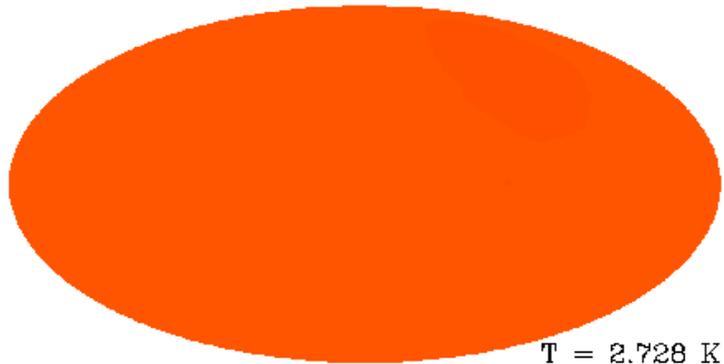
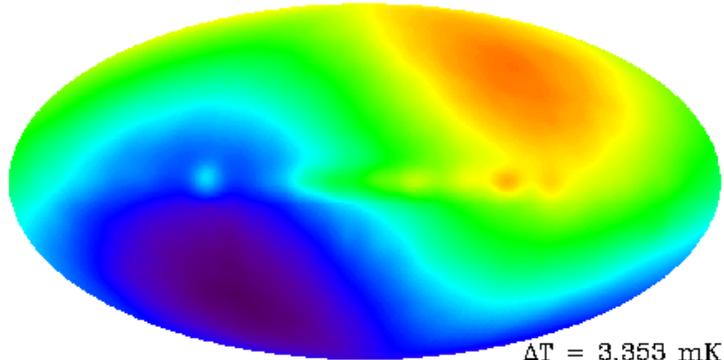


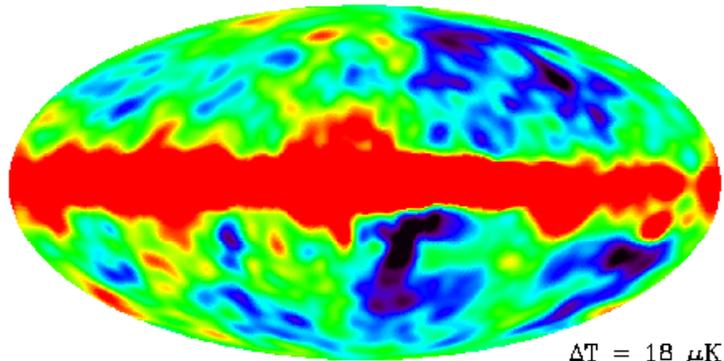
# CMB spectrum: dipole anisotropy



$T = 2.728 \text{ K}$



$\Delta T = 3.353 \text{ mK}$



$\Delta T = 18 \text{ } \mu\text{K}$

Dipole anisotropy in COBE data can be explained as a Doppler effect between the frame of reference of the solar system and that at rest with the observable CMB.

$$v' = \gamma(1 - \beta \cos \theta)v, \quad \text{with } \beta \equiv v/c$$

$$\text{and } \gamma \equiv 1/\sqrt{1 - \beta^2}$$

$$T(\theta) = T_0 / \gamma(1 - \beta \cos \theta) \approx T_0 + T_0 \beta \cos \theta$$

A fit to the image  $T_0 \beta = 3353 \pm 24 \mu\text{K}$   
And with  $T_0 = 2.735 \text{ K}$

$$\vec{v}_{sun} - \vec{v}_{CMB} = 369 \pm 3 \text{ km s}^{-1}$$

Taking into account the movement around the MW, and the movement of the LG

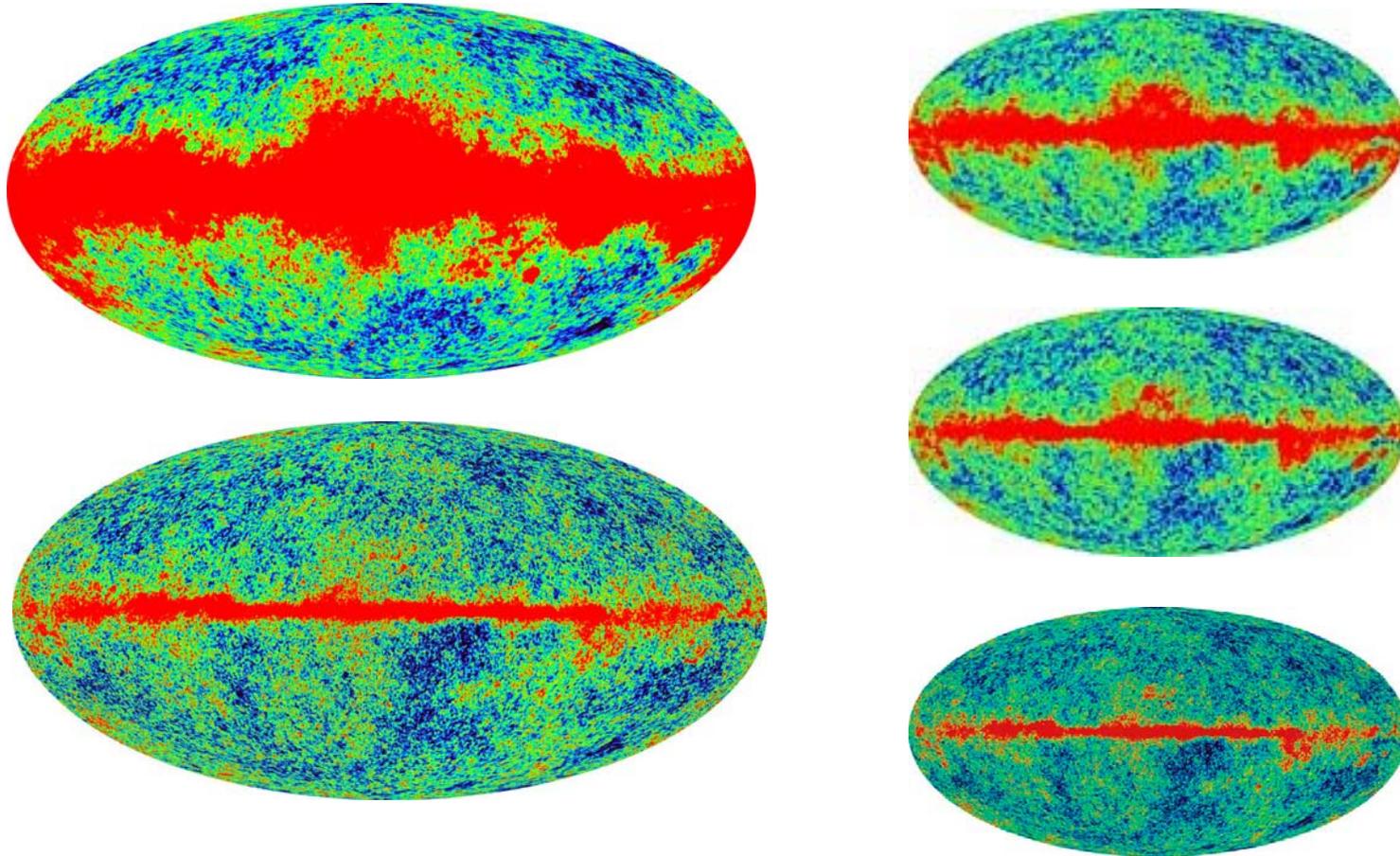
towards  $(l, b) \approx (277^\circ, 30^\circ)$

Signature of local attractors.

$$\vec{v}_{LG} - \vec{v}_{CMB} \approx 620 \pm 45 \text{ km s}^{-1}$$

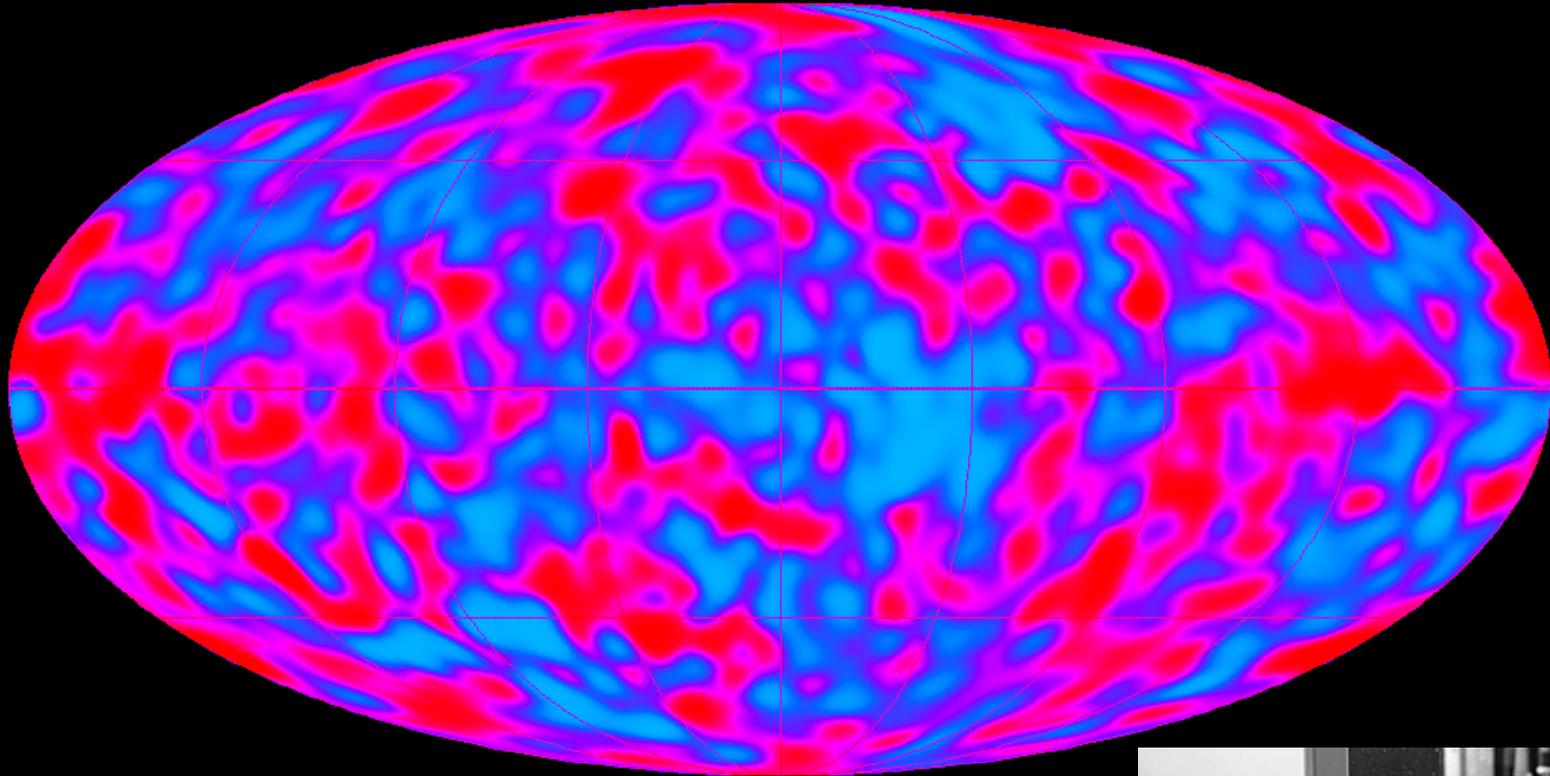
(Following E. Wright's CMB review paper)

# CMB spectrum: removing the galaxy



WMAP: 23 to 90 GHz image of the CMB after dipole subtraction. The galaxy emission is dominated by dust

# CMB spectrum: removing the galaxy

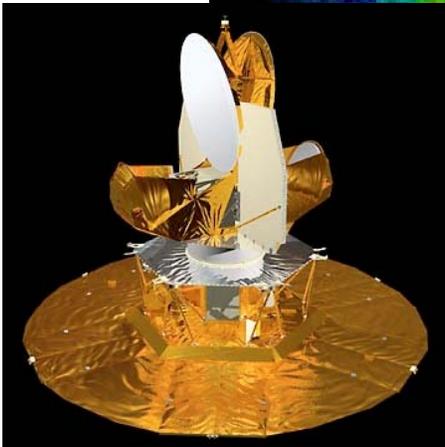
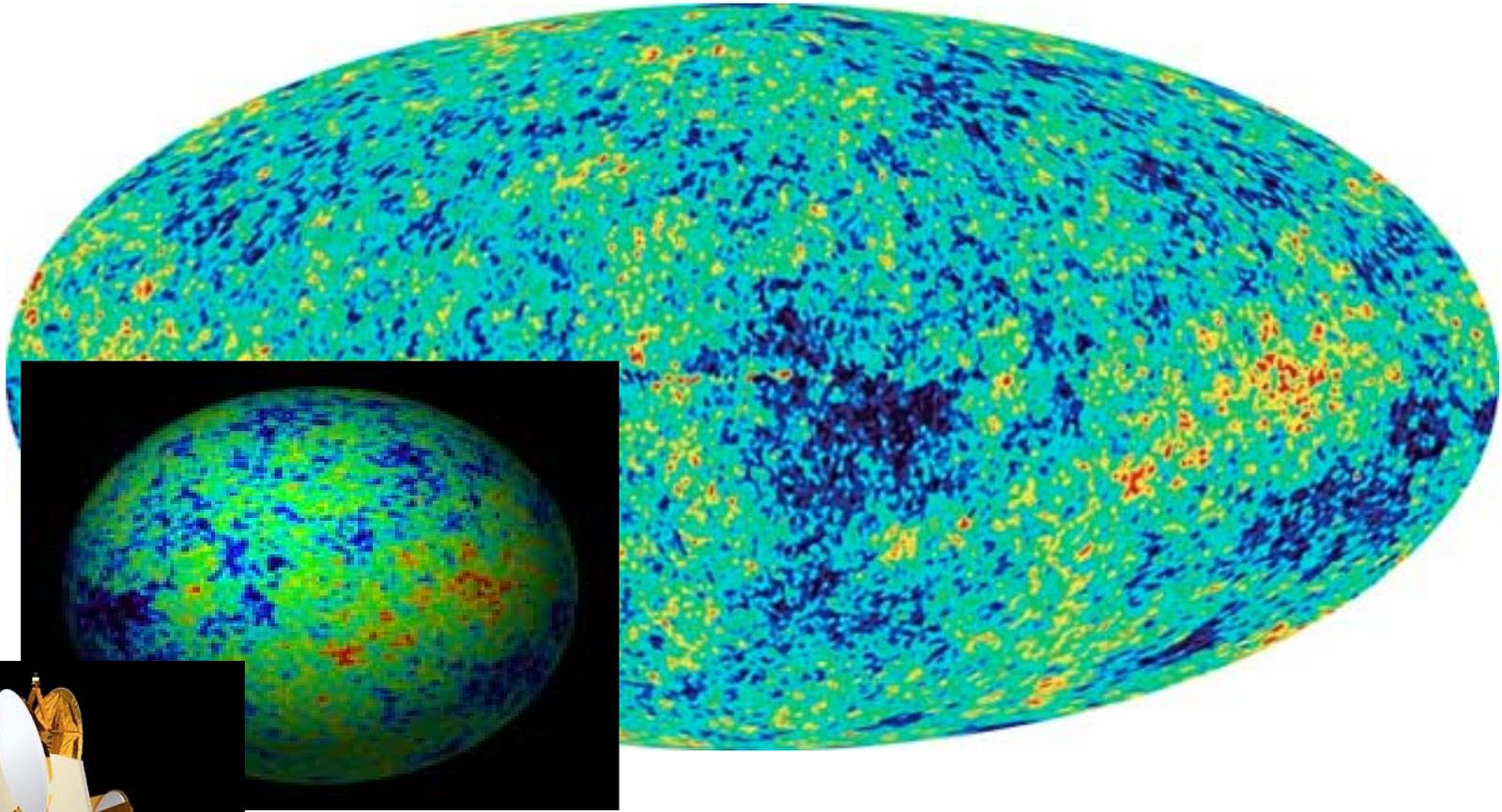


Cosmic Background  
Explorer COBE  
(1992):

$$\Delta T/T = 10^{-5}$$



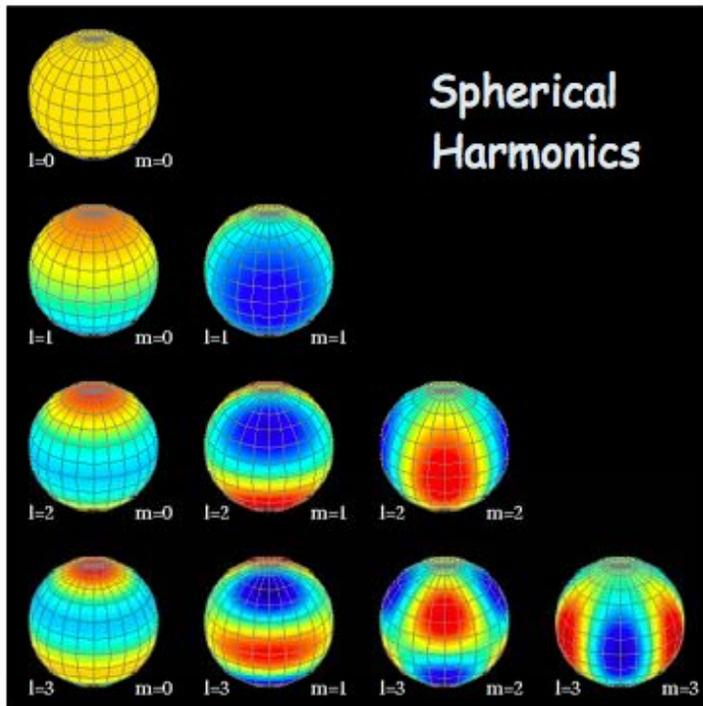
# CMB spectrum: removing the galaxy



WMAP full sky view

# CMB spectrum: statistical properties

$T(l,b)$  can be fully specified by either the angular correlation function  $C(\theta)$  or its Legendre transformation, the angular power spectrum  $C_l$ ,

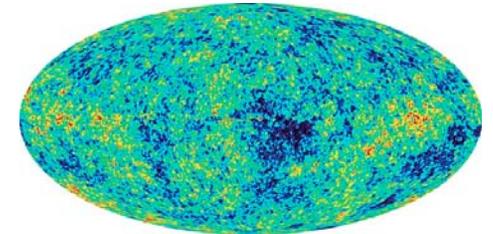


Write the temperature fluctuations as a series of spherical harmonics:

$$\frac{\delta T(\theta, \phi)}{T} = \sum_{l=0}^{\infty} \sum_{m=-l}^l a_{lm} Y_{lm}(\theta, \phi)$$

The correlation function, a CMB map derived function:

$$C(\theta) = \left\langle \frac{\delta T(\hat{n})}{T} \frac{\delta T(\hat{n}')}{T} \right\rangle_{\hat{n}\hat{n}'=\theta}$$



$$a_{\ell m} = \int \frac{\Delta T(\theta, \phi)}{T_{\text{CMB}}} Y_{\ell m}(\theta, \phi) d\Omega$$

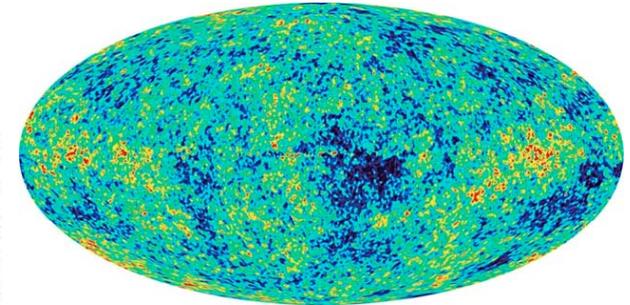
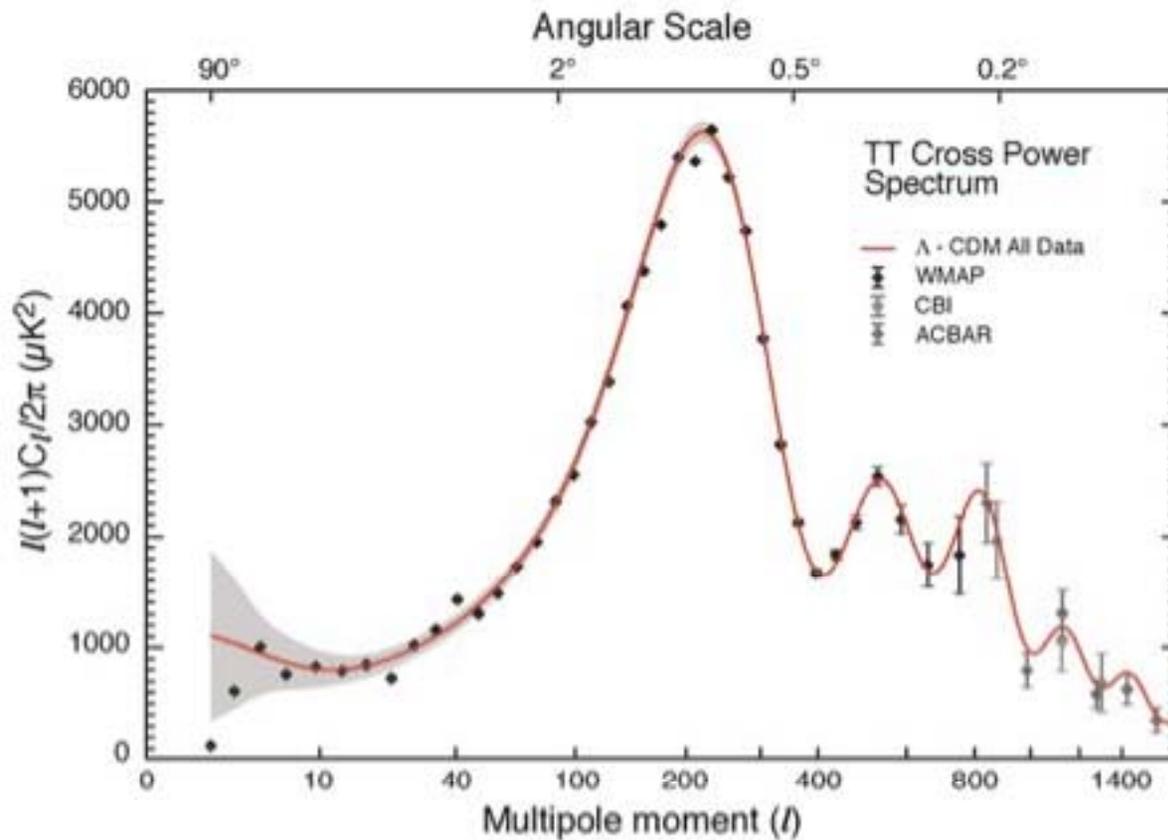
$$C_\ell = \langle |a_{\ell m}|^2 \rangle$$

Which are related by  $C(\theta) = \frac{1}{4\pi} \sum_l (2l+1) C_l P_l(\cos \theta)$  where  $P_l$  are Legendre polynomials of order  $l$ .

A term  $C_l$  is a measure of angular fluctuations on the angular scale  $\theta \sim 180^\circ/l$ .

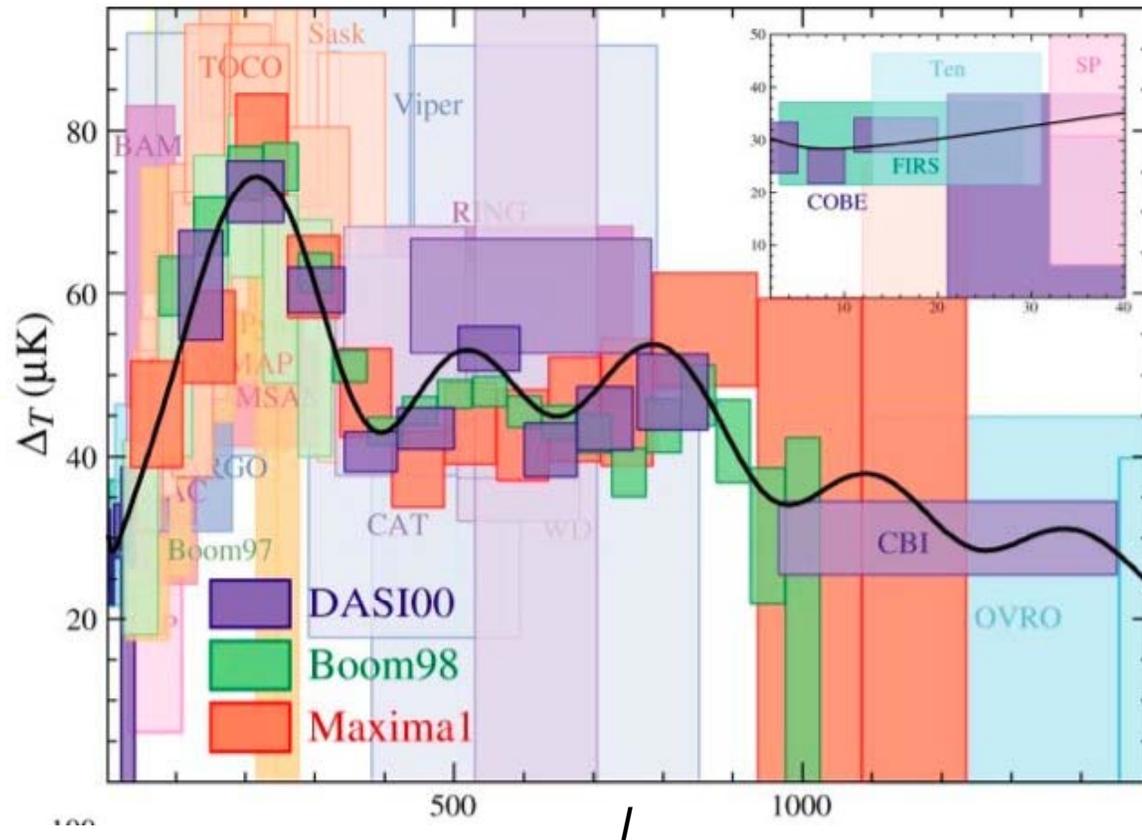
In order to have equal power for all scales the spherical harmonics impose  $C_l = \text{cte}/l(l+1)$ . If the sky had equal power on all scales  $l C_l/(l+1)$  should be a constant.

# CMB spectrum: power spectrum



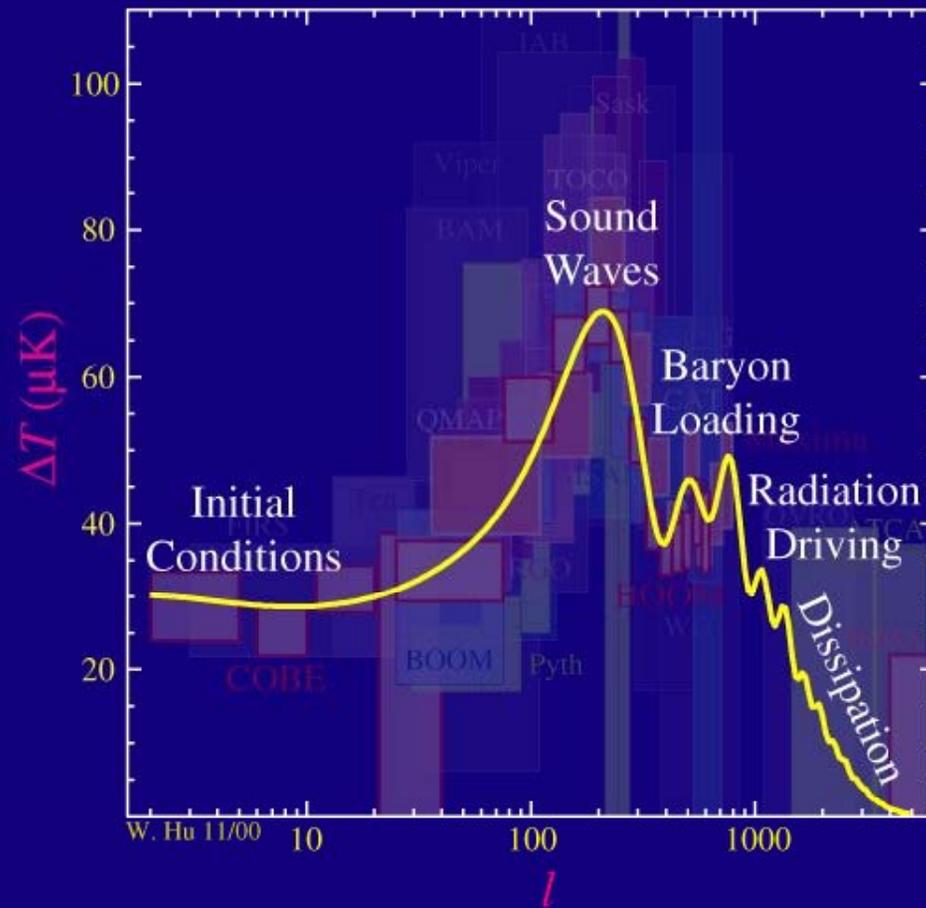
Why is there a major peak at  $l \sim 200$  ( $\theta \sim 1^\circ$ )?  
And what are the secondary peaks?

# CMB spectrum: power spectrum



First peak had already been constrained by an array of 1992-2000 missions, and sampled in its full amplitude by Boomerang (1998) & Maxima (2000)

# CMB spectrum: power spectrum



**Soundscape:** COBE measured the temperature fluctuations  $\Delta T$  on the largest angular scales which correspond to multipoles as roughly  $l=100/\text{angle (degrees)} \sim 2-20$ . The current generation of experiments are measuring multipoles  $l > 100$  where the acoustic peaks are expected to dominate the scene (yellow curve). The physical landscape described in these pages begins with sound waves and proceeds through baryon loading, radiation driving, and dissipation by diffusion damping. In the background, are the measurements as of January 2001.

(From Hu's webpage)

# CMB spectrum: power spectrum

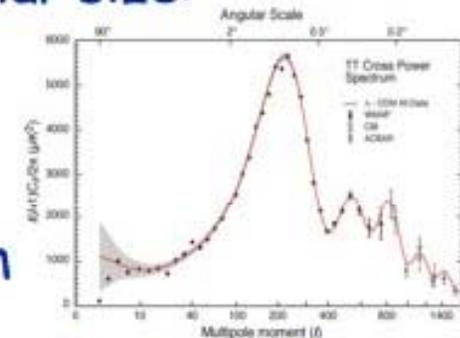
At the time of last scattering the Universe was matter dominated and the Hubble distance was:

$$\frac{c}{H(z_{ls})} = \frac{c}{H_0(z+1)^{3/2}} = \frac{3 \times 10^8 \text{ m s}^{-1}}{1.24 \times 10^{-18} \text{ s}^{-1} (1101)^{3/2}} \approx 6.6 \times 10^{21} \text{ m} \approx 0.2 \text{ Mpc}$$

Seen from Earth this has an angular size:

$$\theta_H = \frac{c/H(z_{ls})}{d_A} \approx \frac{0.2 \text{ Mpc}}{13 \text{ Mpc}} \approx 0.015 \text{ rad} \approx 1^\circ$$

This is the first peak of the power spectrum



The Hubble distance at the time of decoupling is  $\sim$  the maximum length information (photons, matter) could have traveled. The fluctuations at  $\theta > \theta_H$  and  $\theta < \theta_H$  must reveal different things.

# CMB spectrum: power spectrum

At the time of last scattering the Universe, the non-baryonic dark matter dominates:

$$\Omega_{dm} : \Omega_{\gamma} : \Omega_m = 6.4 : 1.4 : 1$$

If the dark matter density distribution at the time of last scattering has a spatially varying component  $\delta\rho$ , then there is a spatially varying gravitational potential  $\delta\Phi$  given by Poisson's equation:

$$\nabla^2(\delta\Phi) = 4\pi G\delta\rho$$

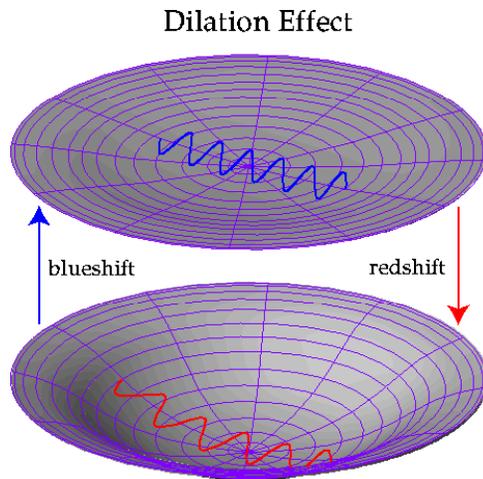
# CMB spectrum: power spectrum $\theta > \theta_H$

If at the time of last scattering a CMB photon is at a local minimum (in a "potential well"), it will spend energy to climb out of the well and it will be redshifted.

If it is at a local maximum, it will roll down the maximum and gain energy (it will be blueshifted). In general, Sachs and Wolfe (1967) showed that:

$$\frac{\delta T}{T} = \frac{1}{3} \frac{\delta \Phi}{c^2}$$

All these are valid for angular scales  $\theta > \theta_H$ , because the photon-baryon fluid has no time to move further than  $\theta_H$ . At smaller scales, we need to consider the fact that the photon-baryon fluid has time to move substantially under the influence of the gravity of dark matter.



# CMB spectrum: power spectrum $\theta < \theta_H$

The photon-baryon fluid moves under the influence of the non baryonic matter that dominates. When the fluid finds itself in a potential of dark matter, it flows toward the bottom of the well.

As it falls its pressure rises. As the pressure rises, it slows down, stops and inverts the fluid fall. The fluid expands, and a series of oscillations takes place, called "acoustic oscillations".

The oscillations stop at recombination, when photons decouple from baryons. How many oscillations take place? The larger the well the fewer the oscillations. For  $\theta \sim \theta_H$  there is time for  $\sim 1$  oscillation.

If the photon-baryon fluid is compressed in a well at the time of recombination, the decoupled photons will be hotter than average.

If the photon-baryon fluid is expanded at the time of recombination, the decoupled photons will be cooler than average.

Graphic by Wayne Hu, <http://background.uchicago.edu/~whu/beginners/introduction.html>

The highest peak at  $\theta \sim \theta_H$  ( $l \sim 200$ ) represents the wells where the photon-baryon fluid had just reach maximum compression at the recombination time.

(From M. Georganopoulos' lecture lib)

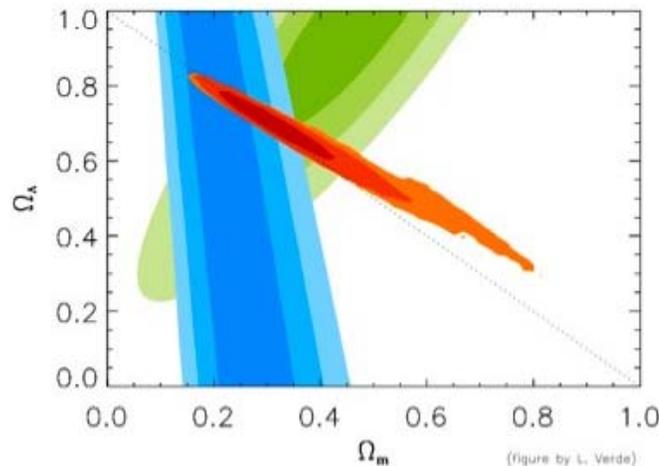
# CMB spectrum: 1st peak of power spectrum

In a negatively curved Universe, the angular size of an object of known intrinsic size at a given redshift is smaller than it is in a positively curved Universe.

If the Universe were negatively curved, the first peak would be seen at  $l > 180$  or  $\theta < 1^\circ$ .

If the Universe were positively curved, the first peak would be seen at  $l < 180$  or  $\theta > 1^\circ$ .

The observe peak at  $l \sim 200$  is consistent with  $k=0$  or  $\Omega_0=1$



The observe peak at  $l \sim 200$  is consistent with  $k=0$  or  $\Omega_m + \Omega_\Lambda = 1$

Supernovae, large scale  
structure, CMB

(From M. Georganopoulos' lecture lib)

# CMB spectrum: 1st peak of power spectrum

The amplitude of the first peak depends on the sound speed of the photon baryon fluid:

$$c_s = c \sqrt{w_{pb}}$$

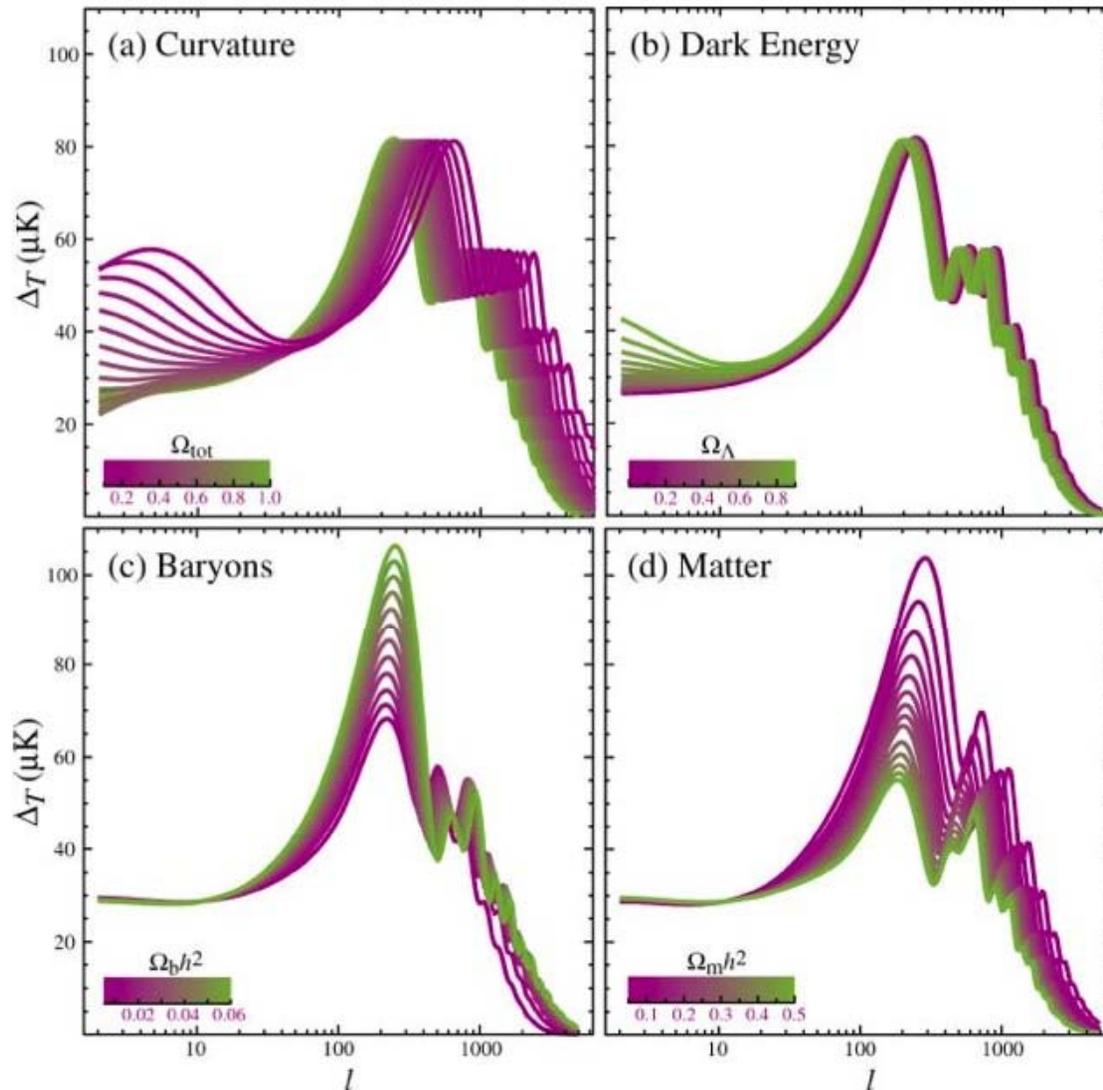
The equation of state,  $p_{pw} = w_{pb} \rho$  depends of the baryon to photon ratio. To reproduce the power spectrum we need

$$\Omega_{bary,0} = 0.04 \pm 0.02.$$

How nice! This is in good agreement with the nucleosynthesis result!

There is much more to the CMB power spectrum.

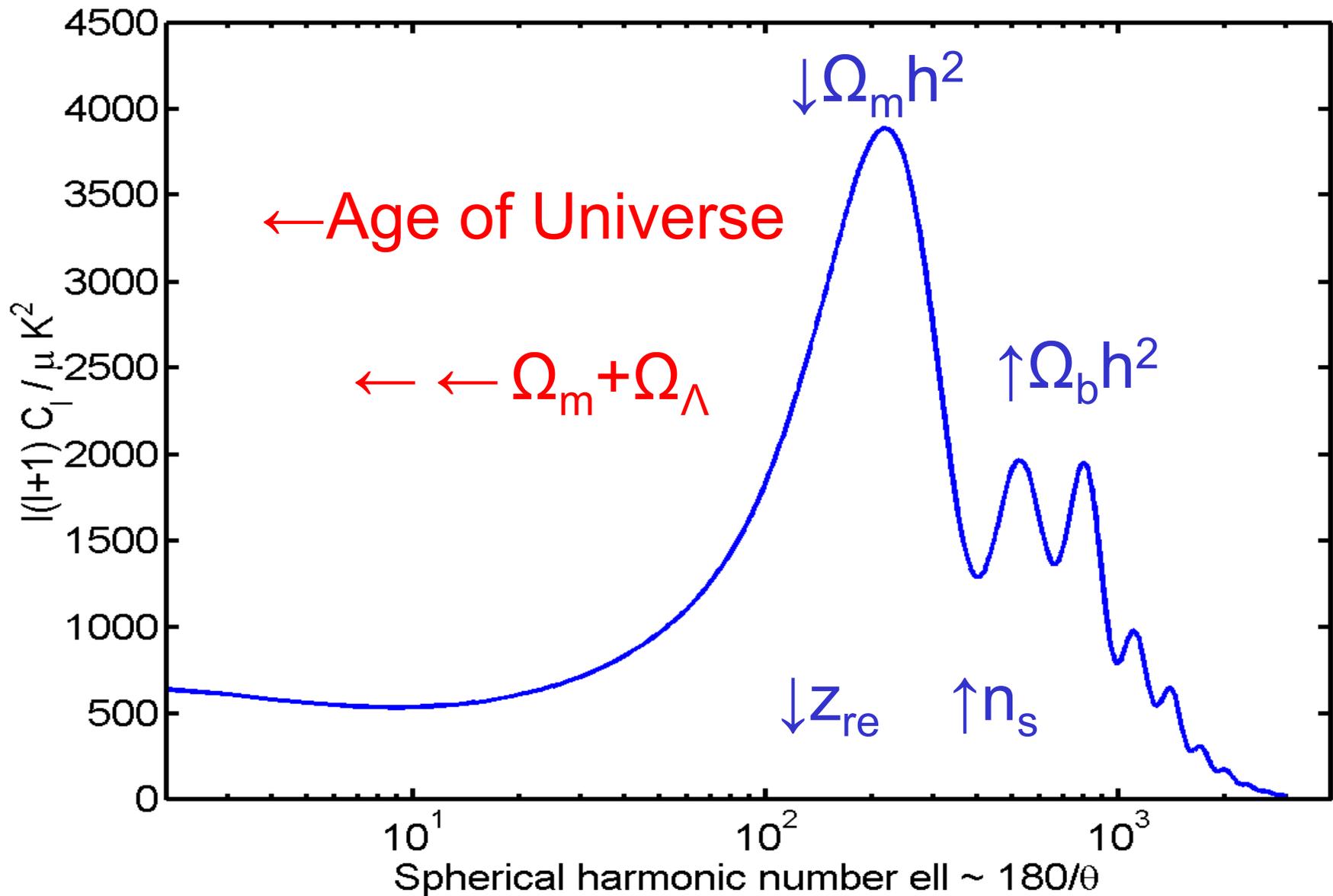
# CMB spectrum: cosmic dependences



Varied around a fiducial model  $\Omega_{\text{tot}}=1$ ,  $\Omega_{\Lambda}=0.65$ ,  $\Omega_{\text{B}}=0.02h^2$ ,  $\Omega_{\text{m}}=0.147h^2$ ,  $n=1$ ,  $z_{\text{ri}}=0$ ,  $E=0$

(Hu & Dodelson 2002)

# CMB spectrum: cosmic dependences



# WMAP spectrum: precision cosmology

Table 2: Power Law  $\Lambda$ CDM Model Parameters and 68% Confidence Intervals. The Three Year fits in this Table assume no SZ contribution,  $A_{SZ} = 0$ , to allow direct comparison with the First Year results. Fits that include SZ marginalization are given in Table 5 (first column) and represent our best estimate of these parameters.

Parameter	First Year Mean	WMAPext Mean	Three Year Mean	First Year ML	WMAPext ML	Three Year ML
$100\Omega_b h^2$	$2.38^{+0.13}_{-0.12}$	$2.32^{+0.12}_{-0.11}$	$2.23 \pm 0.08$	2.30	2.21	$10^2\Omega_b h^2 = 2.233^{+0.072}_{-0.091}$
$\Omega_m h^2$	$0.144^{+0.016}_{-0.016}$	$0.134^{+0.006}_{-0.006}$	$0.126 \pm 0.009$	0.145	0.138	0.128
$H_0$	$72^{+5}_{-5}$	$73^{+3}_{-3}$	$74^{+3}_{-3}$	68	71	73
$\tau$	$0.17^{+0.08}_{-0.07}$	$0.15^{+0.07}_{-0.07}$	$0.093 \pm 0.029$	0.10	0.10	0.092
$n_s$	$0.99^{+0.04}_{-0.04}$	$0.98^{+0.03}_{-0.03}$	$0.961 \pm 0.017$	0.97	0.96	0.958
$\Omega_m$	$0.29^{+0.07}_{-0.07}$	$0.25^{+0.03}_{-0.03}$	$0.234 \pm 0.035$	0.32	0.27	0.24
$\sigma_8$	$0.92^{+0.1}_{-0.1}$	$0.84^{+0.06}_{-0.06}$	$0.76 \pm 0.05$	0.88	0.82	0.77

Table 6:  $\Lambda$ CDM Model

Parameter	WMAP+ SDSS	WMAP+ LRG	WMAP+ SNLS	WMAP + SN Gold	WMAP+ CFHTLS
$100\Omega_b h^2$	$2.233^{+0.062}_{-0.086}$	$2.242^{+0.062}_{-0.084}$	$2.233^{+0.069}_{-0.088}$	$2.227^{+0.065}_{-0.082}$	$2.255^{+0.062}_{-0.083}$
$\Omega_m h^2$	$0.1329^{+0.0056}_{-0.0073}$	$0.1337^{+0.0044}_{-0.0061}$	$0.1295^{+0.0056}_{-0.0072}$	$0.1349^{+0.0056}_{-0.0071}$	$0.1408^{+0.0034}_{-0.0050}$
$h$	$0.709^{+0.024}_{-0.032}$	$0.709^{+0.016}_{-0.023}$	$0.723^{+0.021}_{-0.030}$	$0.701^{+0.020}_{-0.026}$	$0.687^{+0.016}_{-0.024}$
$A$	$0.813^{+0.042}_{-0.052}$	$0.816^{+0.042}_{-0.049}$	$0.808^{+0.044}_{-0.051}$	$0.827^{+0.045}_{-0.053}$	$0.846^{+0.037}_{-0.047}$
$\tau$	$0.079^{+0.029}_{-0.032}$	$0.082^{+0.028}_{-0.033}$	$0.085^{+0.028}_{-0.032}$	$0.079^{+0.028}_{-0.034}$	$0.088^{+0.026}_{-0.032}$
$n_s$	$0.948^{+0.015}_{-0.018}$	$0.951^{+0.014}_{-0.018}$	$0.950^{+0.015}_{-0.019}$	$0.946^{+0.015}_{-0.019}$	$0.953^{+0.015}_{-0.019}$
$\sigma_8$	$0.772^{+0.036}_{-0.048}$	$0.781^{+0.032}_{-0.045}$	$0.758^{+0.038}_{-0.052}$	$0.784^{+0.035}_{-0.049}$	$0.826^{+0.022}_{-0.035}$
$\Omega_m$	$0.266^{+0.025}_{-0.036}$	$0.267^{+0.018}_{-0.025}$	$0.249^{+0.024}_{-0.031}$	$0.276^{+0.023}_{-0.031}$	$0.299^{+0.019}_{-0.025}$

Data Set Constraints on Geometry and Vacuum Energy

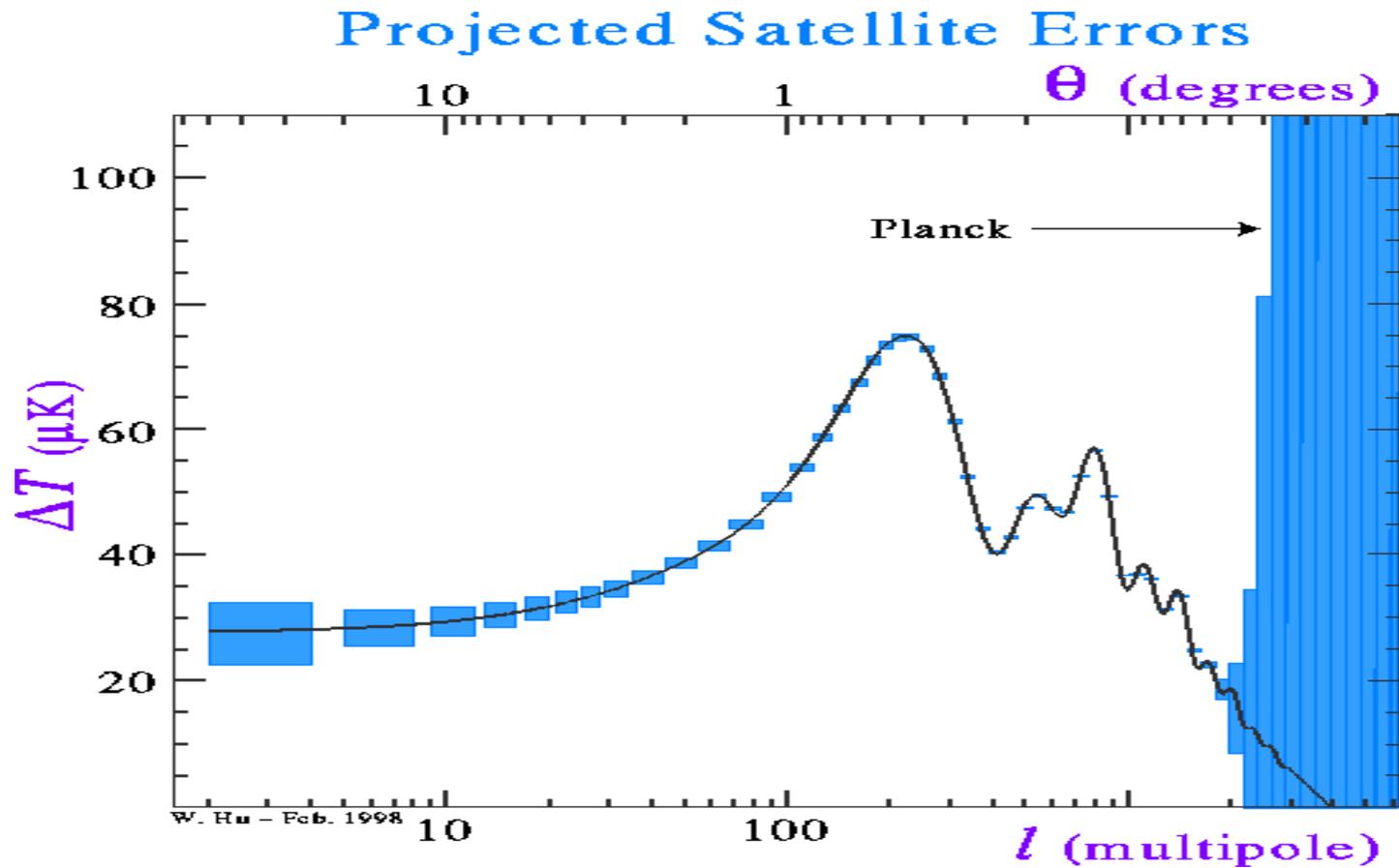
Data Set	$\Omega_K$	$\Omega_\Lambda$
WMAP + $h = 0.72 \pm 0.08$	$-0.003^{+0.013}_{-0.017}$	$0.758^{+0.035}_{-0.058}$
WMAP + SDSS	$-0.037^{+0.021}_{-0.015}$	$0.650^{+0.055}_{-0.048}$
WMAP + 2dFGRS	$-0.0057^{+0.0061}_{-0.0088}$	$0.739^{+0.026}_{-0.029}$
WMAP + SDSS LRG	$-0.010^{+0.011}_{-0.015}$	$0.728^{+0.020}_{-0.028}$
WMAP + SNLS	$-0.015^{+0.020}_{-0.016}$	$0.719^{+0.021}_{-0.029}$
WMAP + SNGold	$-0.017^{+0.022}_{-0.017}$	$0.703^{+0.030}_{-0.038}$

# CMB spectrum: the future



(From Planck website)

# CMB spectrum: the future



(From Planck website)