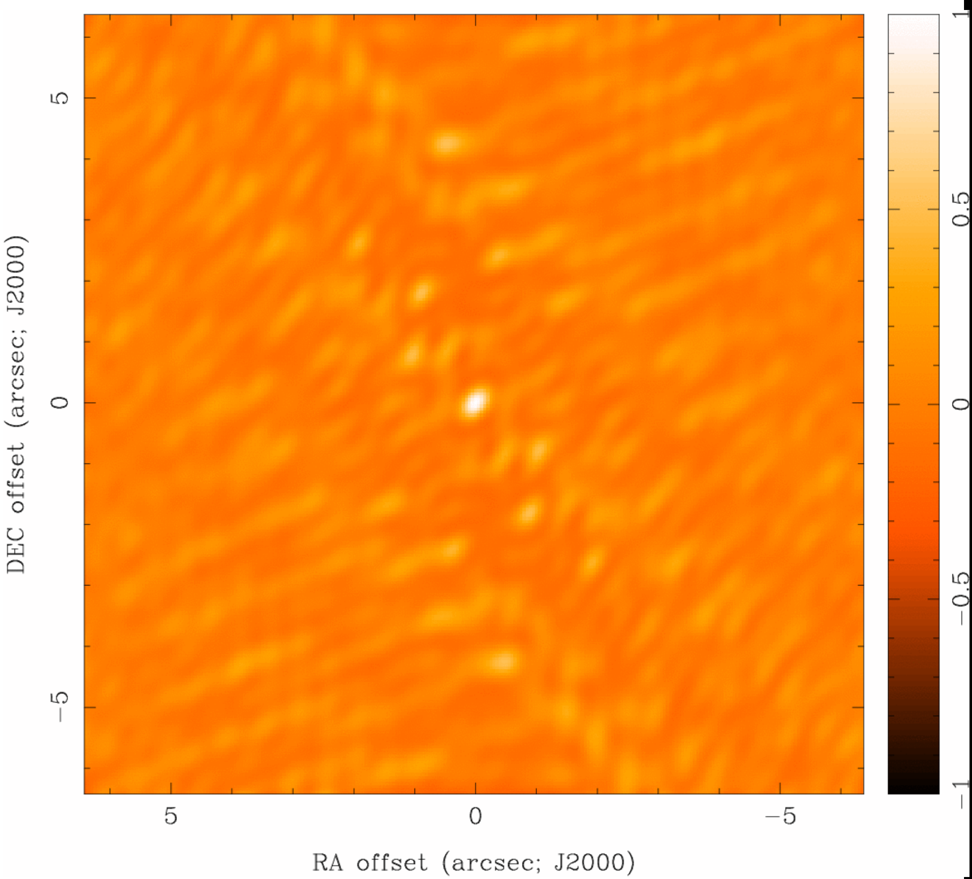




DIRTY BEAM SHAPE AND N ANTENNAS

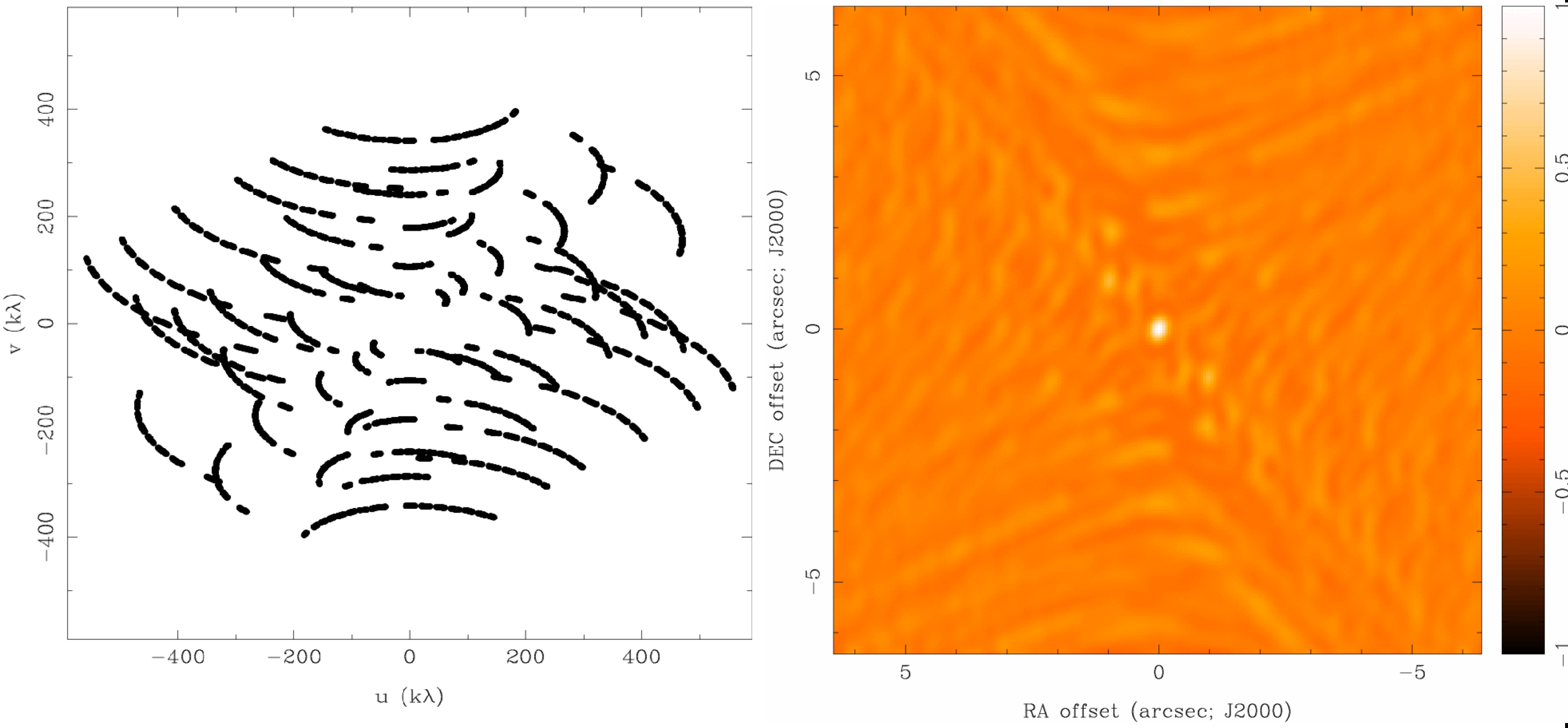
8 Antennas x 120 Samples





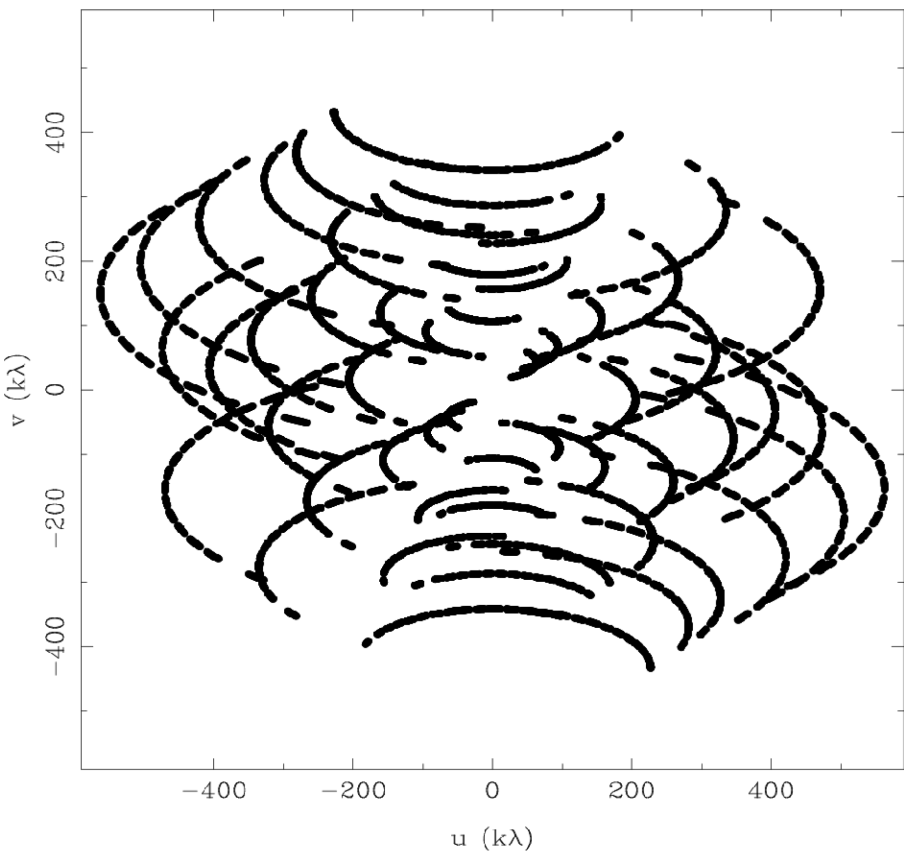
DIRTY BEAM SHAPE AND N ANTENNAS

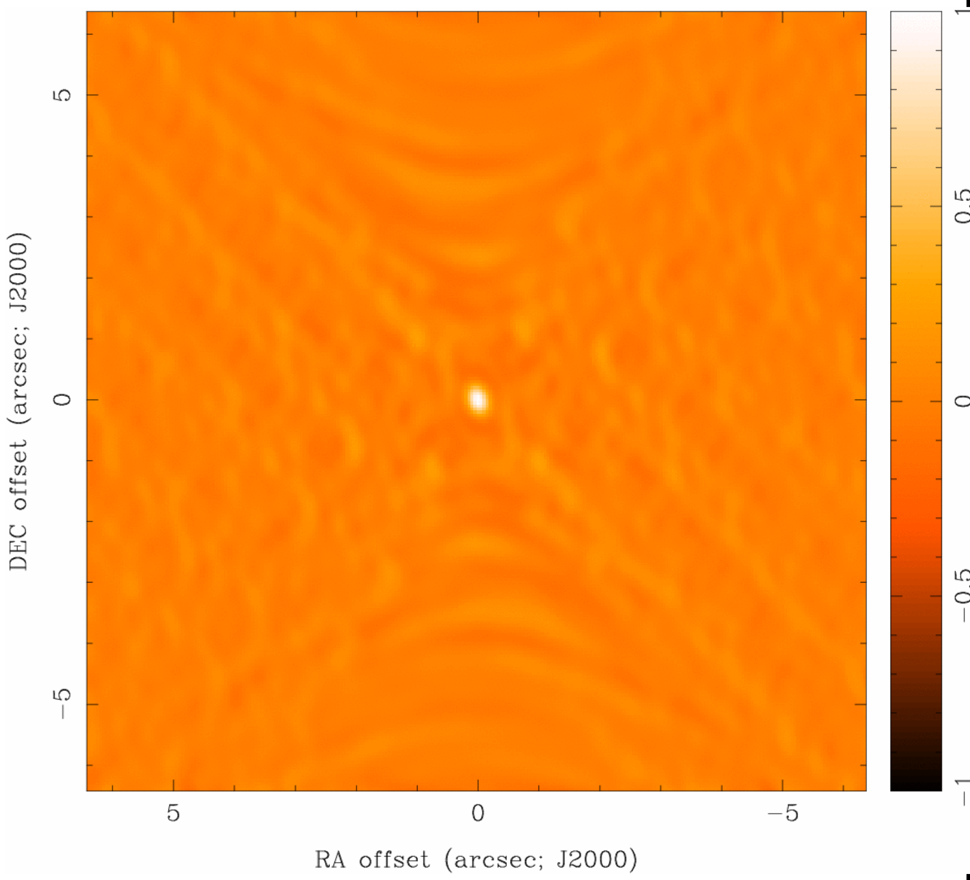
8 Antennas x 240 Samples



DIRTY BEAM SHAPE AND N ANTENNAS

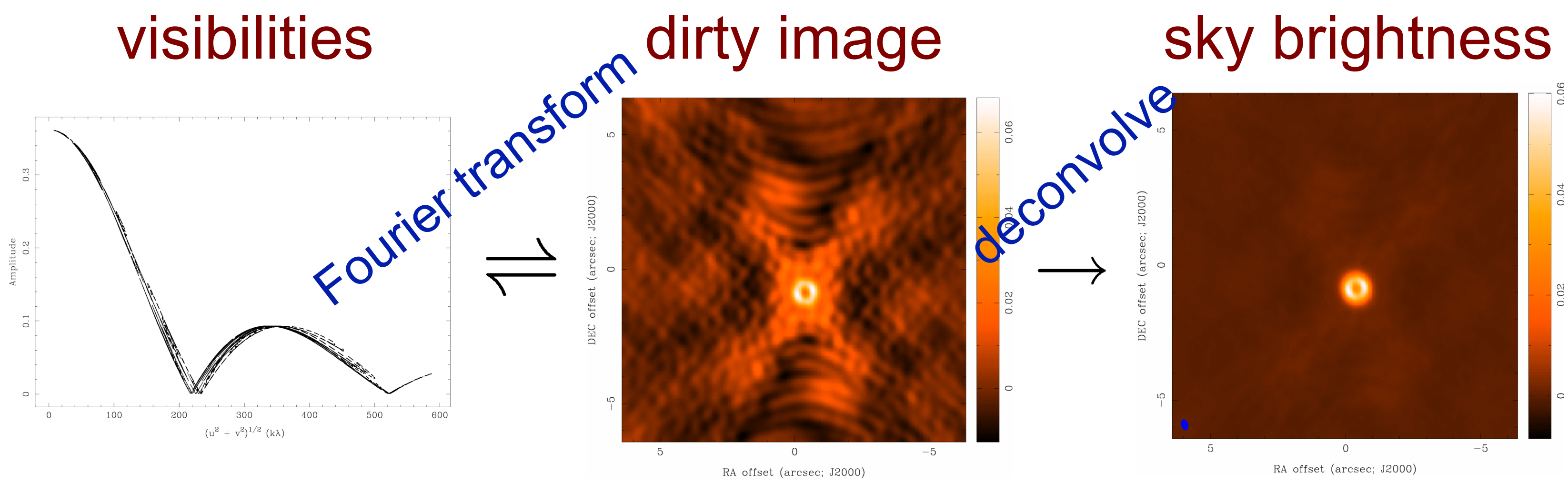
8 Antennas x 480 Samples





FROM VISIBILITIES TO IMAGES

- uv-plane analysis
  → only feasible with simple sources (point sources, disks,…)
- image plane analysis
  → dirty image $I^D(l,m)$ = Fourier transform { V(u,v) }
  → deconvolve b(l,m) from $I^D(l,m)$ to determine (model of) I(l,m)



DECONVOLUTION

- ∃ an infinite number of $I(l,m)$ compatible with sampled $V(u,v)$
- noise → undetected/corrupted structure in $I(l,m)$
- no unique prescription for extracting optimum estimate of true sky brightness from visibility data

→ Deconvolution

- uses non-linear techniques effectively interpolate/extrapolate samples of $V(u,v)$ into unsampled regions of the (u,v) plane
- aims to find a sensible model of $I(l,m)$ compatible with data
- requires a priori assumptions about $I(l,m)$

SYNTHESIZED BEAM

Discrete sampling: $I'(l,m) = \iint W(u,v)V(u,v)e^{2\pi i(uv+vy)}\,du\,dv$

The **weighting function W(u,v)** is 0 where V is not sampled

I'(l,m) is FT of the product of W and V, which is the convolution of the FT of V and W:

$$I'(l,m) = b(l,m) \otimes I(l,m)$$

$$b(l,m) = \iint W(u,v)e^{2\pi i(ul+vm)}\,du\,dv$$

b(l,m) **is the synthesized beam, analogous of the point-spread function in an optical telescope**.

WEIGHTING FUNCTION

Measured flux: $$I'(l,m) = b(l,m) \otimes I(l,m)$$

Synthesized beam: $$b(l,m) = \iint W(u,v) e^{2\pi i (ul+vm)} \, du \, dv$$

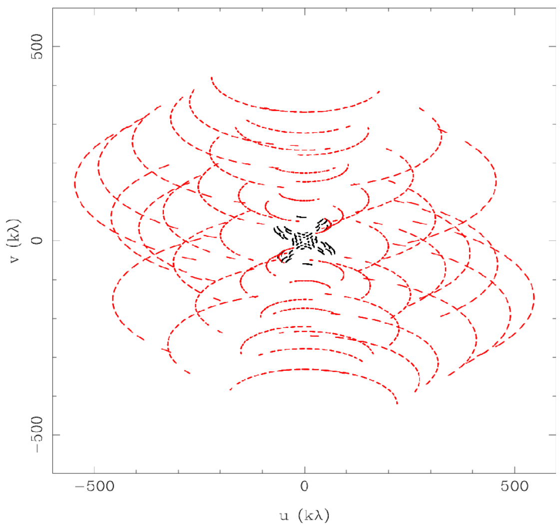
You can change the angular resolution and sensitivity of the final image by changing the weighting function W(u,v)

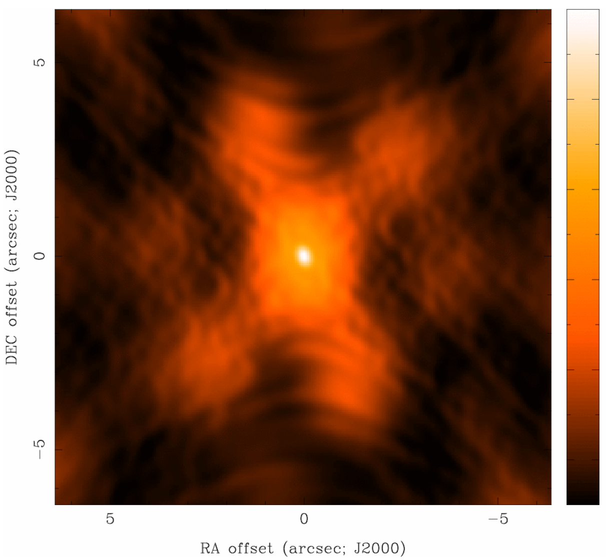
DIRTY BEAM SHAPE AND WEIGHTING

"Natural" weighting

- $W(u,v) = 1/\sigma^2(u,v)$ at points with data and zero elsewhere, where $\sigma^2(u,v)$ is the noise variance of the (u,v) sample

- Advantage: maximizes point source sensitivity → lowest rms in image, highlighting extended structures
- Disadvantage: generally gives more weights to the short baseline (large spatial scales), where there are more measurements of V → degrades the resolution

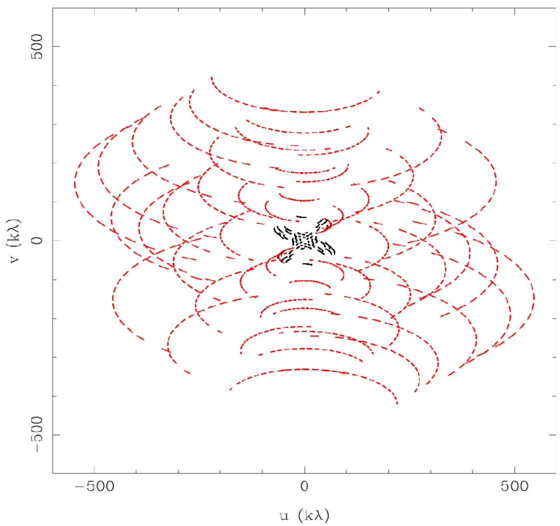


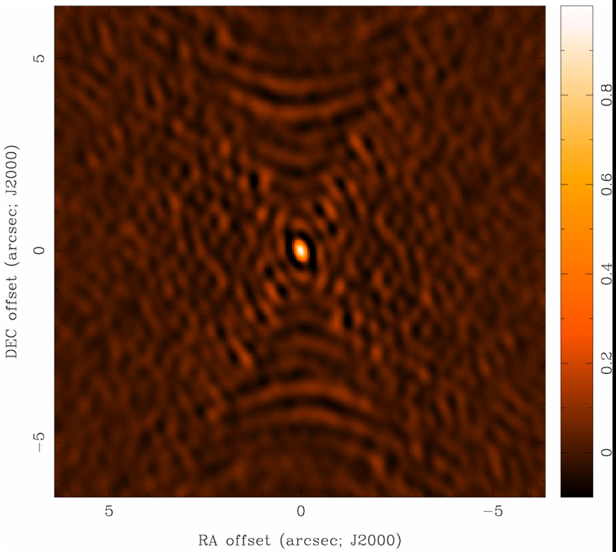


DIRTY BEAM SHAPE AND WEIGHTING

"Uniform" weighting

- W(u,v) is inversely proportional to local density of (u,v) points, so sum of weights in a (u,v) cell is a constant (or zero)
- Advantages:
  - fills (u,v) plane more uniformly → lower sidelobes
  - gives more weight to long baselines → higher angular resolution
- Disadvantages:
  - degrades point source sensitivity → higher rms in image
  - problematic with sparse sampling: cells with few data points have same weight as cells with many data points





DIRTY BEAM SHAPE AND WEIGHTING

"Robust" (Briggs) weighting

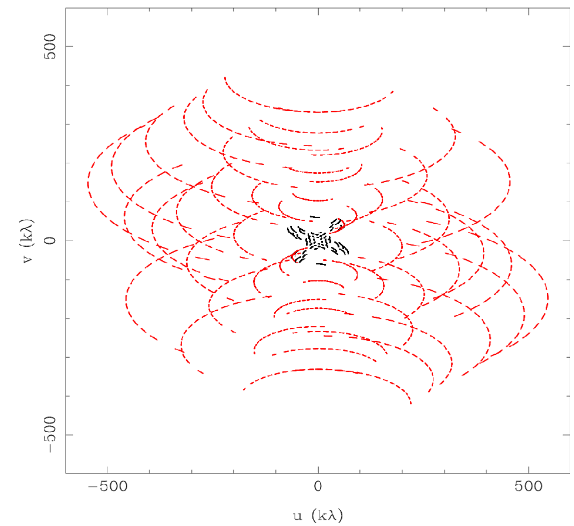
- variant of "uniform" that avoids giving too much weight to cell with low natural weight
- being S_N the natural weight of a cell, W(u,v) depends on a given threshold S:

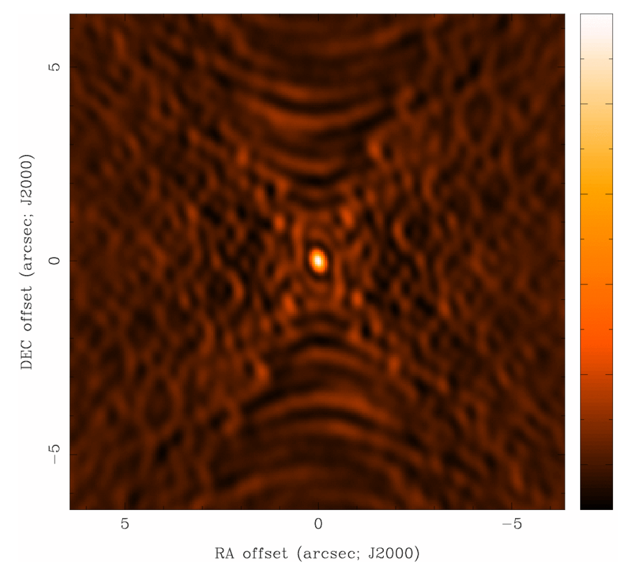
$$W(u,v) = \frac{1}{\sqrt{1+S_N^2/S_{thresh}^2}}$$

large threshold → natural weighting

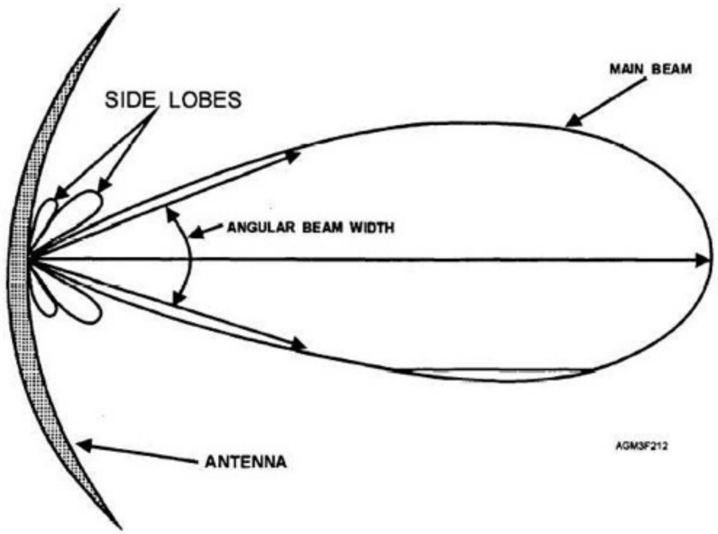
small threshold → uniform weighting

- Advantage: allows for continuous variation between highest angular resolution and optimal point source sensitivity





FIELDS-OF-VIEW AND MOSAICS



HPBW of the primary beam → "field-of-view" of the single-pointing interferometric image:

FOV ~ λ/D

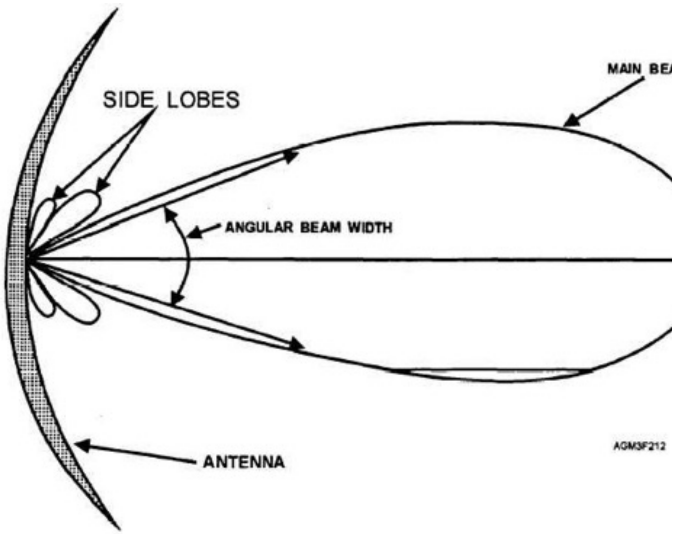
Included in the correlator output:

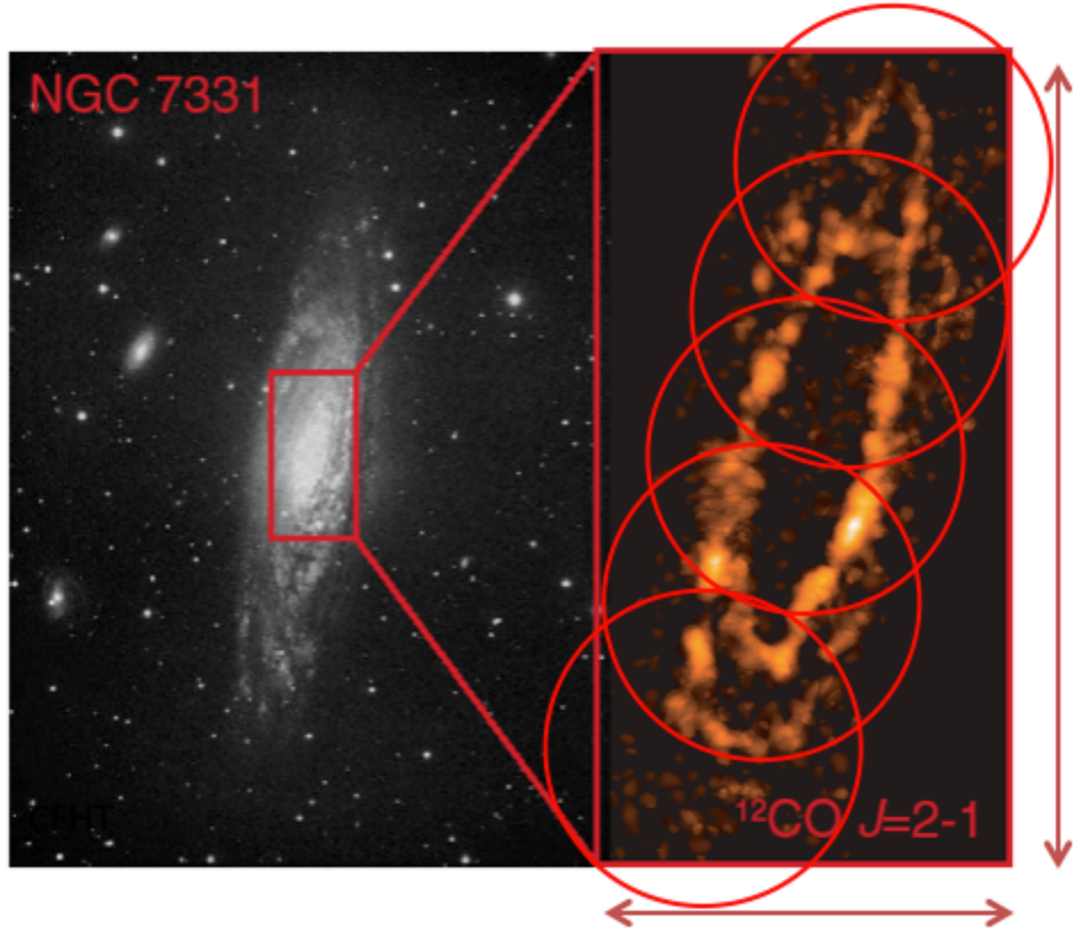
$$\mathcal{V}(u,v) = \iint A(l,m) I(l,m) e^{-2\pi i(ul+vm)} dl dm.$$

Power pattern of one antenna

- Smaller dish → larger FOV
- If source larger than FOV → mosaic !

FIELDS-OF-VI
MOSAICS



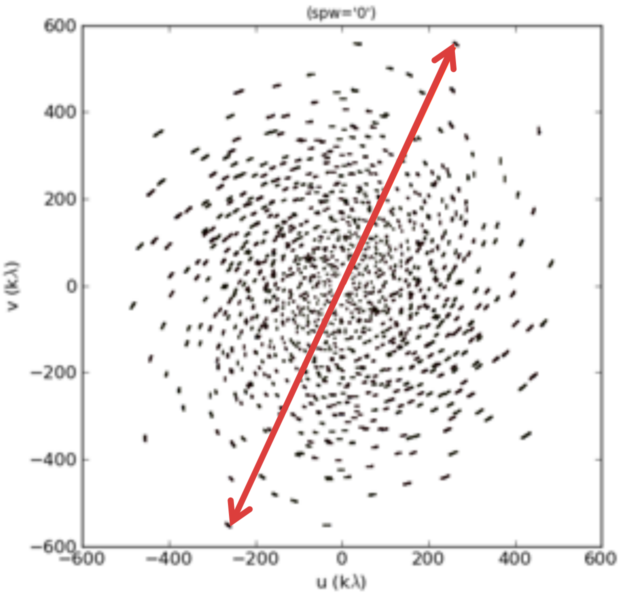


Power pattern of one antenna

- Smaller dish → larger FOV
- If source larger than FOV → mosaic !

ANGULAR RESOLUTION

- Synthesized beam = the way the interferometer "sees" a point source
- Angular resolution = FWHM of synthesized beam

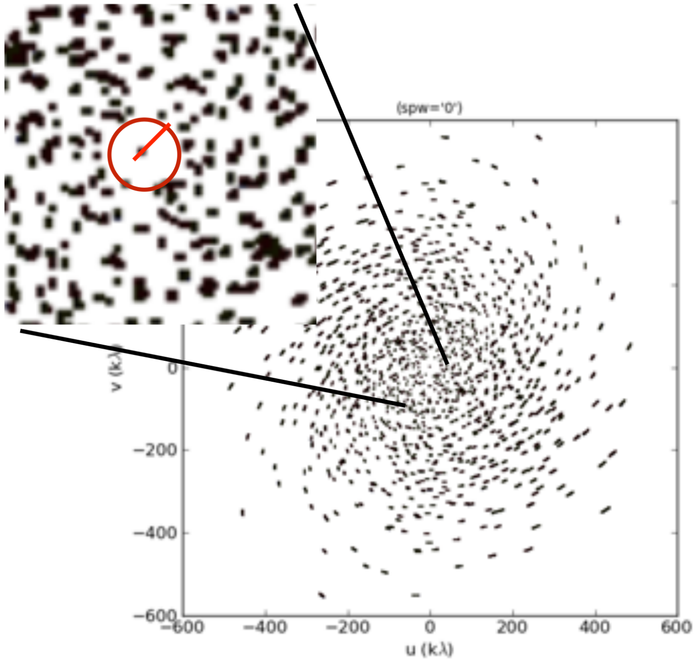


$\theta_{max} \sim \lambda/D_{max}$

- Larger Dmax → higher resolution (image details)
- Ok for compact objects
- BUT careful with extended sources !

MAXIMUM RECOVERABLE SCALE

- Depends on minimum distance between antennas

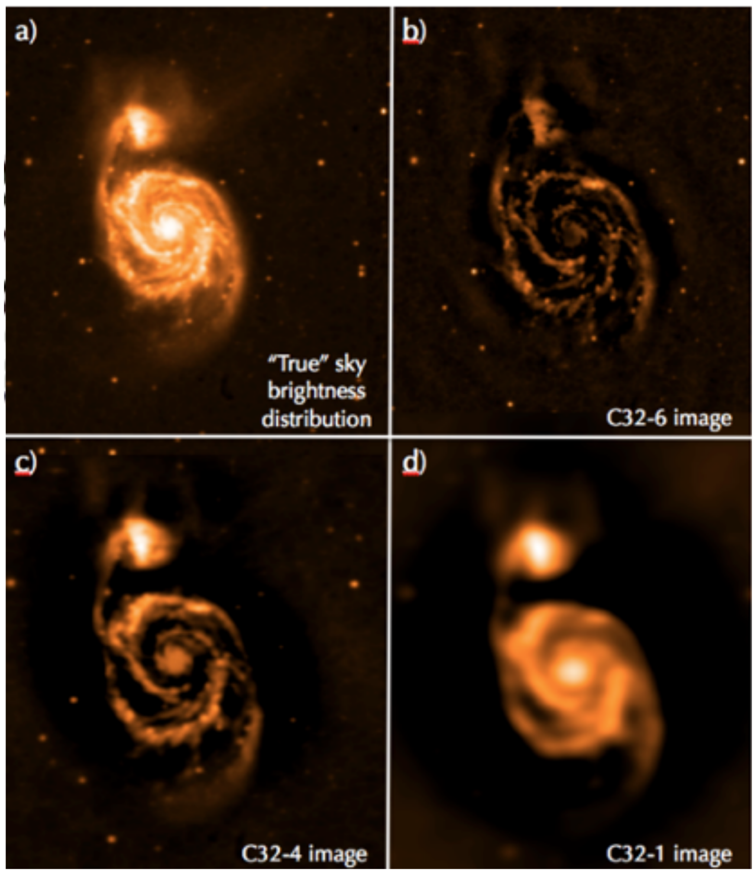


$\theta_{MRS} \sim 0.6\ \lambda / L_{min}$

- Compact configurations → more sensitive to extended structures

MAXIMUM RECOVERABLE SCALE

- Depends on minimum distance between antennas



$$\theta_{MRS} \sim 0.6\ \lambda / L_{min}$$

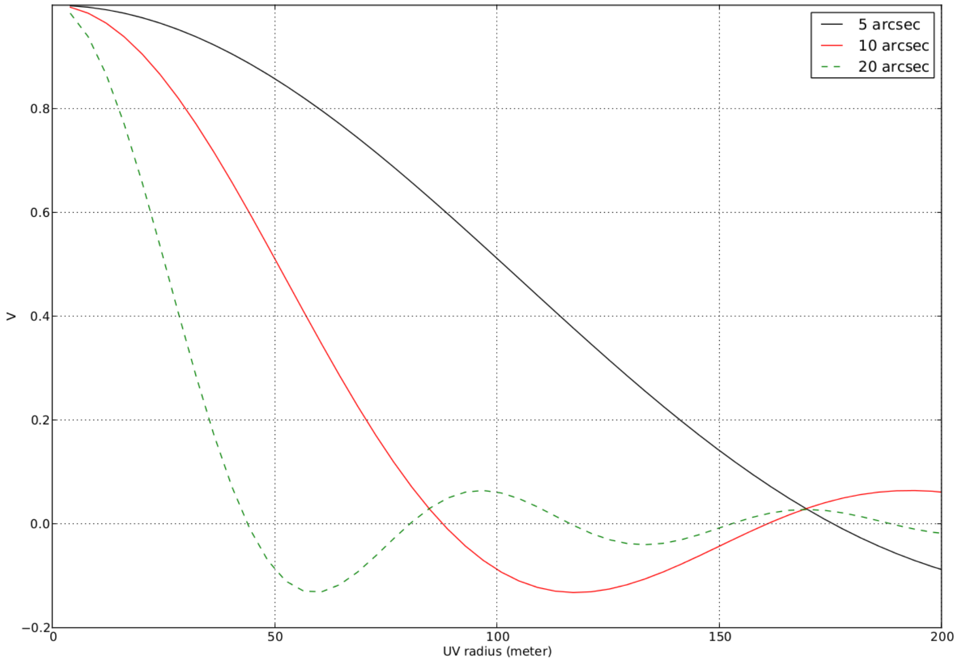
- Compact configurations → more sensitive to extended structures
- Physical limit: the antenna diameter → zero-spacing problem

⬇

- Need to combine with smaller array dishes (ALMA ACA) or single-dish observations

ZERO-SPACING PROBLEM

- An astronomical source is *filtered* if its Fourier transform has substantial power on angular scales outside the region of the uv-plane sampled by a given configuration
- If the source only has structures on size scales larger than the shortest observed baselines, one can “resolve-out” the source entirely



- Smallest disk: large amplitudes up to baselines of 180m
- Most extended disk: large amplitudes only up to 40m
- Structures larger than 20’’ not detectable with array with baselines larger than 40m

OTHER IMPORTANT PARAMETERS...

- **Flux vs brightness** (Rayleigh-Jeans)
  - S = flux density (Jy, Jy/beam^{-1})
  - T_B = brightness temperature (K)

$$I_\nu(\theta, \varphi) = \frac{2k\nu^2}{c^2} T_B(\theta, \varphi).$$

$$S_\nu = \frac{2k\nu^2}{c^2} \int T_B d\Omega.$$

$$\left(\frac{T}{1\ \mathrm{K}}\right) = \left(\frac{S_\nu}{1\ \mathrm{Jy}}\right) \left[13.6 \left(\frac{300\ \mathrm{GHz}}{\nu}\right)^2 \left(\frac{1''}{\theta_{max}}\right) \left(\frac{1''}{\theta_{min}}\right)\right]$$

Synthesized beam

- **Sensitivity**

$$\Delta S \propto \frac{T_{sys}}{D^2 \left[n_p N(N-1)\, \Delta\nu\, \Delta t\right]^{1/2}}$$

Improve ΔS

- Lower T_{sys}
- Increasing integration time
- Increasing number of antennas (collecting area)

GOALS / QUESTIONS

Goals

- Measure the signal emitted from a particular region in the sky
- Obtain high spatial images of the source (cont and/or lines)
- Determine chemical and/or physical properties

Questions

- How does interferometry work?
- Calibration of interferometers →
- Image fidelity

↓

Depends strongly on uv coverage, deconvolution and weigthing

- No need to subtract the emission from the atm (OFF)
- Phase calibration is very important, since it provides the location of the source !

FURTHER READING

- "Interferometry and Synthesis in Radio Astronomy"
  R. Thompson, J. Moran and G. W. Swenson, Jr.

- "Synthesis Imaging in Radio Astronomy II"
  G. B. Taylor, C. L. Carilli and R. A. Perley

- IRAM interferometry schools:
  http://www.iram-institute.org/EN/content-page-67-7-67-0-0-0.html

- NRAO synthesis imaging workshop:
  https://science.nrao.edu/science/meetings/2014/14th-synthesis-imaging-workshop

- ALMA Science Portal documentation: Primer and tech handbook
  https://almascience.eso.org/documents-and-tools