4. Nucleosynthesis

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Radiation era

We have that $\rho_M \propto R^{-3}$

$\rho_{rad} \propto R^{-4}$

There must be a z at which $\rho_M = \rho_{rad}$

Taking into account that nucleosynthesis predicts $n_\nu = 0.68\, n_\gamma$, then $\Omega_{rad} = 4.2 \times 10^{-5}\, h^{-2}$

$$1 + z_{eq} = 23900\,\Omega_m h^2 \quad \Rightarrow \quad z_{eq} \approx 3100$$

Therefore the thermal history of the Universe can be divided in two main eras: a *radiation dominated era* ($z \gg z_{\rm eq}$) and a *matter dominated era* ($z \ll z_{\rm eq}$). In the radiation dominated era, in which we can neglect the curvature and Λ terms in Friedmann's equation, we have:

$$R \propto t^{1/2} \,.$$

By differentiating this relation with respect to time and using (12) we have:

$$t = \left(\frac{3}{32\pi G \rho_\gamma} \right)^{1/2} \,. \tag{52}$$

Using $\rho_\gamma = \boxed{\pi^2 k_{\rm b} T^4 / 15 h^3 c^5}$ we finally obtain the important relation between cosmic time and the temperature of the Universe in the radiation dominated era:

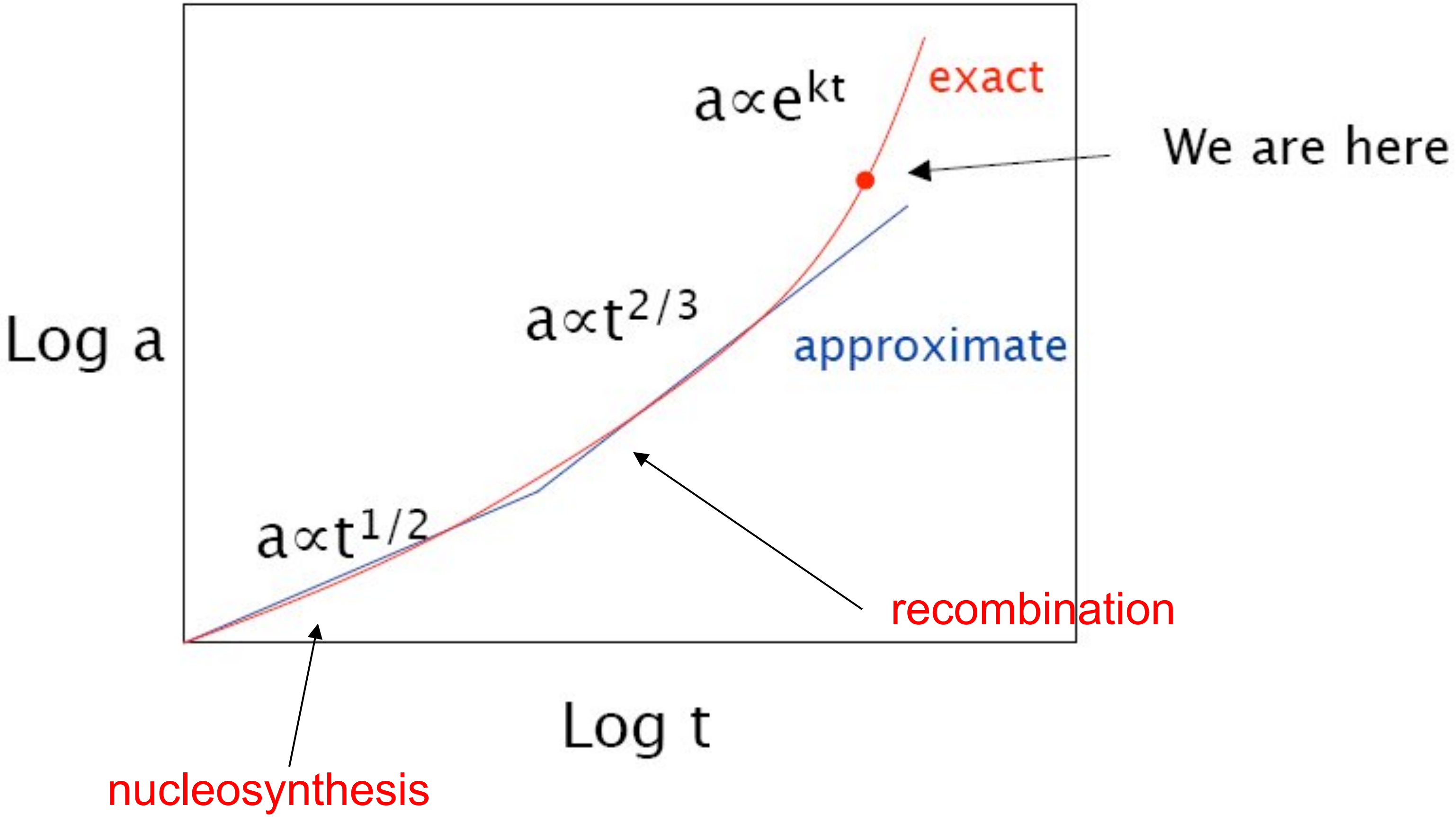
$$T_{\rm Kelvin} \simeq \boxed{1.3} \times 10^{10} t_{\rm sec}^{-1/2} \tag{53}$$

ie., at $t = 1$ sec the Universe had $T \sim 10^{10}$ K! It is evident that the Universe at early times was hot enough for nucleosynthesis to occur, as it had been supposed originally by Gamow. The era of nucleosynthesis takes place around $\sim 10^9$ K.

(From M. Plionis' notes or Peacock 1999)

Epochs in the evolution of the Universe

Quite sudden transition from radiation to matter dominated Universe at z~3100 (t_{eq} ~ 60,700 yr):



Epochs in the evolution of the Universe

Before t_{eq} the Universe was radiation dominated, $a \propto t^{1/2}$:

$$\frac{T}{T_{eq}} = \left(\frac{t_{eq}}{t} \right)^{1/2}$$

T_{eq} ~ 9730 K
t_{eq} ~ 47,000 yr

At t=1 sec, T=2 x 10 ^{10} K -> kT=2 MeV

This is of the order of the nuclear binding energy. Before this time photons were energetic enough to destroy any nuclei.

Before that time we had a sea of separate photons, protons, neutrons, electrons, and neutrinos

Nucleosynthesis took place at that time.

(From M. Georganopoulos' lecture lib)

Epochs in the evolution of the Universe

Before nucleosynthesis?

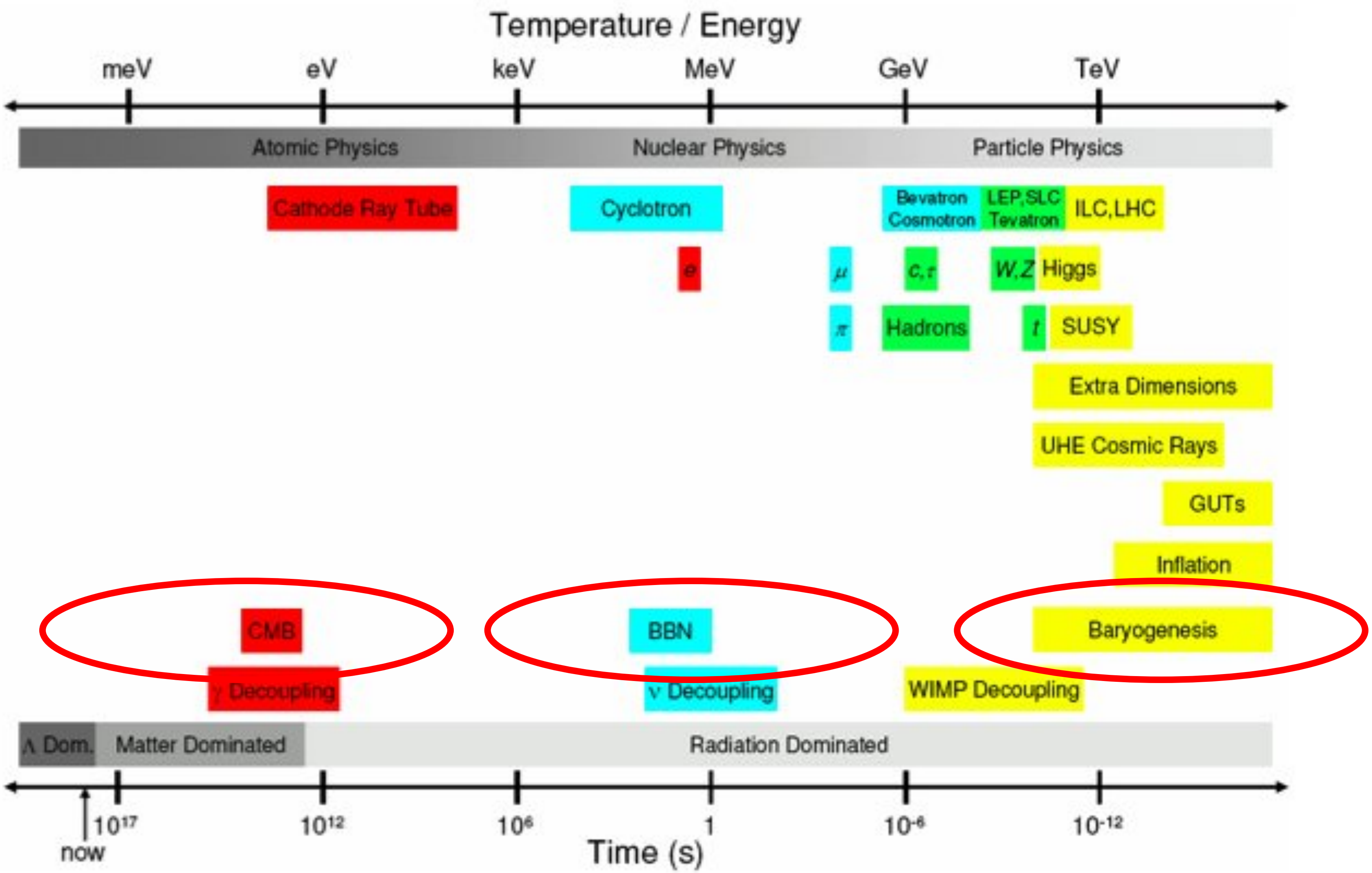
At $t=10^{-4}$ sec, $T=10^{12}$ K -> kT=100 MeV
Protons and neutrons can no longer exist.
Instead their constituent quarks wander in a dense sea in equilibrium with the photons.

The era $t \sim 10^{-4}$ sec when quarks combined to form protons and neutrons is called the quark-hadron phase transition.

Even before that? A quark see back to $t \sim 10^{-12}$ sec, as far as we can probe with our 1 ~TeV accelerators. Before that the physics is not clear.

(From M. Georganopoulos' lecture lib)

PARTICLE PHYSICS AT THE ENERGY FRONTIER



Big Bang Nucleosynthesis (BBN)

Order of magnitude considerations

At t=1 sec, T=2 x 10 ^{10} K -> kT=2 MeV

This is close to the binding energy of deuterium (one proton and one neutron)

$$p + n \leftrightarrow D + 2.22\ MeV$$

At energies higher than that, nuclei cannot survive

Note the similarity with the recombination era!

Remember in recombination the actual T_{rec}=3740 K

We can use this to estimate the nucleosynthesis temperature and time of D formation:

$$T_{nuc} \approx T_{rec} \frac{2.22\ MeV}{13.6\ eV} = 6 \times 10^{8}\ K \Rightarrow t_{nuc} \approx 300\ s$$

D production was over a few minutes after the Big Bang!

(From M. Georganopoulos' lecture lib)

BBN: the neutron to proton decay

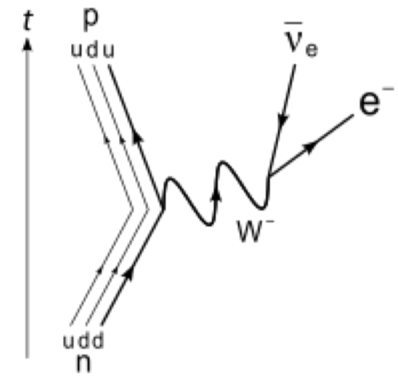
Neutron proton energy difference:

$$Q_n = m_n c^2 - m_p c^2 = 1.29\ MeV$$

Neutrons are unstable with decay time τ_n=~~890~~ s

881.5 s

$$n \rightarrow p + e^- + \bar{\nu}_e$$



Surviving fraction f:

$$f = \exp(-t/\tau_n)$$

After 1hr only 2% survive!

Neutrons survive practically for ever if they are locked in nuclei. All free neutrons will decay after a few τ_n

BBN must have taken place in the first few minutes!!!

(From M. Georganopoulos' lecture lib)

BBN: the neutron/proton ratio

Let's go back at a time when the mean photon energy was

E_{mean}=10 MeV, t=0.1 s.

At such high energies photons pair-produce:

$$\gamma + \gamma \leftrightarrow e^- + e^+$$

Neutrons and protons are in equilibrium via the reactions:

$$n + \nu_e \leftrightarrow p + e^-, \quad n + e^+ \leftrightarrow p + \bar{\nu}_e$$

Boltzmann's equation gives their number densities:

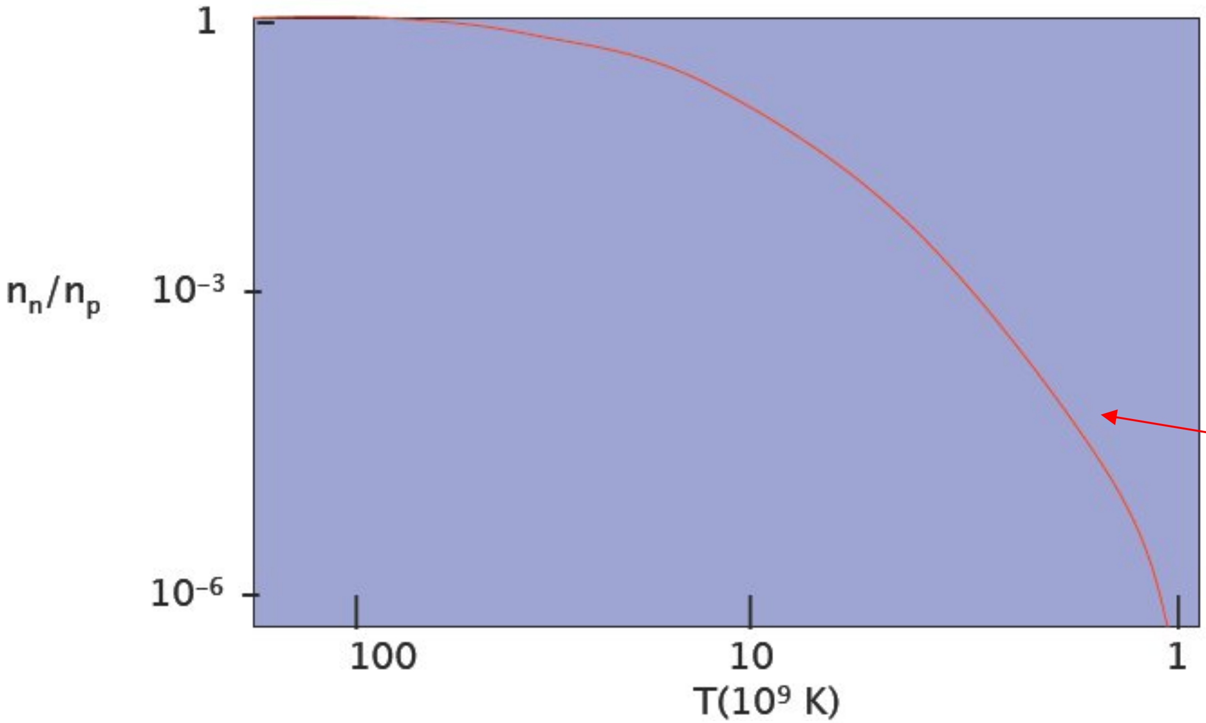
$$n_n = g_n \left(\frac{m_n kT}{2\pi\hbar^2} \right)^{3/2} \exp\left(-\frac{m_n c^2}{kT} \right) \qquad n_p = g_p \left(\frac{m_p kT}{2\pi\hbar^2} \right)^{3/2} \exp\left(-\frac{m_p c^2}{kT} \right)$$

(From M. Georganopoulos' lecture lib)

BBN: the neutron/proton ratio

Taking the ratio and setting $g_p = g_n = 2$, $m_p/m_n \approx 1$, we obtain:

$$\frac{n_n}{n_p} = \exp\left(-\frac{Q_n}{kT}\right), \quad Q_n = (m_n - m_p)c^2 = 1.29\ MeV$$



0.02s 2s 225s

As the temperature drops the lighter protons are favored.

Are we forgetting something?

(From M. Georganopoulos' lecture lib)

m_n= 1.674927471 x 10^{-27} kg
m_p= 1.672621898 x 10^{-27} kg

BBN: the neutron/proton ratio freezeout

In the early hot Universe, neutrons and protons are created and destroyed

$$n + \nu_e \leftrightarrow p + e^-, \quad n + e^+ \leftrightarrow p + \bar{\nu}_e$$

by means of the weak force, which has a typical cross-section:

$$\sigma_w \approx 10^{-47} \left(\frac{kT}{1\,MeV} \right)^2 m^2 \propto T^2 \propto a^{-2} \propto t^{-1}$$

The neutrino number density $n_\nu \propto a^{-3} \propto t^{-3/2}$

So the reaction rate is: $\Gamma = n_\nu c \sigma_w \propto t^{-5/2}$

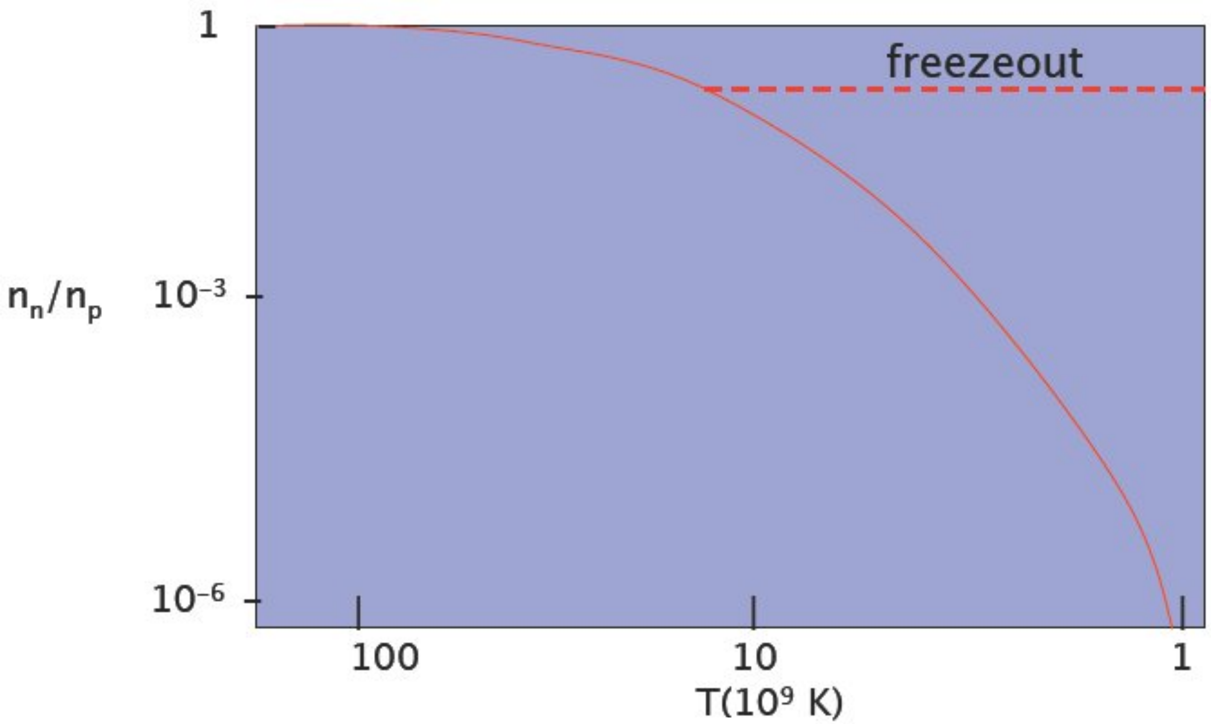
And, equating this to the Hubble parameter H α t^{-1}

$$kT_{freeze} = 0.8\ MeV \Rightarrow T_{freeze} = 9 \times 10^9\ K, \underline{t_{freeze} \approx 1\,\mathrm{sec}}$$

The neutron/proton ratio freezes out when the interaction rate is larger than the local Hubble time

Neutrino decoupling!

BBN: the neutron/proton ratio freezeout



0.02s   2s   225s

$$\frac{n_n}{n_p} = \exp\left(-\frac{Q_n}{kT_{freeze}}\right) \approx$$

$$\exp\left(\frac{-1.29\ MeV}{0.8\ MeV}\right) \approx 0.2$$

<u>One neutron for every five protons</u>

The neutron/proton ratio freezes out when the interaction rate is larger than the local Hubble time

(From M. Georganopoulos' lecture lib)

BBN: Deuterium production

Possible mechanisms:

1. $p + p \rightarrow D + e^{+} + \nu_{e}$ — Problems: Weak nuclear force (low cross-section), Coulomb barrier for protons

2. $n + n \rightarrow D + e^{-} + \overline{\nu}_{e}$ — Problems: Weak nuclear force, low neutron density

3. $p + n \leftrightarrow D + \gamma$ — Strong force, high cross section, dominant reaction for D-production

(From M. Georganopoulos' lecture lib)

BBN: Deuterium production

Best case scenario for BBN: fuse every neutron with a proton. Assume all neutrons end up in helium ^4He (2n+2p).

For every 2 neutrons there are 10 protons. There is one ^4He and 8 protons left. The mass fraction Y_p is the ratio of mass in ^4He over the total mass. Then

$$Y_{p,\max} = \frac{4}{12} = 0.33$$

In general if $f=n_n/n_p<1$, $Y_{p,max}=2f/(f+1)$

The observed value is $Y_p=0.24$. Why this discrepancy?

(From M. Georganopoulos' lecture lib)

BBN: Deuterium production

Best case scenario for BBN: fuse every neutron with a proton. Assume all neutrons end up in helium ^{4}He (2n+2p).

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In general if $f=n_n/n_p<1$, $Y_{p,max}=2f/(f+1)$

The observed value is $Y_p=0.24$. Why this discrepancy? Mostly because some neutrons decay before they form D and, through this He.

(From M. Georganopoulos' lecture lib)

BBN: Deuterium production

The time is now ~ 1 sec, the neutrinos have decoupled form p, n, e and they follow their way, in a manner similar to that of photons after recombination.

Photons are, of course, still coupled to matter and have enough energy to participate in the first major step in BBN:

$$p + n \leftrightarrow D + \gamma, \quad \varepsilon_\gamma \geq (m_n + m_p - m_D)c^2 = B_D = 2.22\ MeV$$

Assume thermodynamic equilibrium. Write down Boltzmann's equations for p, n, D, and take the ratio:

$$\frac{n_D}{n_p n_n} = \frac{g_D}{g_p g_n}\left(\frac{m_D}{m_p m_n}\right)^{3/2}\left(\frac{kT}{2\pi\hbar^2}\right)^{-3/2}\exp\left(\frac{[m_p + m_n - m_D]c^2}{kT}\right)$$

(From M. Georganopoulos' lecture lib)

BBN: Deuterium production

Using $g_D = 3, g_n = g_p = 2, \quad m_p = m_n = m_D/2, \quad (m_n + m_p - m_D)c^2 = B_D$

we obtain:

$$\boxed{\frac{n_D}{n_p n_n} = 6\left(\frac{m_n kT}{\pi\hbar^2}\right)^{-3/2} \exp\left(\frac{B_D}{kT}\right)}$$

At T->0 we have only D, while at T->∞ we have only n and p.

Deuterium production, as recombination is a gradual process.

Nucleosynthesis temperature T_{nuc}: the temperature at which the neutrons have been locked into D nuclei: $n_D=n_n$ .Then:

$$1 = n_p 6\left(\frac{m_n kT}{\pi\hbar^2}\right)^{-3/2} \exp\left(\frac{B_D}{kT}\right)$$

But how much is n_p?

(From M. Georganopoulos' lecture lib)

BBN: Deuterium production

Here, a familiar quantity comes in:
η, the time invariant baryon to photon ratio

We know that at t~1 sec 5 out of 6(~83%) baryons were protons.
We also know that by the end of nucleosynthesis ~75% of all baryons are unbound protons. Without feeling very guilty, we assume that ~80% of all baryons are "free running" protons:

$$n_p \approx 0.8\, n_{baryons} = 0.8\,\eta\, n_\gamma = 0.8\eta\left[0.243\left(\frac{kT}{\hbar c}\right)^3\right]$$

The last step comes from integrating $\varepsilon(\nu)/h\nu$ of the blackbody over all frequencies.

(From M. Georganopoulos' lecture lib)

BBN: Deuterium production

Using $n_p \approx 0.8\eta\left[0.243\left(\frac{kT}{\hbar c}\right)^3\right]$

we write: $$1 \approx 6.5\eta\left(\frac{kT_{nuc}}{m_n c^2}\right)^{3/2} \exp\left(\frac{B_D}{kT_{nuc}}\right)$$

Setting: $m_n c^2 = 939.6\ MeV,\ B_D = 2.22\ MeV,\ \eta = 5.5\times10^{-10}$

we obtain: $\boxed{kT_{nuc} \approx 0.066\ MeV,\quad T_{nuc} \approx 7.6\times10^8\ K,\ t_{nuc} \approx 200\ s}$

Compare this to the neutron decay time τ_n=890 s. At $t=t_{nuc}$ only a fraction $\exp(-t_{nuc}/\tau_n)$=0.8 of the neutrons survives.

$$f = \frac{n_n}{n_p} \approx \frac{0.8^{\times 0.2}}{1+(1-.8)} = 0.133 \Rightarrow Y_{p.\max} = \frac{2f}{f+1} \approx 0.235$$

Closer to the observed Y=0.24!

$n_n = (0.2 \text{ x } n_p) \text{ x } 0.8$ $\quad$ $n_p = (1+0.2) \text{ x } n_p$

(From M. Georganopoulos' lecture lib)

BBN: Heavier nuclei

So, by t~200 s we managed to form a substantial amount of D. What's next?

Several nuclear reactions take place producing ^{3}H and ^{3}He:

$$D + p \leftrightarrow {}^3He + \gamma, \quad D + n \leftrightarrow {}^3H + \gamma, \quad D + D \leftrightarrow {}^4He + \gamma,$$

$$D + D \leftrightarrow {}^3H + p, \quad D + D \leftrightarrow {}^3He + n$$

^{3}H and ^{3}He are then converted to ^{4}He by reactions such:

$$^3H + p \leftrightarrow {}^4He + \gamma, \quad {}^3He + n \leftrightarrow {}^4He + \gamma,$$

$$^3H + D \leftrightarrow {}^4He + n, \quad {}^3He + D \leftrightarrow {}^4He + p$$

All these involve the strong force (note, no neutrinos!), and are therefore fast. Once you start building D, you very quickly convert it to ^{4}He.

(From M. Georganopoulos' lecture lib)

BBN: Heavier nuclei

There are no stable nuclei with mass number A=5.
Small amounts of ^6Li and ^7Li form through:

$$^4He + D \leftrightarrow {}^6Li + \gamma, \quad {}^4He + {}^3H \leftrightarrow {}^7Li + \gamma$$

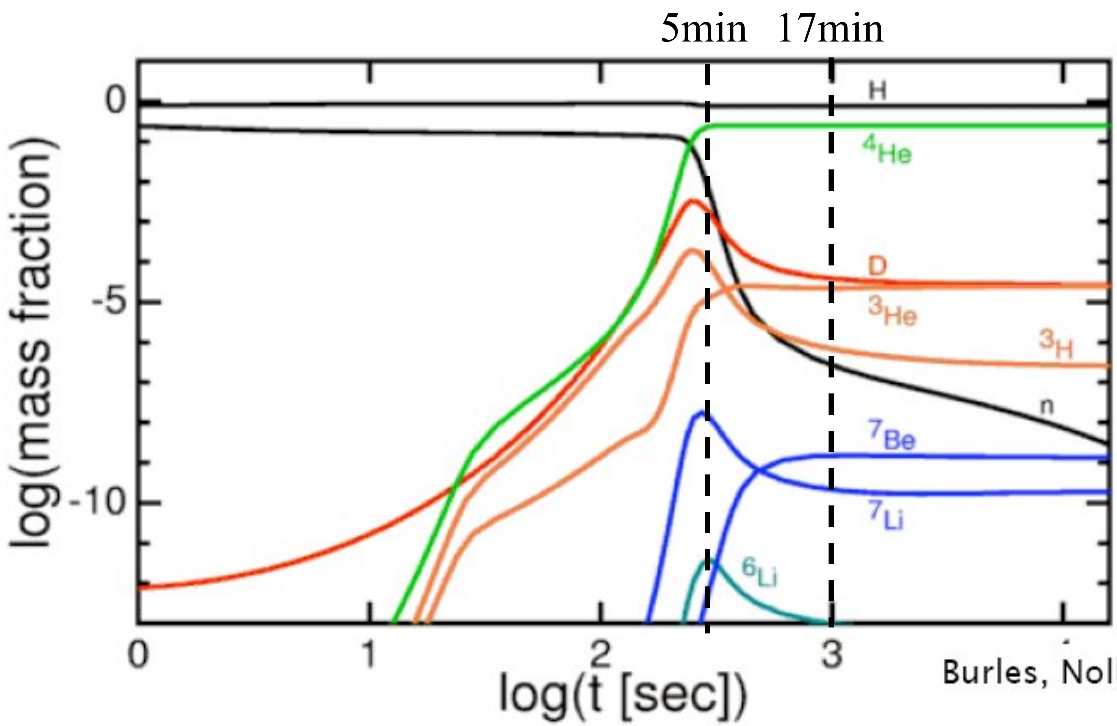
Small amounts of ^7Be form through:

$$^4He + {}^3He \leftrightarrow {}^7Be + \gamma$$

There are no stable nuclei with A=8.
As soon as we form D, He follows rapidly,
but after that heavier elements are produced much slower and with much smaller efficiency.

(From M. Georganopoulos' lecture lib)

BBN: Heavier nuclei



Burles, Nollett & Turner (1999)

The mass fraction of each element can be calculated numerically as the Universe expands and cools. The evolution and final mass fraction ratios depend on the baryon to photon ration η.

<u>Nucleosynthesis is practically over after a couple of hours.</u>

(From M. Georganopoulos' lecture lib)

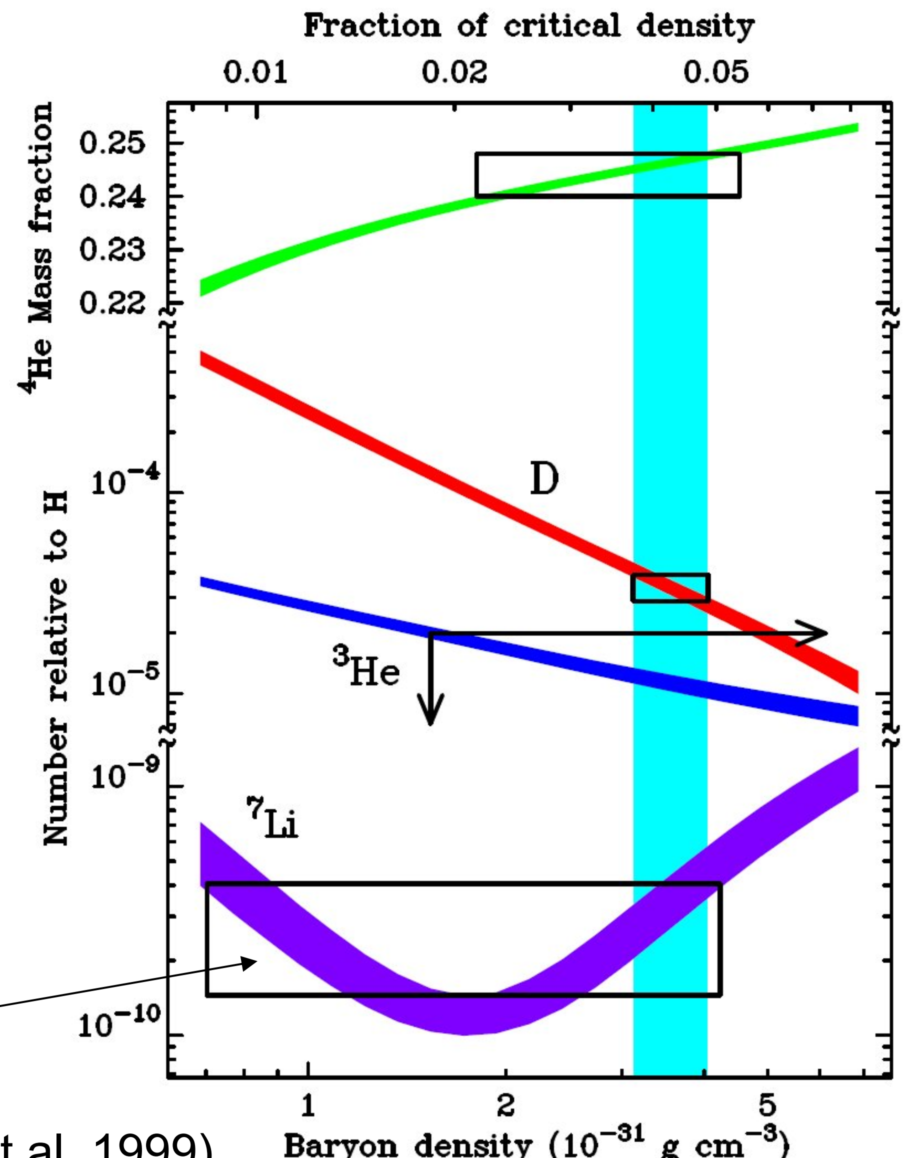
BBN: measurement of Ω_B

Impressive agreement over 9 orders of magnitude.

Higher η implies a higher T_{nuc}, and nucleosynthesis would start before, increasing ^{4}He, and reducing D and ^{3}He

^{7}Li is produced through fusion of ^{4}He and ^{3}H and also through e^- capture by ^{7}Be. As η increases, the 2nd channel gets more efficient.

This has dramatically changed in the last decade.



(Burles et al. 1999)

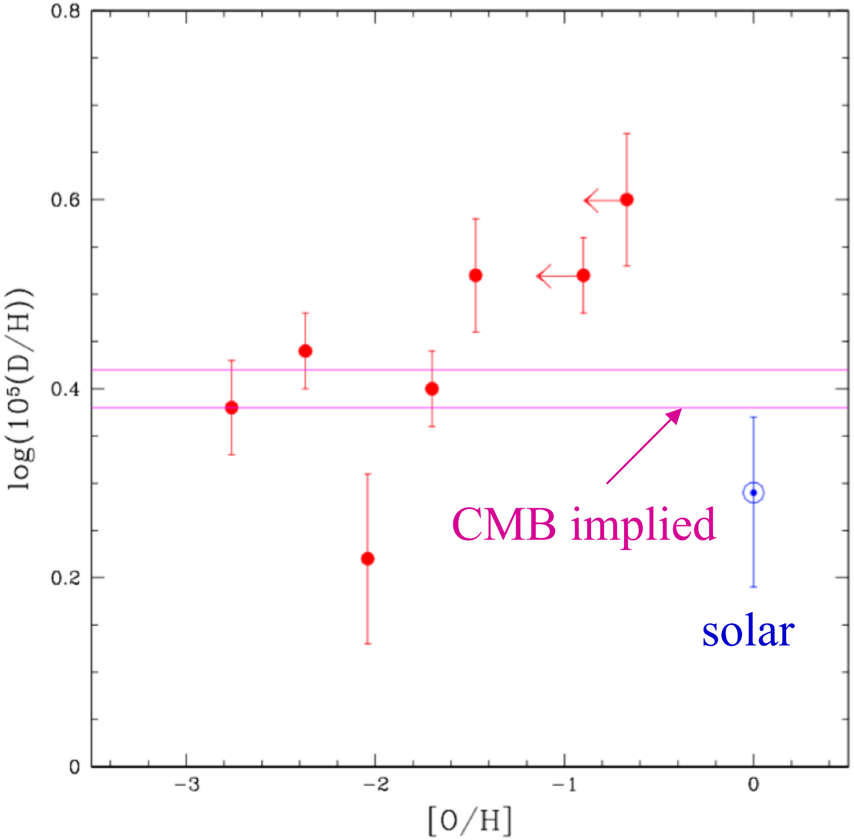
BBN: measurement of Ω_B

BBN only produces D, ^{3}H, ^{3}He,^{4}He, ^{6}Li, ^{7}Li, ^{7}Be

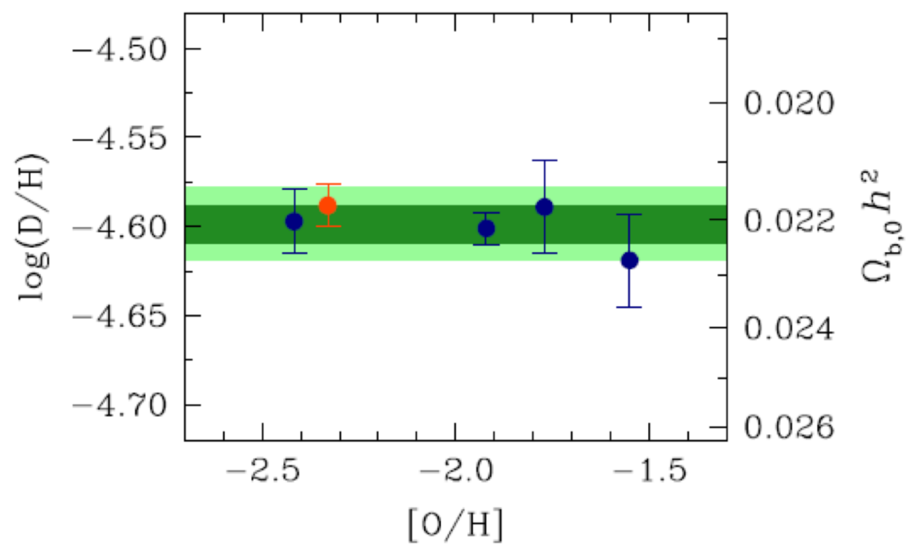
Deuterium: measured in QSO absorption-line systems, comparing the HI and DI column densities.

• advantage: it only gets destroyed in stars $(D/H)_{obs} < (D/H)_P$

• disadvantage, very difficult to measure, only 7 systems.



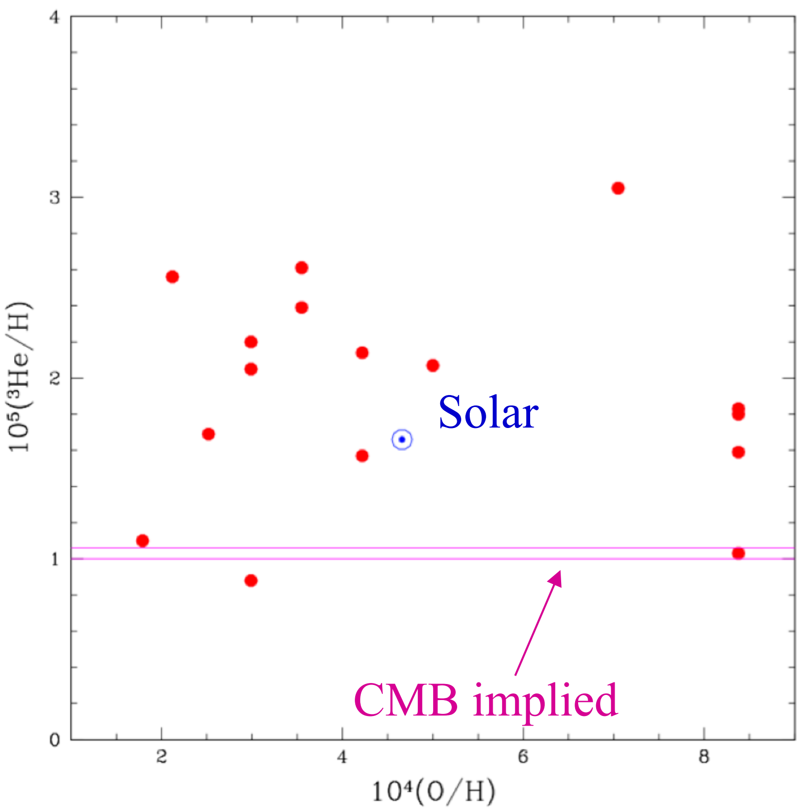
η_{10} (SBBN) = 5.80 ± 0.28

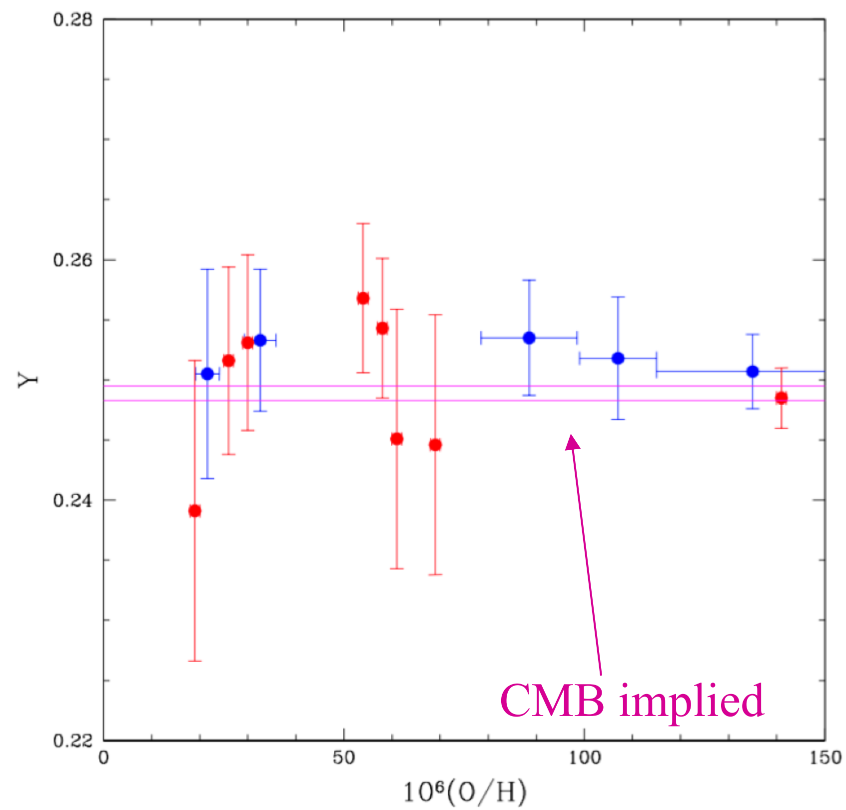


(Pettini et al. 2008, Steigman 2009, Cooke et al 2014: D/H=(2.53±0.04)x10^{-5})

BBN: measurement of Ω_B

^{3}He and ^{4}He: measured in galactic and extragalactic HII regions
• advantage: it can be measured in many systems
• disadvantage: it increases, one has to look for "primeval" regions
but ^{3}He only good signal in galactic polluted regions.

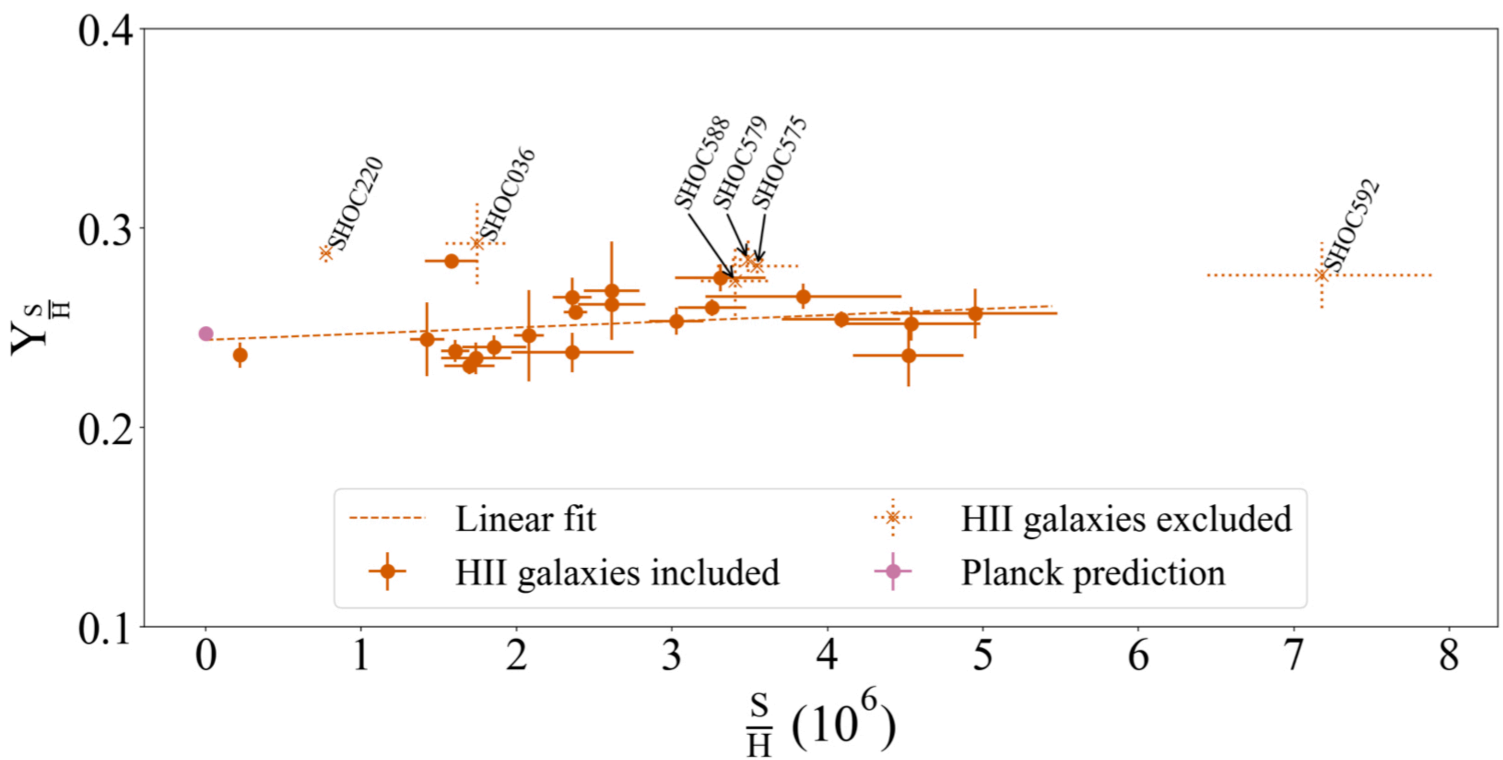




(Steigman 2009)

BBN: measurement of Ω_B

^{3}He and ^{4}He: Fernández et al. (2018) measured in 27 HII galaxies using S to estimate metallicity $Y_p = 0.244 \pm 0.006$ (Y_p[O-N-S] = 0.245 ± 0.007). In agreement with Peimbert et al. (2016) using O to trace metallicity.



(Fernández et al. 2018)

BBN+CMB: measurement of Ω_B

CMB-Planck+WP+BAO +BBN 95% confidence consistency, excluding ^{7}Li (Nollet & Steigman 2013):

$\eta_{10} = 6.13 \pm 0.07$

$\Omega_B h^2 = 0.0224 \pm 0.0003$

$N_{eff} = 3.46 \pm 0.17$

Planck (Ade et al. 2013):

$\eta_{10} = 6.11 \pm 0.08$

$\Omega_B h^2 = 0.0223 \pm 0.0003$

$N_{eff} = 3.30 \pm 0.27$

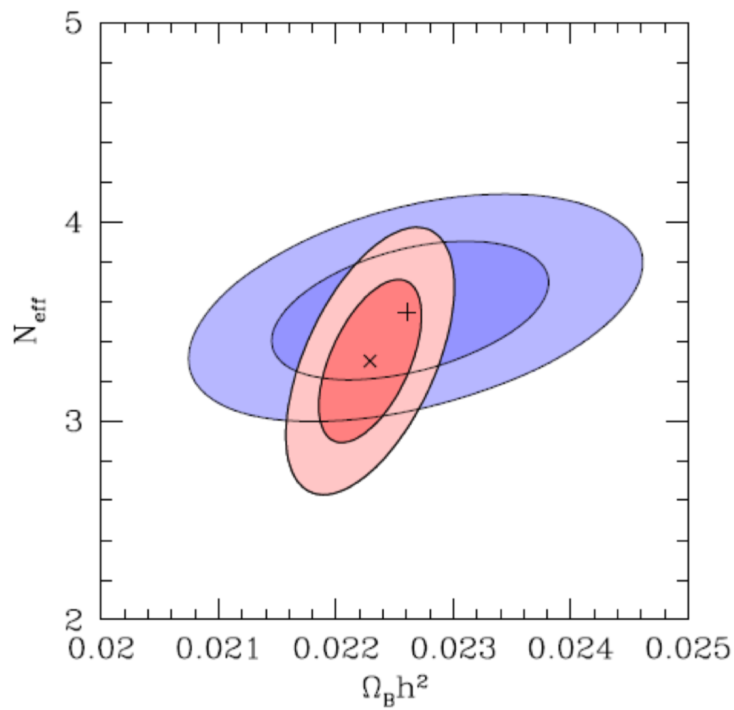
and BBN (Nollet & Steigman 2013)

$\eta_{10} = 6.19 \pm 0.21$

$\Omega_B h^2 = 0.0226 \pm 0.0008$

Neff = 3.56 ± 0.23

are consistent.

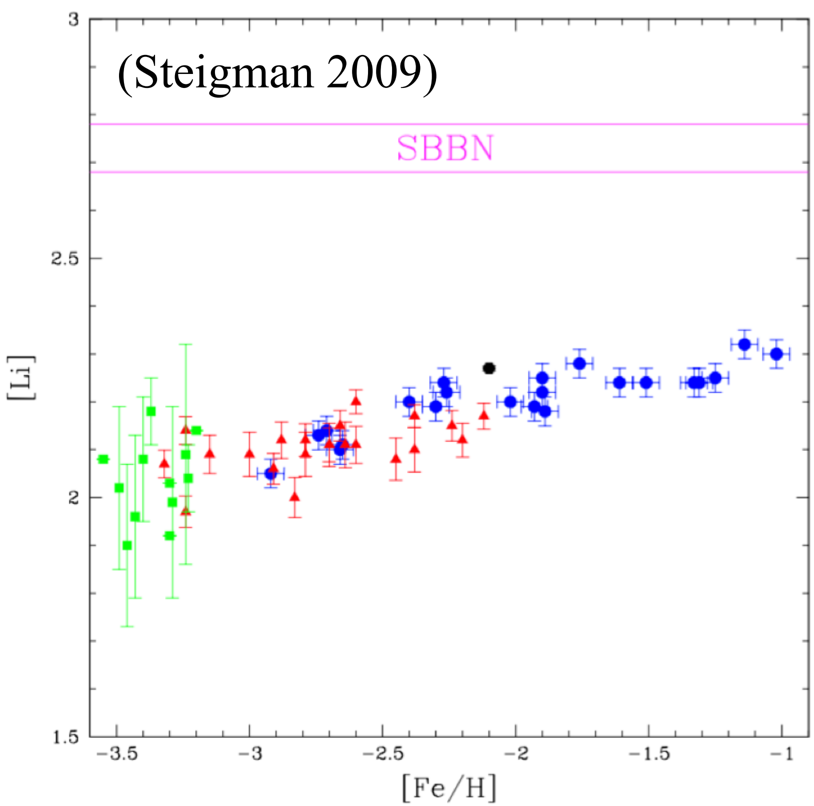


BBN: measurement of Ω_B

BBN only produces D, ^{3}H, ^{3}He,^{4}He, ^{6}Li, ^{7}Li, ^{7}Be

^{7}Li: measured in low-Z galactic halo and globular cluster stars

• disadvantage: produced by cosmic-ray nucleosynthesis, destroyed in the interior of stars, net effect observed of production is expected



Most outstanding problem in Standard Big Bang Nucleosynthesis a factor of ~3 difference between observations and CMB or predictions derived from SBBN interpretation of lighter elements. Some solutions have been proposed (e.g. Korn et al. 2006)

Latest measurements and numerical estimates seem to converge, but a x2.27 discrepancy remains. (Singh et al. 2017)

NOT THE END …

Happy Birthday!

Kurt Donald **Cobain** (Aberdeen, Washington, 20 de febrero de 1967-Seattle, Washington, 5 de abril de 1994)