# Complex permittivity measurements using cavity perturbation technique with substrate integrated waveguide cavities

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Cavity perturbation technique is widely used in the measurements of complex dielectric permittivity of materials due to its accuracy and ease of configuration. This paper presents the theoretical formulas for the evaluation of complex permittivity of materials using cavity perturbation technique with substrate integrated cavity resonators. With the proposed formulas, the use of various planar cavities is possible by taking into account the dielectric characteristics of the substrate in which the cavity is implemented. Simulations and measurements are performed on various dielectric samples to validate the proposed theory. The maximum deviation in the measured dielectric permittivity values is below 6% compared to the literature values. The implemented substrate integrated cavity is then analyzed in terms of sensitivity, showing a good performance.  [doi:10.1063/1.3442512]

## I. INTRODUCTION

Complex permittivity ($\varepsilon = \varepsilon' - j\varepsilon''$) measurement of materials plays a very important role in several industrial, scientific, and medical applications.[1–4] Resonant structures (coaxial, dielectric, and cavity resonators) are most commonly used to characterize materials at microwave frequencies.[1,2] Microwave cavities are popular because they present the advantage of high $Q$ factors and good sensitivity.[1,2] In cavity perturbation technique (CPT) for complex permittivity measurements, the sample under test is located at a position of maximum electric field $\boldsymbol{E}$, and it creates a small perturbation that is reflected in variations in the resonant response of the device. The changes in resonance frequency $f_s$ and quality factor $Q_s$ are related to the properties of the sample through CPT. This characterization method has been widely used due to its accuracy and good sensitivity, and has been exploited by several authors.[1–4]

Recently, with the development of substrate integrated waveguide (SIW) technology, which is realized from the planar substrate, it is possible to design SIW cavities with the advantages of high $Q$ factor, low profile, and low cost.[5] SIW cavities can be employed for material dielectric characterization using CPT. Saeed *et al.*[4] performed complex dielectric permittivity measurements of liquids using a SIW cavity designed with RT/duroid 5880 substrate ($\varepsilon_r = 2.2$ and $\tan \delta = 0.0009$). The authors[4] used conventional expressions of CPT to characterize the binary mixtures of isobutanol and isopropanol, and the results were compared with theoretical data. Conventional formulas of CPT are linear approximations that fit well in a considerable range of sample permittivity values, and are derived assuming that the cavity is air filled.[2] In Ref. 4, the dielectric properties of the SIW cavity (different from that of air) were not considered in the characterization analysis.

This paper presents the derivation of formulas that can be used for CPT with SIW cavities, where the substrates have different permittivity values, and consequently, wider design flexibility in material characterization can be obtained with accurate results. Simulations and measurements are performed to support the theoretical expressions.

This paper is organized as follows. Section II presents the theoretical derivation of CPT for SIW formulas. Section III shows the design and simulation of SIW cavities with various sample permittivity values to support the theoretical formulas. Section IV presents experimental measurements and characterization of samples, and sensitivity analysis of the implemented SIW cavity.

## II. THEORETICAL CPT FOR SIW

The proposed CPT formulas for permittivity characterization using SIW cavities consider the dielectric properties of the medium filling the cavity. The fundamental expression of CPT for complex permittivity measurements using SIW cavities is

$$\frac{\omega_2 - \omega_1}{\omega_1} = -\left(\frac{\varepsilon_2 - \varepsilon_1}{2\varepsilon_1}\right)\frac{\iiint_{Vs}\boldsymbol{E}_1^* \cdot \boldsymbol{E}_2 dV}{\iiint_{Vc}|\boldsymbol{E}_1|^2 dV}, \tag{1}$$

where $\omega_1$ and $\omega_2$ are the complex angular resonant frequencies of the cavity before and after perturbation, respectively, $\varepsilon_1$ is the complex dielectric coefficient of the medium filling the cavity, $\varepsilon_2$ is the effective complex dielectric coefficient of the medium after the perturbation, and $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ represent the electric field before and after the sample perturbation, respectively, inside the cavity. Equation (1) is derived assuming that the sample is located at a maximum $\boldsymbol{E}$-field position, and the perturbation inside the cavity is small.[2]

As $\omega$ and $\varepsilon$ are complex values, Eq. (1) can be separated in real and imaginary parts, which are related to the material real permittivity coefficient and dielectric losses, respectively.[2] The final expressions for the CPT using SIW cavities are

$$\varepsilon_s' = \frac{A\varepsilon_r' V_c}{V_s}\left(\frac{f_0 - f_s}{f_s}\right) + \varepsilon_r', \tag{2}$$

$$\varepsilon_s'' = \frac{BV_c}{V_s}\left(\frac{\varepsilon_r'^2 + \varepsilon_r''^2}{\varepsilon_r'}\right)\left(\frac{Q_0 - Q_s}{Q_0 Q_s}\right) + \frac{\varepsilon_s' \varepsilon_r''}{\varepsilon_r'}, \tag{3}$$

where $\varepsilon_s'$ and $\varepsilon_s''$ correspond to real and imaginary permittivities of the sample, respectively, $\varepsilon_r'$ is the substrate relative permittivity, and $\varepsilon_r''$ is related to the substrate loss tangent[2] as $\varepsilon_r''=\varepsilon_r' \tan \delta$; $f_0$ and $f_s$ are the resonant frequencies before and after the small perturbation, respectively. $A$ and $B$ are related to the cavity configuration, mode of operation of the cavity, and shape and position of the sample inside the cavity.[2,4] As it is difficult to obtain $A$ and $B$ analytically, these parameters are obtained experimentally by using standard samples of known dielectric properties.[2,4]

## III. SIMULATIONS FOR CPT ANALYSIS USING SIW CAVITIES

### A. Design of SIW cavity

The resonant frequency of a rectangular SIW cavity which is operating in $TE_{m0l}$ mode[4,5] is given by

$$f_0 = \frac{c}{2\sqrt{\varepsilon_r \mu_r}}\sqrt{\left(\frac{m}{a_{\mathrm{eff}}}\right)^2 + \left(\frac{l}{d_{\mathrm{eff}}}\right)^2}, \tag{4}$$

where $c$ is the speed of light in a vacuum, $\varepsilon_r$ and $\mu_r$ are the dielectric permittivity and permeability constants, respectively, $m$ and $l$ are related to the operating mode, and $a_{\mathrm{eff}}$ and $d_{\mathrm{eff}}$ are the effective width and effective length of the cavity, respectively. As the SIW cavity can be closed by metallic posts,[5] $a_{\mathrm{eff}}$ and $d_{\mathrm{eff}}$ are given by

$$a_{\mathrm{eff}} = a - \frac{D^2}{0.95b} \quad \text{and} \quad d_{\mathrm{eff}} = d - \frac{D^2}{0.95b}, \tag{5}$$

where $a$ and $d$ are the overall width and length dimensions of the cavity, $D$ is the metallic post diameter, and $b$ is the separation between the center of adjacent posts. For a good performance of the lateral post walls, the following design rules[5] are adopted: $D<\lambda_g/5$ and $b \leq 2D$, where $\lambda_g$ is a guided wavelength in the medium filling the cavity.

### B. CPT analysis for SIW cavities (simulations)

In order to validate the theory, three cavities with different dielectric substrates are designed at 7 GHz with a $TE_{103}$ resonant mode. Cavity 1 is designed using a RT/duroid 5880 substrate[6] with $\varepsilon_r=2.2$, $\tan \delta=0.0009$, and thickness $h=3.175$ mm, based on the SIW design criteria. Figure 1 shows the layout of the designed resonant cavity. The dimensions of the cavity are $a=21$ mm and $d=63$ mm; the metallic posts have $D=1$ mm and $b=2$ mm. The cavity is excited at one of the ***E***-field maximum position. A coaxial probe is used for the excitation. The cavity is analyzed using a full-wave electromagnetic (EM) simulator.[7] The simulated values are $f_0=7.1884$ GHz and $Q_0=392.81$, which correspond to the parameters before perturbation of the cavity in CPT. Cavity 2 is designed using a RT/duroid 6006 substrate ($\varepsilon_r'=6.15$, $\tan \delta=0.0019$, and thickness $h=2.5$ mm), and cavity 3 is designed with a RT/duroid 6010.2LM substrate ($\varepsilon_r'=10.2$, $\tan \delta=0.0023$, and $h=1.9$ mm). The sample under test consists of a 2 mm diameter cylinder placed at an ***E***-field maximum position, as shown in Fig. 2.

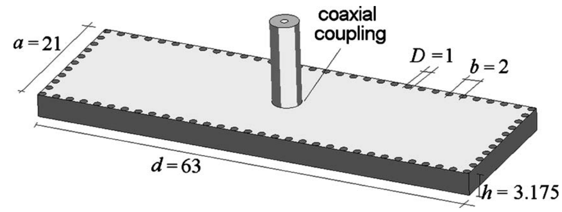


FIG. 1. Layout of the SIW cavity 1.

Complex permittivity characterization based on simulations consists of two parts. In the first part, $\varepsilon_s'$ characterization is carried out by having no dielectric losses in the sample ($\varepsilon_s''=0$). In the second part, $\varepsilon_s''$ characterization is performed by keeping the parameter $\varepsilon_s'$ constant which is equal to 1. Samples of different complex dielectric permittivity values ranging from $\varepsilon_s'=1$ to 25 and $\varepsilon_s''=0$ to 0.1 have been chosen for characterization. Resonant frequencies and $Q$ factors for all the samples are obtained. The complex $\varepsilon$ values of the samples are estimated by using the proposed formulas of CPT for SIW [Eqs. (2) and (3)]. The parameters $A$ and $B$ are obtained by calibration with the introduced standard permittivity quantities $\varepsilon_a'$ and $\tan \delta_a$. This procedure is followed for all the cavities. Table I tabulates the values of $\varepsilon_s$ obtained from the simulations using the proposed formulas of CPT for SIW; the corresponding values of $A$ and $B$ for each characterization are also shown. In Table I, $\varepsilon_a'$ and $\tan \delta_a$ are the introduced values in the simulations, while $\varepsilon_s'$ and $\tan \delta$ are referred to the characterized values from the proposed CPT for SIW formulas.

From Table I, it can be seen that CPT for SIW gives accurate results. The errors of $\varepsilon_s'$ values using CPT for SIW are 3.4%, 1.8%, and 2.7% for cavities 1, 2, and 3, respectively, while for sample $\tan \delta$ quantities the errors are 1.6%, 5.9%, and 2.5%, respectively.

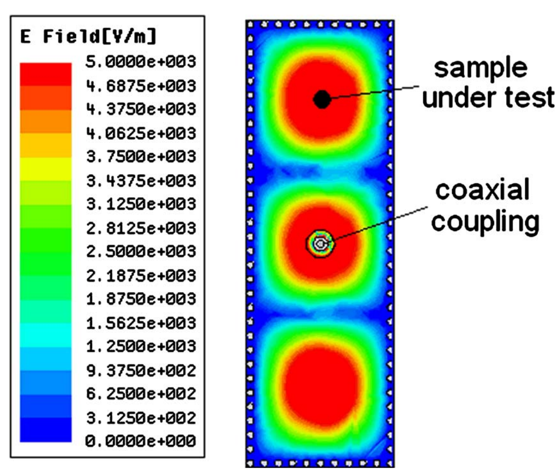


FIG. 2. (Color online) ***E***-field distribution of the SIW cavity and allocation of the sample.

TABLE I. Characterized $\varepsilon_s$ values from simulations using CPT for SIW.

| | $\varepsilon_s'$ | | |
|---|---|---|---|
| $\varepsilon_a'$ | Cavity 1 $A=0.491$ $B=0.291$ | Cavity 2 $A=0.589$ $B=0.315$ | Cavity 3 $A=0.64$ $B=0.289$ |
| 1 | 1.115 | 1.002 | 1.652 |
| 7 | 7.008 | 7.123 | 7.309 |
| 13 | 13.375 | 13.612 | 13.025 |
| 19 | 19.442 | 19.284 | 19.083 |
| 25 | 25.047 | 24.824 | 25.357 |
| tan $\delta_a$ | | tan $\delta$ | |
| 0 | −0.001 | 0.001 | 0 |
| 0.02 | 0.02 | 0.016 | 0.021 |
| 0.04 | 0.04 | 0.04 | 0.04 |
| 0.06 | 0.06 | 0.058 | 0.062 |
| 0.08 | 0.079 | 0.084 | 0.08 |
| 0.1 | 0.104 | 0.101 | 0.095 |

## IV. EXPERIMENTAL MEASUREMENTS

### A. Fabrication of the cavity

To validate the proposed theory and simulations, cavity 1 is fabricated using a Rogers RT/duroid 5880 substrate with thickness $h=3.175$ mm. The lateral walls are created by metallic posts and the structure is excited by means of a coaxial probe at the center of the structure (maximum $\boldsymbol{E}$ field). The cavity is tested using an Agilent-PNA series vector network analyzer (VNA) (E8361A). Measured values of resonant frequency and $Q$ factor are 7.17625 GHz and 517.2072, respectively. A 2.4 mm diameter hole, at a position of maximum $\boldsymbol{E}$ field, is chosen for allocation of the sample.

### B. Characterization of samples

Samples with different permittivity properties are characterized for the resonant frequencies $f_s$ and quality factors $Q_s$. The samples are shaped in a cylindrical form (2.4 mm diameter) and are allocated in the hole of the cavity. The samples chosen for characterization are Teflon, acrylic, polyamide, nylon, Rogers RO4003C,[6] wood, quartz, Rogers RT/duroid 6010.2LM,[6] and Rogers RT/duroid 6010.8LM.[6]

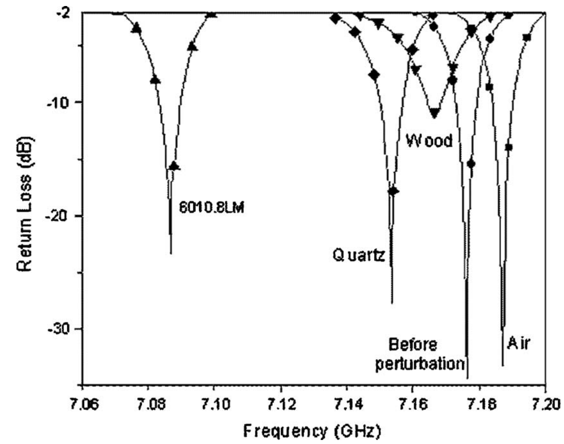


FIG. 3. Experimental response of the cavity with various materials.

The samples are introduced in the cavity and their responses are measured using the VNA. Figure 3 shows the measured return losses for the cavity before perturbation, and after perturbation with air, wood, quartz, and RT/duroid 6010.8LM samples. For material characterization, the calibration constants $A$ and $B$ are obtained by using the standard samples of RT/duroid 6010.2LM and 6010.8LM, as their characteristics are well documented.[6] It can be observed, from Fig. 3 that samples with higher dielectric permittivity values produce lower resonant frequencies of the SIW cavity. Also it can be noticed that materials with higher losses (such as wood) produce lower $Q$ factors. Table II shows the measured permittivity values $\varepsilon_s$ using the proposed CPT for SIW formulas and comparisons are made by using conventional CPT expressions.[4]

From Table II, it is clear that conventional CPT formulas yield negative results for samples whose $\varepsilon_s'$ are lower than that of the substrate. For tan $\delta$ calculations, using the conventional CPT expression, it generates negative results when the losses of the sample are lower than that of the cavity substrate. It is also noticeable that the calculated $\varepsilon_s'$ quantities using conventional CPT formulas only agree for values higher than those of the cavity substrate, while using CPT for SIW the agreement is for the whole linear region. The error for the real permittivity characterization using CPT for SIW

TABLE II. Characterized $\varepsilon_s$ values of materials from measurements.

| | $\varepsilon_s'$ | | | tan $\delta$ | | |
|---|---|---|---|---|---|---|
| Sample | Theoretical | Proposed CPT for SIW $A=1.073$ | Conventional CPT[a] $A=2.98$ | Theoretical | Proposed CPT for SIW $B=0.027$ | Conventional CPT[a] $B=0.1$ |
| Air | 1 | 1.0303 | −1.475 | 0 | 0.0015 | 0.0011 |
| Teflon | 2.1 | 1.8509 | −0.4389 | 0.001 | 0.0021 | 0.002 |
| Acrylic | 2.7 | 2.1248 | −0.093 | | 0.0032 | 0.0039 |
| Polyamide | 2.5 | 2.2951 | 0.122 | 0.004 | 0.0024 | 0.0026 |
| RO4003C | 3.38 | 2.9637 | 0.9663 | 0.0027 | 0.0018 | 0.0015 |
| Nylon | 4 | 3.0302 | 0.231 | | 0.0028 | 0.0031 |
| Wood | 3 | 3.1078 | 1.1483 | | 0.0217 | 0.0347 |
| Quartz | 4.2 | 4.1986 | 2.5256 | 0 | 0.0006 | −0.0004 |
| 6010.2LM | 10.2 | 10.2267 | 10.1373 | 0.0023 | 0.0023 | 0.0023 |
| 6010.8LM | 10.8 | 10.7999 | 10.8612 | 0.0023 | 0.0023 | 0.0023 |

[a]Reference 4.

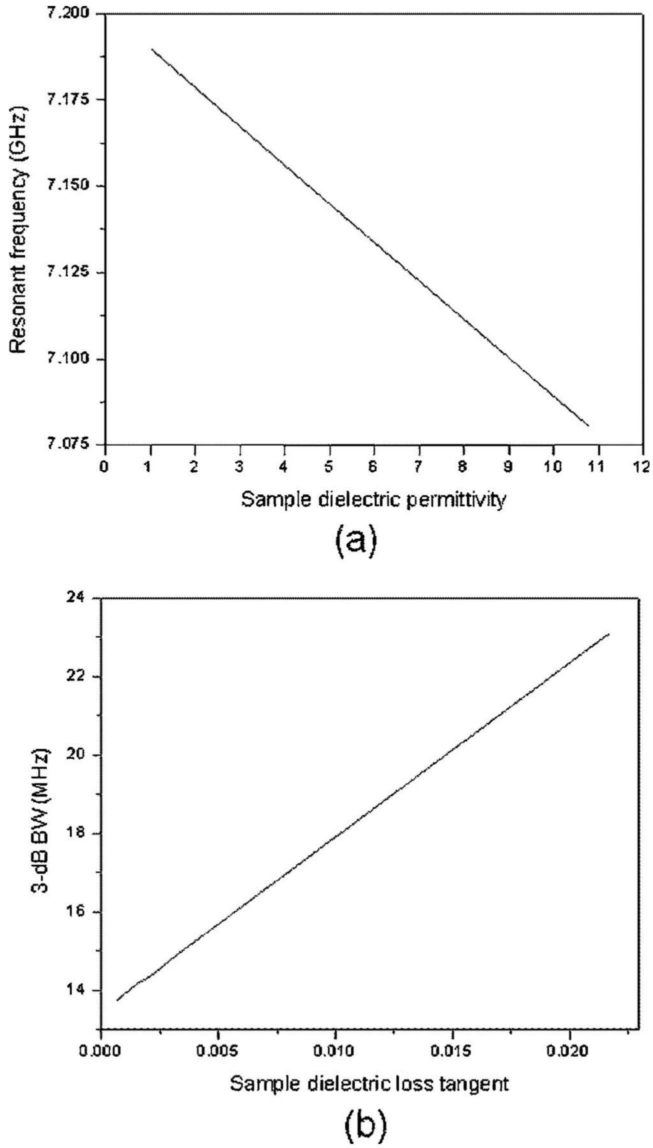


FIG. 4. Experimental sensitivity of cavity 1: (a) resonant frequency vs sample dielectric permittivity and (b) 3 dB bandwidth vs sample dielectric loss tangent.

is within ±6%, while using conventional CPT it results to ±46%.

### C. Sensitivity analysis

It is known[2,4] that the shift in resonant frequency increases with the increase in the dielectric constant of material. The experimental variations in resonant frequency versus sample dielectric permittivity and 3 dB bandwidth versus sample dielectric loss tangent are plotted in Fig. 4.

It can be observed from Fig. 4 that a change in the sample dielectric permittivity of 0.5 generates a shift of 5.6 MHz in the resonant frequency; also a variation of 5.5 MHz in the 3 dB bandwidth is observed for a change in sample tan $\delta$ of 0.01, indicating a high sensitivity in the cavity. Figure 5 shows the simulated and experimental fractional change in the resonant frequency $F=(f_0-f_s)/f_s$ with the varying dielectric constant of the samples using cavity 1.

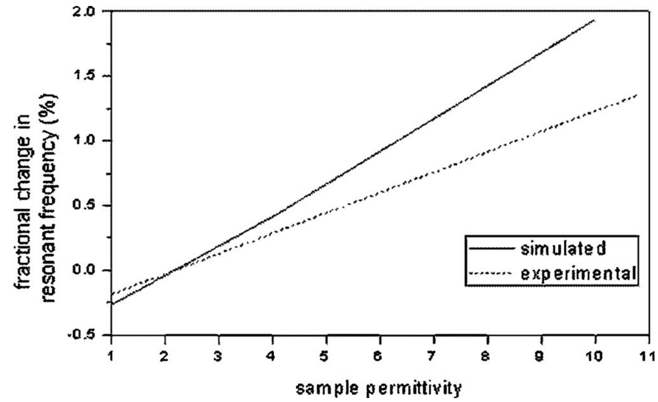


FIG. 5. Fractional change in resonant frequency vs sample permittivity values.

It is noticeable from Fig. 5 that the fractional change in resonant frequency obtained from simulations is higher than that of experimental measurements. The sensitivity $s = dF/d\varepsilon_s'$ from simulations with cavity 1 results of $s = 0.2338\%$, while the experimental sensitivity is of $s = 0.1576\%$. The differences in both the sensitivity values are due to the manufacturing tolerances, sample shape, and position of the sample hole.

## V. CONCLUSIONS

The analysis of complex permittivity of materials using CPT for SIW cavity resonator has been presented. With the derivation of CPT formulas for SIW, it is possible to take into account the relative permittivity and the losses of the cavity substrate, while in conventional CPT it is assumed that the cavity is air filled and without losses. Simulations and measurements of different permittivity materials have been performed for the validation of theoretical expressions. Characterization of materials with different permittivity properties using CPT for SIW presented high accuracy for three different permittivity SIW cavities. Sample variations of less than 0.5 and 0.01 in permittivity constant and loss tangent, respectively, can be clearly characterized with the designed SIW cavity. The proposed CPT formulas produce errors of less than 6% using different permittivity SIW cavities. Also, the proposed expressions allow the use of different dielectric permittivity SIW cavities, which provide the advantages of ease of fabrication, lower dimensions, and more flexibility of design and construction in material characterization systems.

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