



The Thermal Beginning of the Universe

The Cosmic Microwave Background Radiation (CMB)

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CMB discovery time-line

-1946-1949 Gamow, Alpher, Bethe, and Herman's model of nucleosynthesis predicts relic millimeter radiation, but the model has difficulties to produce elements heavier than Li, and therefore neglected.

-1965 Arno Penzias & Robert Wilson's serendipitous discovery of a constant excess isotropic noise with an antenna at Bell Labs (Nobel 1978).



-late 60's Many groups made measurements of the intensity of the radiation and its temperature, collectively showing the spectrum is that of a BB (to 10% accuracy)

-1969 Tentative detection of a dipole anisotropy by E. Conklin (8GHz differential radiometer), confirmed in 1971-1977 by Henry, Corey, Wilkinson, and Smoot et al.

-1989 COBE (COsmic Background Explorer) launched, in 1990 first results confirming BB spectrum.

-1992 COBE's detection of non-dipole anisotropies (nobel prize 2006 for PI's Smoot & Mather).

-2003 WMAP results on precision cosmology through CMB anisotropies.

-2013 Planck's results

CMB discovery time-line

Arno Penzias & Robert Wilson **serendipitously** discovered the CMB (Nobel 1978).

In spring, a couple of pigeons nested in the antenna. They got rid of them and carefully cleaned the instrument, but the problem remained. They were about to give up their investigation when an astrophysicist colleague, B. Burke, told them that he had heard predictions about a background radiation with similar

characteristics to their measured discrepancy. He advised them to contact R. Dicke's group at Princeton. R. Dicke and J. Peebles had shown an expanding universe should be filled with this $\sim 3\text{K}$ radiation. In fact, two of their colleagues, P. Roll and D. Wilkinson, were already designing a radiometer to measure it.

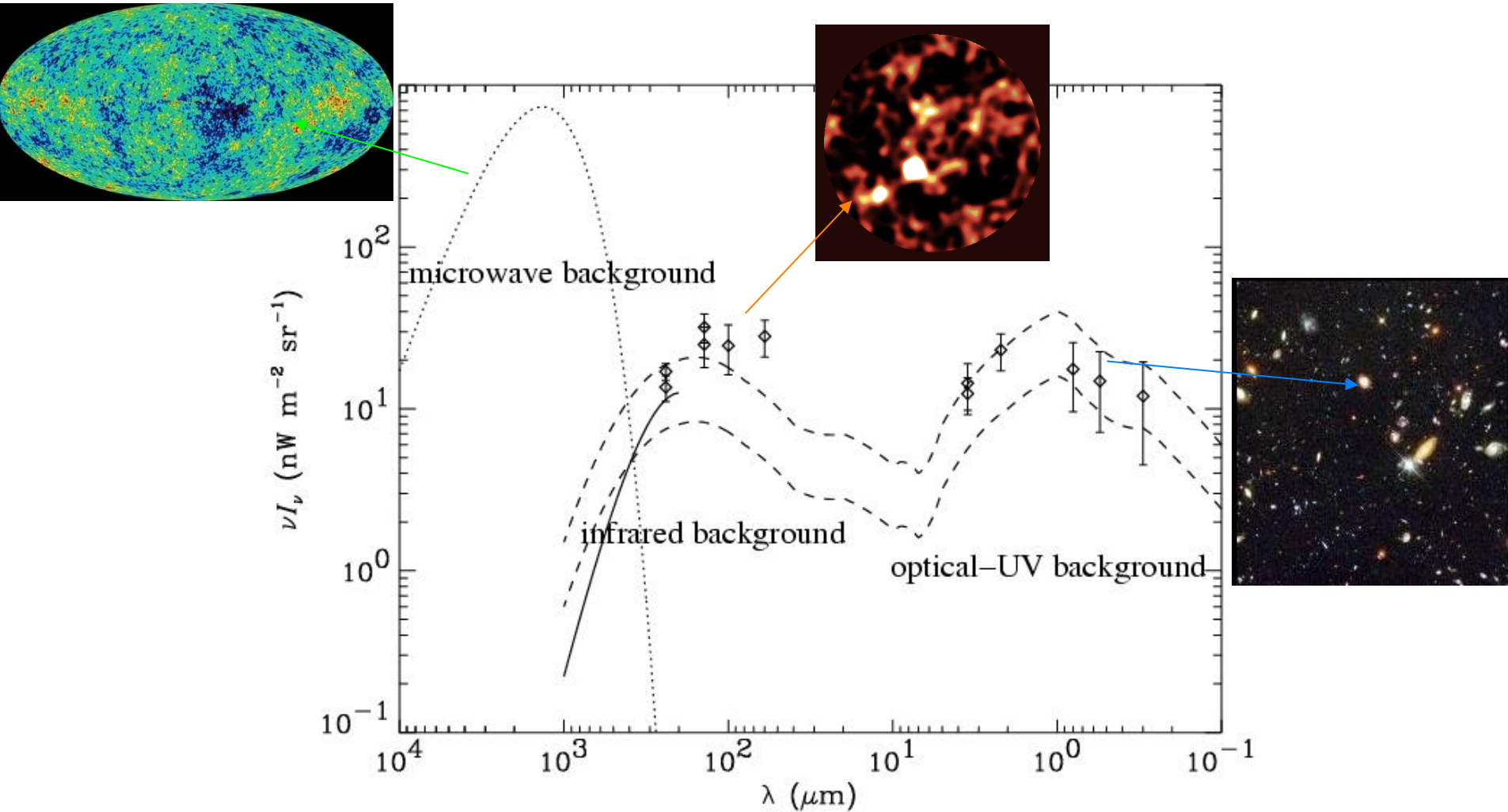


In July of 1965 two articles were published simultaneously: Penzias and Wilson presented their observations, whilst Dicke et al. suggested that this radiation might come from an epoch when the universe was very hot and dense.

It should be noted that, at the same time (1964) but on the other side of the world, two Soviet astrophysicists, Doroshkevich and Novikov, independently predicted the existence of a CMB radiation.

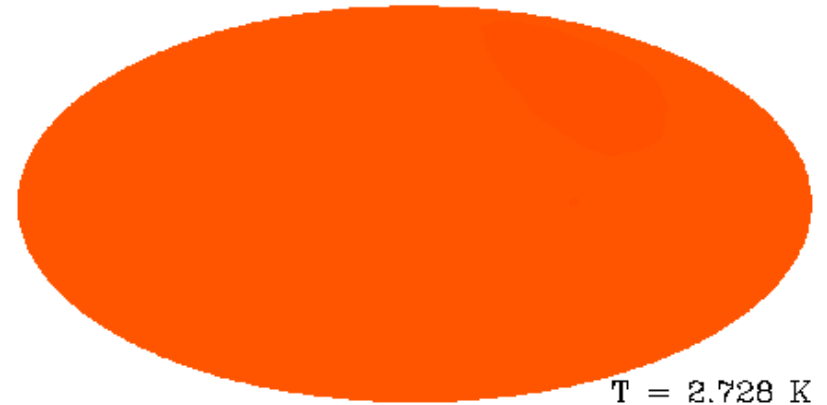
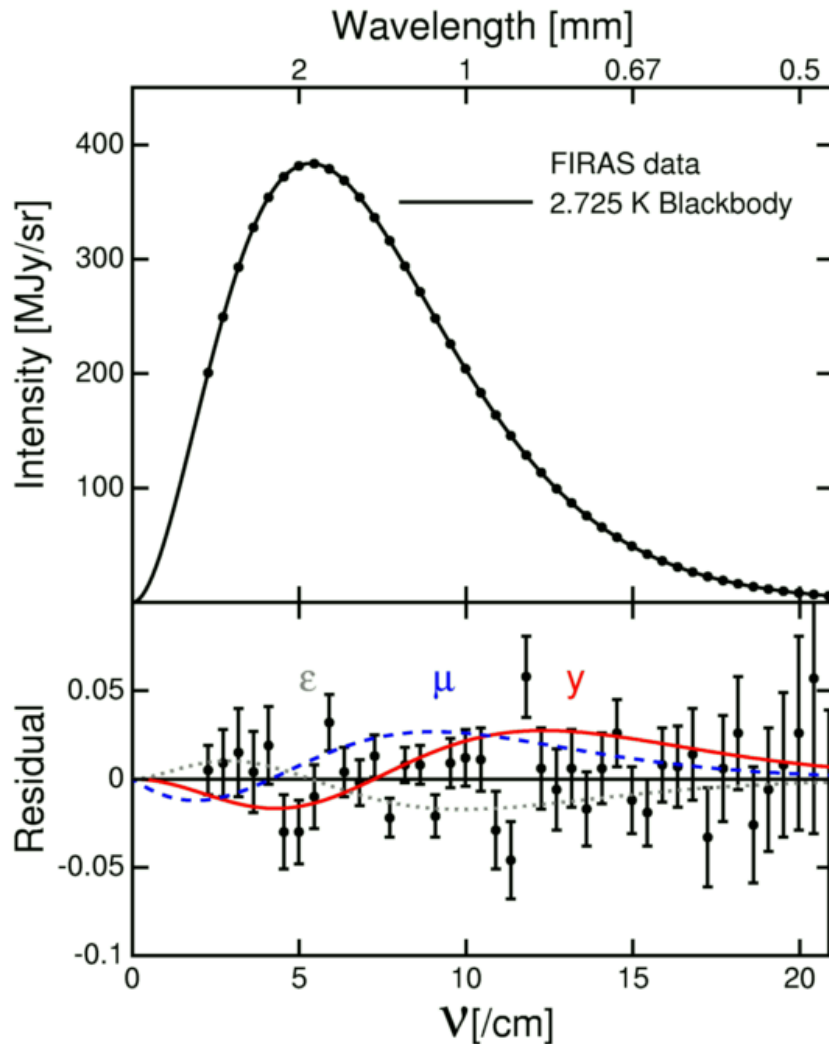
In 1941 Adams (& McKellar) had measured excited $J=1$ CN (cyanogen) absorption lines towards ζ – Ophiuchus, which needed a 2.3K radiation field (CMB!).

Dominant background radiation



The photons of the CMB are still the largest contributors to the radiation energy in the Universe.

CMB spectrum



An almost perfect BB was measured by the FIRAS instrument aboard COBE when it was compared to a very good BB calibrator.

$$T = 2.725 \pm 0.002 \text{ K } (< 0.1\%)$$

CMB spectrum

The energy density of this radiation is

$$\epsilon_{rad} = \rho_{rad} c^2 = a T^4 = 4.17 \times 10^{-14} \text{ Jm}^{-3} \Rightarrow \rho_{rad} = 4.63 \times 10^{-31} \text{ Kgr m}^{-3}$$
$$a \equiv 4\sigma/c = 7.565 \times 10^{-16} \text{ Jm}^{-3} \text{ K}^{-4}$$

Dividing this by the critical density

$$\rho_c = \frac{3H^2}{8\pi G} = 1.88 h^2 \times 10^{-26} \text{ Kgr m}^{-3}$$

we obtain the CMB density parameter

$$\Omega_{rad} = 2.47 \times 10^{-5} h^{-2}$$

Because $\epsilon \propto T^4$, and $\epsilon \propto a^{-4} \Rightarrow$

$$T \propto 1/a$$

The radiation was hotter and had a higher energy density in the past
But was it a blackbody as well?

CMB spectrum: z evolution

$$\epsilon(f)df = \frac{8\pi h}{c^3} \frac{f^3 df}{\exp(hf/kT) - 1}$$

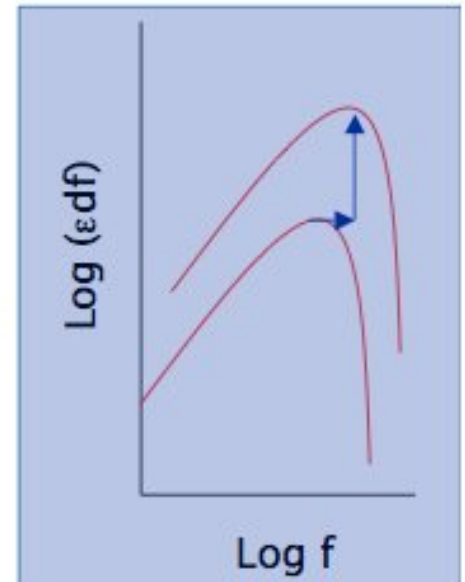
Going to earlier times, as a decreases, T increases. The denominator is invariant for $f/T = \text{constant}$.

If a_1 is the scale factor at given instance in the past, the peak of the emission will be at $f_{1,\text{peak}} = f_{\text{peak}} / a_1$, since $T_1 = T / a_1$. Also,

$$f_{1,\text{peak}}^3 df_{1,\text{peak}} = f_{\text{peak}}^3 df_{\text{peak}} / a_1^4$$

The peak of the curve will shift up in frequency by a factor $1/a_1$ and up in energy density by a factor $1/a_1^4$. This is the case not only for the peak frequency but for any frequency:

the blackbody shape is preserved, shifting up in frequency by $1/a_1$ and in energy density by $1/a_1^4$.



$$B(\nu, T) \sim a^{-3}$$

CMB spectrum: z evolution

The almost perfect BB shape of the CMB backs up the expansion of the Universe, and the existence of a hotter earlier universe.

◆ If the CMB was just a tired relic light (Tired Light Cosmology):

$n_\gamma(z=0)=n_\gamma(z=z_e) \sim B_\nu(T_e) \sim (1+z_e)^3 B_\nu(T_0)$ but FIRAS imposes that the factor in front of $B_\nu(T_e)$ is 1 with a precision better than 10^{-4} .

Hence $(1+z_e)^3 = 1 \pm 1 \times 10^{-4} \Rightarrow z_e < 0.000033$ and opaque from that onwards. But we have sources at $z \sim 4$! So this is not a possibility.

◆ If the steady models were correct, there would be no evolution. Today we see CMB + FIR radiation from stars and galaxies. Energy added between 1 month and a few thousand yrs after Black-Body will produce

$$I_{BE}(\nu, T) = \frac{2\pi\nu^3}{c^2} \frac{1}{\exp(h\nu/kT + \mu) - 1}$$

but there are no deviations to the Black-Body spectrum

Sunyaev-Zeldovich effect, SNe, molecular clouds

(Following E. Wright's CMB review paper)
(www.astro.ucla.edu/~wright/)

Photon/baryon ratio

If no particles are created or destroyed, is this ratio conserved?

Yes, both photon and baryon number densities scale as $1/a^3$

What is the current number density of CMB photons?

Their energy density is :

$$\epsilon_{rad} = aT^4 = 2.6 \times 10^5 \text{ eV m}^{-3}$$

Dividing this by their mean energy $E_{mean} = 3kT = 7.05 \times 10^{-4} \text{ eV}$ gives

$$n_{rad} = \frac{\epsilon_{rad}}{E_{mean}} = 3.7 \times 10^8 \text{ m}^{-3}$$

Photon/baryon ratio

What is the current number density of baryons?

Their density parameter, derived from nucleosynthesis, is :

$\Omega_B \approx 0.02 h^{-2}$ This can be converted into an energy density:

$$\varepsilon_B = \rho_B c^2 = \Omega_B \rho_c c^2 \approx 3.4 \times 10^{-11} J m^{-3} = 2.1 \times 10^8 eV m^{-3}$$

Divide this by the proton rest mass, $E_B = 939 \text{ MeV}$: $n_B = 0.22 m^{-3}$

Recall : $\varepsilon_{rad} = 2.6 \times 10^5 eV m^{-3}$, $n_{rad} = 3.7 \times 10^8 m^{-3}$

Currently: >the baryon energy density dominates over that of CMB photons by ~1000.

>the CMB number density dominates over that of baryons by $\sim 10^9$. (valid for all times).

$$\Omega_B / \Omega_\gamma \approx 1000 \quad n_\gamma / n_B \approx 10^9$$

Radiation era

We have that: $\rho_M \propto a^{-3}$ and $\rho_{\text{rad}} \propto a^{-4}$

There must be a z at which $\rho_M = \rho_{\text{rad}}$

Taking into account that nucleosynthesis predicts $n_\nu = 0.68 n_\gamma$, then $\Omega_{\text{rad}} = 4.2 \times 10^{-5} h^{-2}$

$$1 + z_{\text{eq}} = 23900 \Omega_m h^2 \Rightarrow z_{\text{eq}} \approx 3100$$

Therefore the thermal history of the Universe can be divided in two main eras: a *radiation dominated era* ($z \gg z_{\text{eq}}$) and a *matter dominated era* ($z \ll z_{\text{eq}}$). In the radiation dominated era, in which we can neglect the curvature and Λ terms in Friedmann's equation, we have:

$$a \propto t^{1/2} . \quad \text{Friedmann eq:} \quad H^2 \equiv \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G \rho + \Lambda c^2}{3} - K \frac{c^2}{a^2}$$

By differentiating this relation with respect to time and using (12) we have:

$$t = \left(\frac{3}{32\pi G \rho_\gamma} \right)^{1/2} . \quad (52)$$

Using $\rho_\gamma = \pi^2 k_b T^4 / 15 h^3 c^5$ we finally obtain the important relation between cosmic time and the temperature of the Universe in the radiation dominated era:

$$T_{\text{Kelvin}} \simeq 1.3 \times 10^{10} t_{\text{sec}}^{-1/2} \quad (53)$$

ie., at $t = 1$ sec the Universe had $T \sim 10^{10}$ K! It is evident that the Universe at early times was hot enough for nucleosynthesis to occur, as it had been supposed originally by Gamow. The era of nucleosynthesis takes place around $\sim 10^9$ K.

(From M. Plionis' notes or Peacock 1999)

CMB origin

$$T \propto 1/a \rightarrow$$

In very early times, the energy of the CMB photons was much greater than the 13.6 eV required to ionize H. just a soup of photons, e and p (for simplicity, ignore the few He nuclei produced in nucleosynthesis).

All the mater in the Universe was ionized. If a p manages to capture an e and form an H atom, the H atom was immediately ionized by one of the abundant photons with $E > 13.6$ eV.

In this soup of e,p, and photons, the interaction that insures thermodynamic equilibrium (a single temperature for all three species) is electron Thomson scattering:

$$\gamma + e^- \rightarrow \gamma + e^-, \quad \text{Thomson cross section } \sigma_T = 6.65 \times 10^{-29} \text{ m}^2$$

As time was passing, the CMB photons cooled down due to the expansion of the Universe, and eventually they were not able to ionize H, the Universe became neutral.

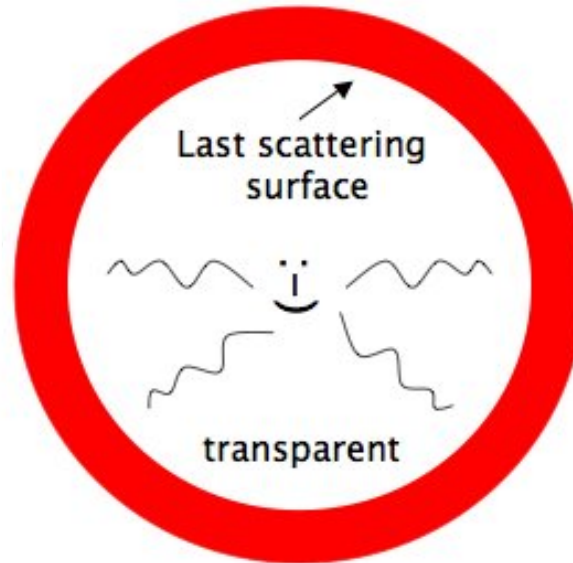
Since there were no free electrons left, the CMB photons stopped getting scattered and, after a last scattering, kept propagating unobstructed.

CMB origin

- ◆ As the universe expands **the temperature and density decrease**, and the energy of the photons is no longer high enough to keep the atoms ionised.
- ◆ Photons start **decoupling** from electrons.
- ◆ **At 10,000 K**, helium is half in the form of He^{2+} , half as He^+ while hydrogen is completely ionised.
- ◆ **At 7000 K**, helium is half neutral and half in the form of He^+ . Hydrogen is still ionised.
- ◆ **At 4000 K**, helium and 50% of the hydrogen are in their neutral forms.
- ◆ **At a temperature of ~ 3000 K**, the number of ionised atoms can be neglected and the photons start travelling freely through the universe. During this recombination epoch (even if the atoms had never previously been combined), the universe was $\sim 3 \times 10^5$ years old and today it is observed as the Last Scattering Surface (LSS).

CMB origin

The last scattering surface.



Every observed is surrounded by a spherical last scattering surface. The CMB photons emerge from the last scattering surface and propagate in a straight line all the way to the observed with no further scatterings.

CMB origin

3 important epochs:

1. **Recombination**, the time when the baryonic component of the Universe became neutral (number of ions=number of neutral atoms)

2. **Photon decoupling**, the time when the rate of photon scattering becomes smaller than H .

In other words, this is the epoch when the time between scatterings for a photon, becomes larger than the Hubble time. When photons decouple, they cease to interact with electrons and the Universe becomes transparent.

3. **Last scattering**. This is the time when a typical CMB photon underwent its last scattering from an electron.

The last scattering time is very close to the photon decoupling time

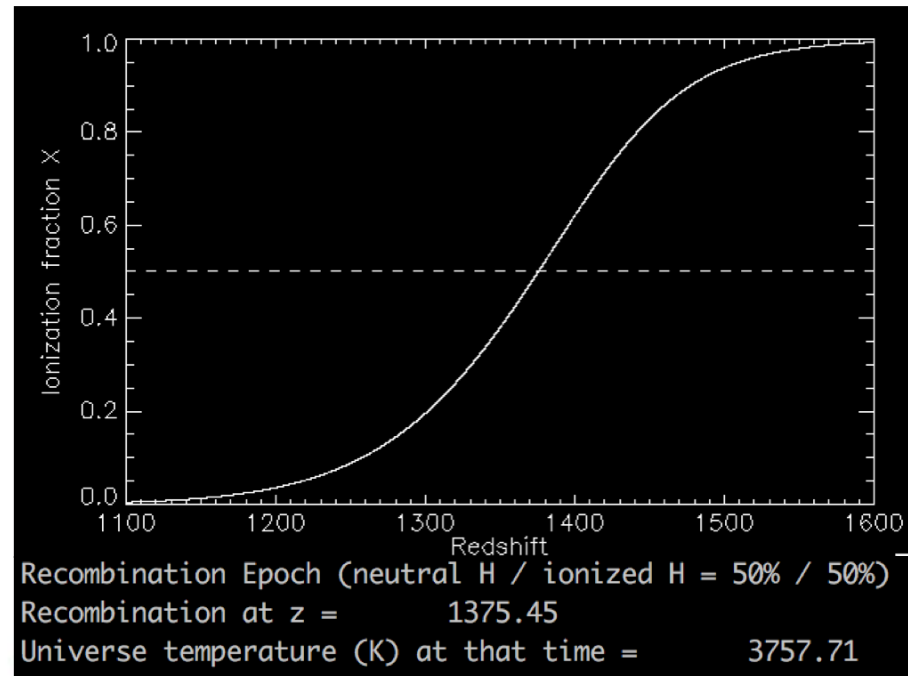
CMB origin: recombination

1. **Recombination**, the time when the baryonic component of the Universe became neutral (number of ions=number of neutral atoms)

the Saha equation gives us the ionization fraction X as a function of the ionization potential Q and the baryon to photon ratio η :

$$\frac{1-X}{X^2} = 3.84\eta \left(\frac{kT}{m_e c^2} \right)^{3/2} \exp\left(\frac{Q}{kT} \right)$$

$$X \equiv n_e / n_B = n_e / (n_H + n_p)$$



Recombination is a gradual process. Defining the moment of recombination at $X=1/2$ we obtain:

$$kT_{rec} = 0.323 \text{ eV} \Rightarrow T_{rec} = 3740 \text{ K}, \quad 1 + z_{rec} = \frac{1}{a_{rec}} = \frac{T_{rec}}{T_{CMB,0}} = \frac{3740 \text{ K}}{2.73 \text{ K}} \approx 1371$$

CMB origin: decoupling

2. **Photon decoupling**, the time when the rate of photon scattering becomes smaller than H .

In other words, this is the epoch when the time between scatterings for a photon, becomes larger than the Hubble time. When photons decouple, they cease to interact with electrons and the Universe becomes transparent.

The photon scattering rate is:

$$\Gamma = \frac{c}{\lambda} = n_e(z) \sigma_e c = X(z) n_{B,0} (1+z)^3 \sigma_e c = 4.4 \times 10^{-21} X(z) (1+z)^3 s^{-1}$$

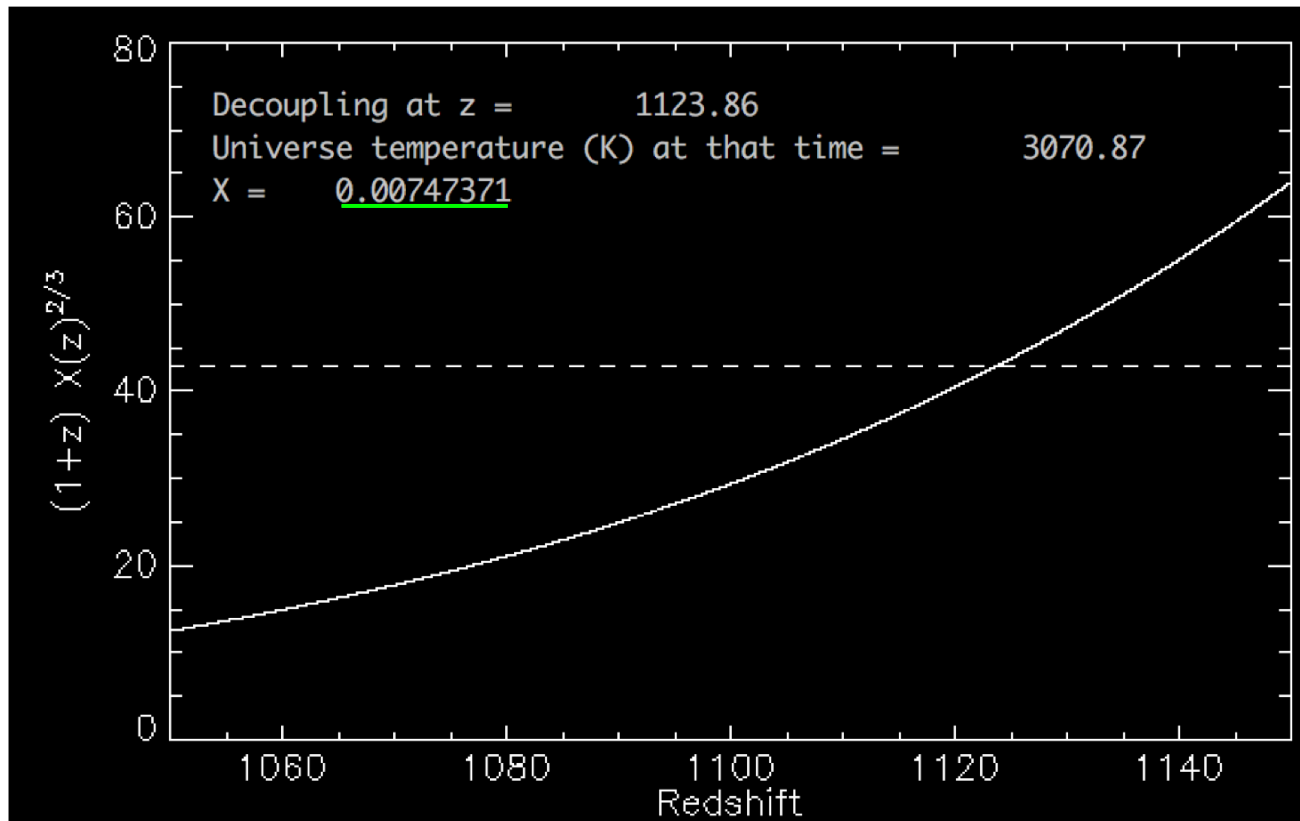
Recombination and decoupling take place during the matter dominated era, so Friedmann's eq. is:

$$\frac{H^2}{H_0^2} = \frac{\Omega_{m,0}}{a^3} = \Omega_{m,0} (1+z)^3 \Rightarrow H = 1.24 \times 10^{-18} (1+z)^{3/2}$$

Setting $\Gamma=H$, we obtain

$$1 + z_{dec} = \frac{43.0}{X(z_{dec})^{2/3}} \Rightarrow z_{dec} = 1130$$

CMB origin: decoupling



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CMB origin: decoupling

In our calculation we used the Saha equation, which assumes that photoionization is always in equilibrium. This is not true when Γ becomes comparable to H . A detailed calculation gives:

$$z_{dec} \approx 1100, T_{dec} \approx 3000 K. *$$

Event	Redshift	Temperature	Time(yr)
Radiation-matter equality	3570	9730	47,000
Recombination	1370	3740	240,000
Photon decoupling	1100	3000	350,000
Last scattering	1100	3000	350,000

- > Before decoupling, photon pressure on the matter smoothed out density fluctuations in the photon baryon field at distances smaller than the horizon distance back then.
- > After, the hydrogen gas was free to collapse under its self gravity (and that of the dark matter) to form structure in the Universe

* In fact ... “it is well fitted by a Gaussian of mean redshift = 1065 and standard deviation in redshift = 80” ... (J.A. Peacock, 1999)

CMB: *Implications*

- ◆ Stimulated rapid advances in theoretical and observational cosmology.
- ◆ Strengthened the Big Bang model.
- ◆ Place constraints on physical and astrophysical processes that could have occurred since the early Universe.
- ◆ A background with an almost perfect thermal spectrum also discarded cosmological models that rejected the expansion of the Universe, and alternative explanations for the CMB emission.
- ◆ **Homogeneity and Isotropy:** The “cosmological principle” is a hypothesis based on simplicity and a Copernican desire not to occupy a preferred position in the universe. THE HORIZON PROBLEM: independent points at the last scattering surface, separated by $\sim 1^\circ$, would not receive information from each other.
- ◆ **Homogeneity?** Then why do the distribution of galaxies, stars, planets and our own existence do not prove it?

CMB spectrum: dipole anisotropy

Dipole anisotropy in COBE data can be explained as a Doppler effect between the frame of reference of the solar system and that at rest with the observable CMB.

$$\nu' = \gamma(1 - \beta \cos \theta)\nu, \text{ with } \beta \equiv v/c$$
$$\text{and } \gamma \equiv 1/\sqrt{1 - \beta^2}$$

$$T(\theta) = T_0 / \gamma(1 - \beta \cos \theta) \approx T_0 + T_0 \beta \cos \theta$$

A fit to the image $T_0 \beta = 3353 \pm 24 \mu\text{K}$
And with $T_0 = 2.725\text{K}$

$$\vec{v}_{sun} - \vec{v}_{CMB} = 369 \pm 3 \text{ km s}^{-1}$$

Taking into account the movement around the MW, and the movement of the MW in the LG, then the LG moves towards $(l, b) \approx (277^\circ, 30^\circ)$.

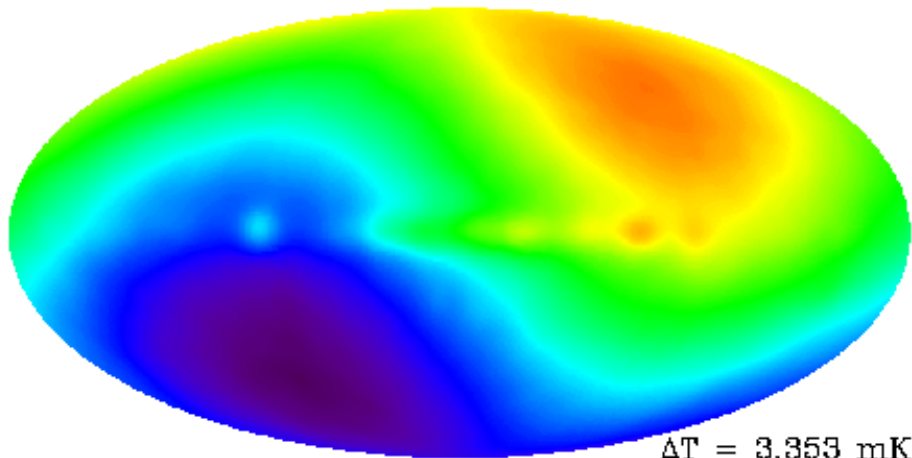
Signature of local attractor.

$$\vec{v}_{LG} - \vec{v}_{CMB} \approx 620 \pm 45 \text{ km s}^{-1}$$

(Following E. Wright's CMB review paper)

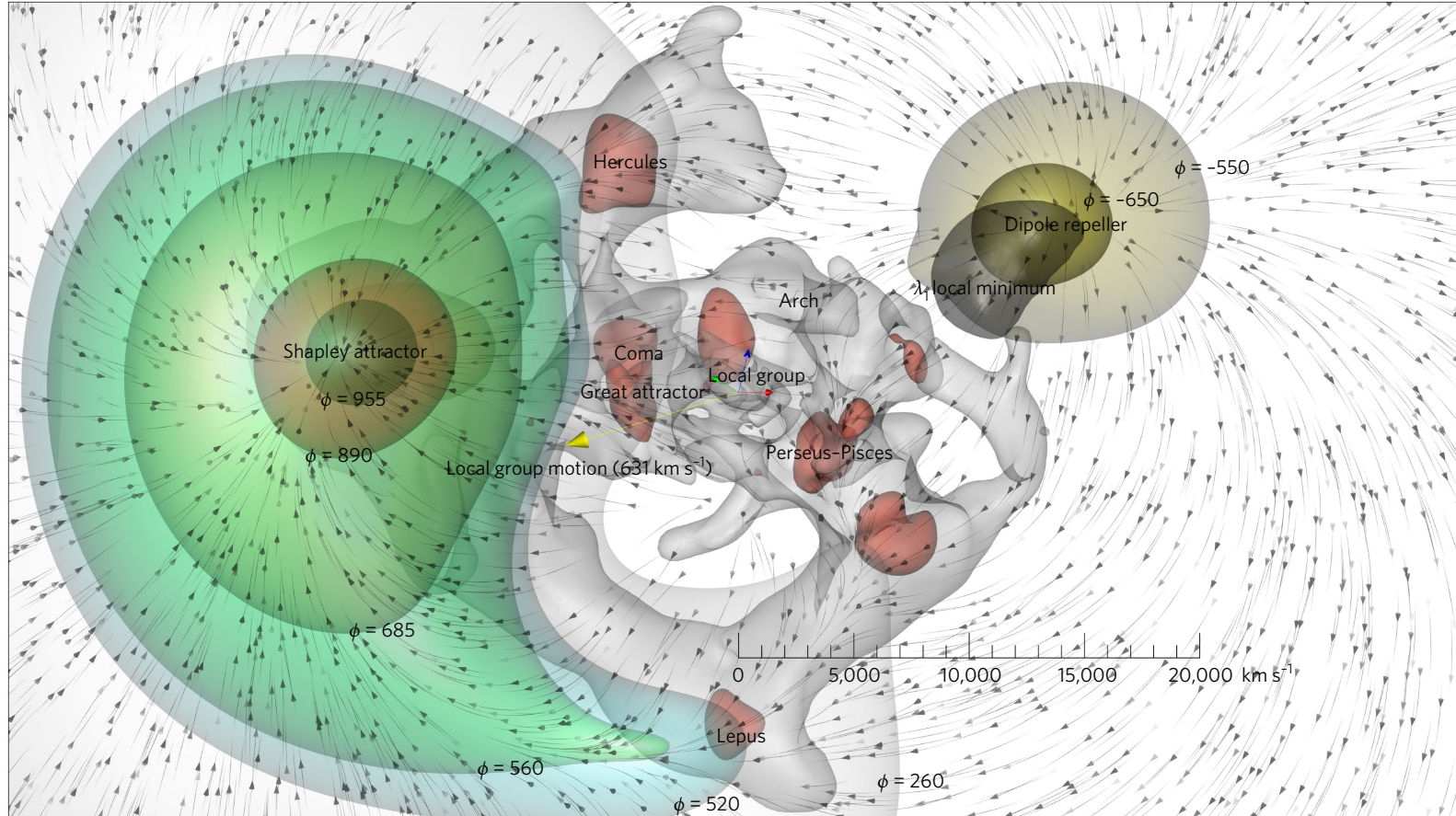


$T = 2.728 \text{ K}$



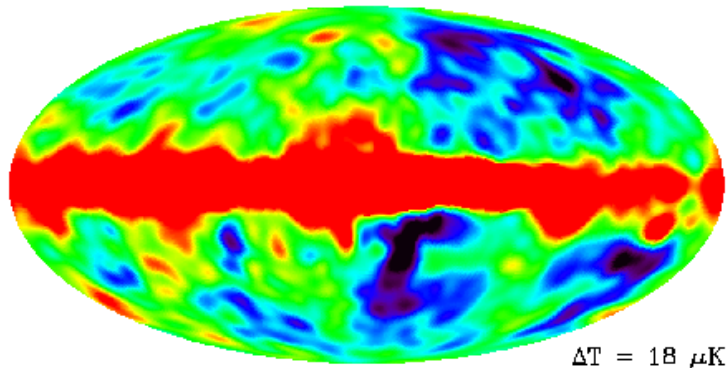
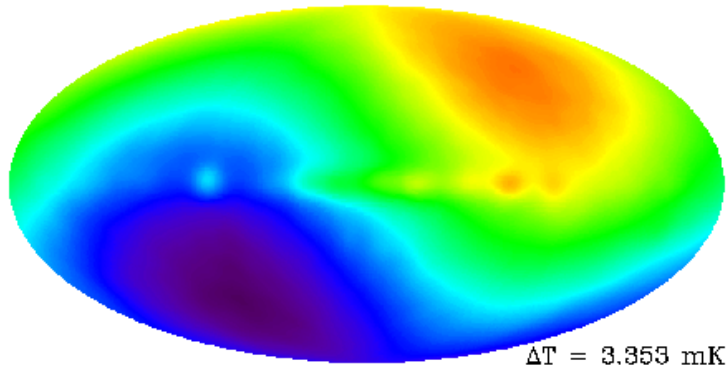
$\Delta T = 3.353 \text{ mK}$

CMB spectrum: dipole anisotropy



Mapping of the velocity field is shown by means of streamlines; red and grey surfaces present the knots and filaments of the V-web; equi-gravitational potential (ϕ) surfaces are shown in green and yellow. The yellow arrow originates at our position and indicates the direction of the CMB dipole (galactic longitude $l = 276^\circ$, galactic latitude $b = 30^\circ$).

CMB spectrum: dipole anisotropy



Dipole anisotropy in COBE data can be explained as a Doppler effect between the frame of reference of the solar system and that at rest with the observable CMB.

$$v' = \gamma(1 - \beta \cos \theta)v, \text{ with } \beta \equiv v/c \text{ and } \gamma \equiv 1/\sqrt{1 - \beta^2}$$

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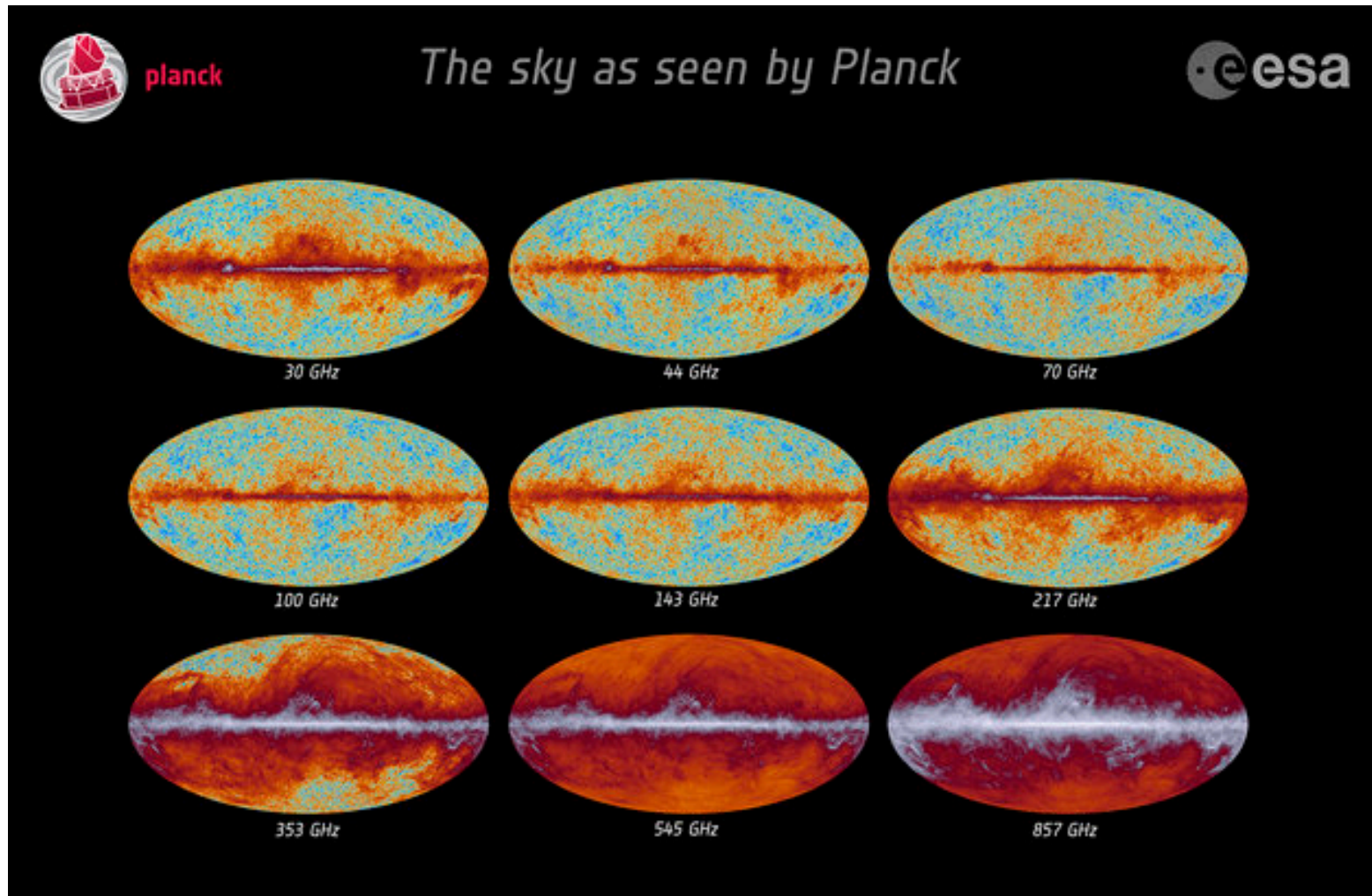
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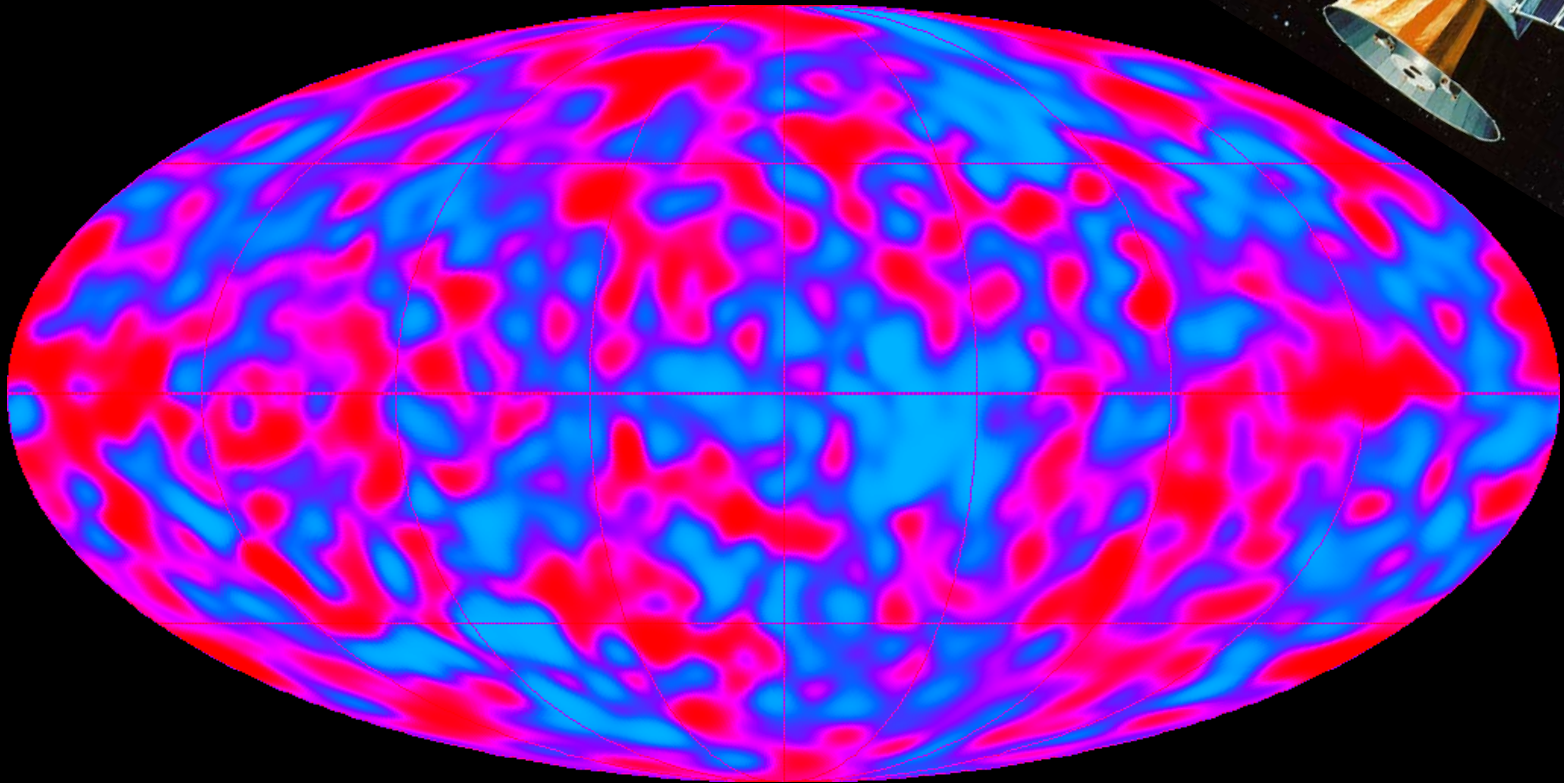
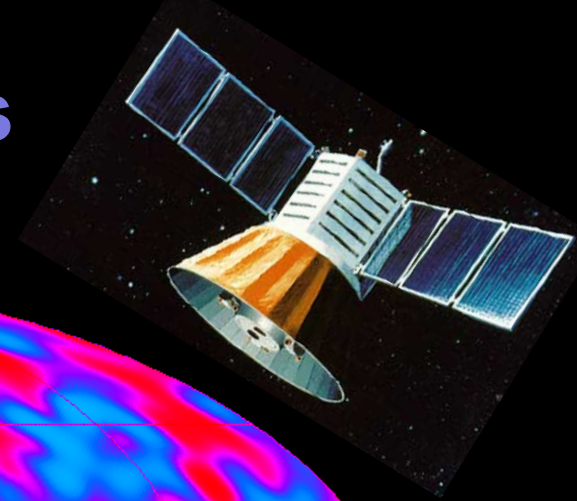
(Following E. Wright's CMB review paper)

CMB: removing the galaxy



Planck: 30 GHz (10mm) to 857 GHz (350 μ m) image of the CMB after dipole subtraction. The galaxy emission is dominated by dust.

CMB: observations



**Cosmic Background
Explorer *COBE*
(1992):**

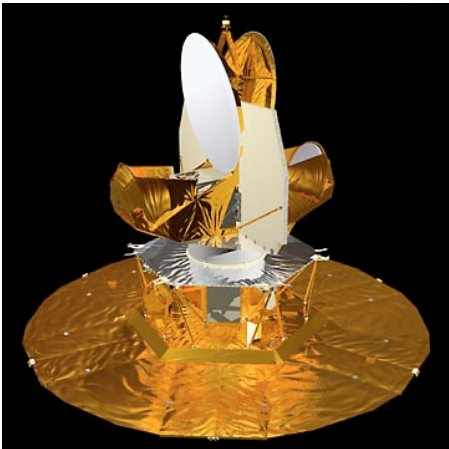
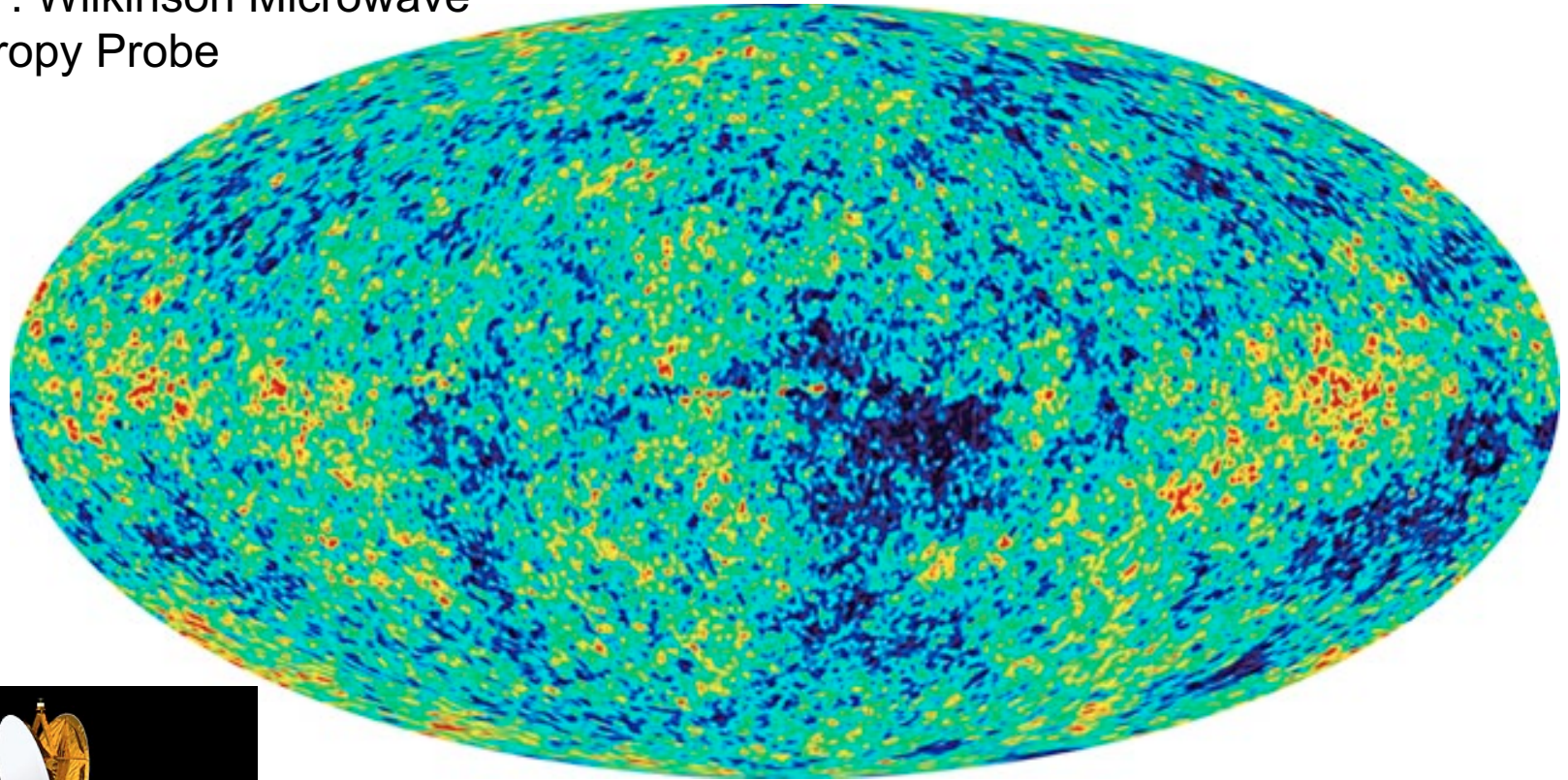
$$\Delta T/T = 10^{-5}$$

**FWHM
 $\sim 7\text{deg}$**



CMB: observations

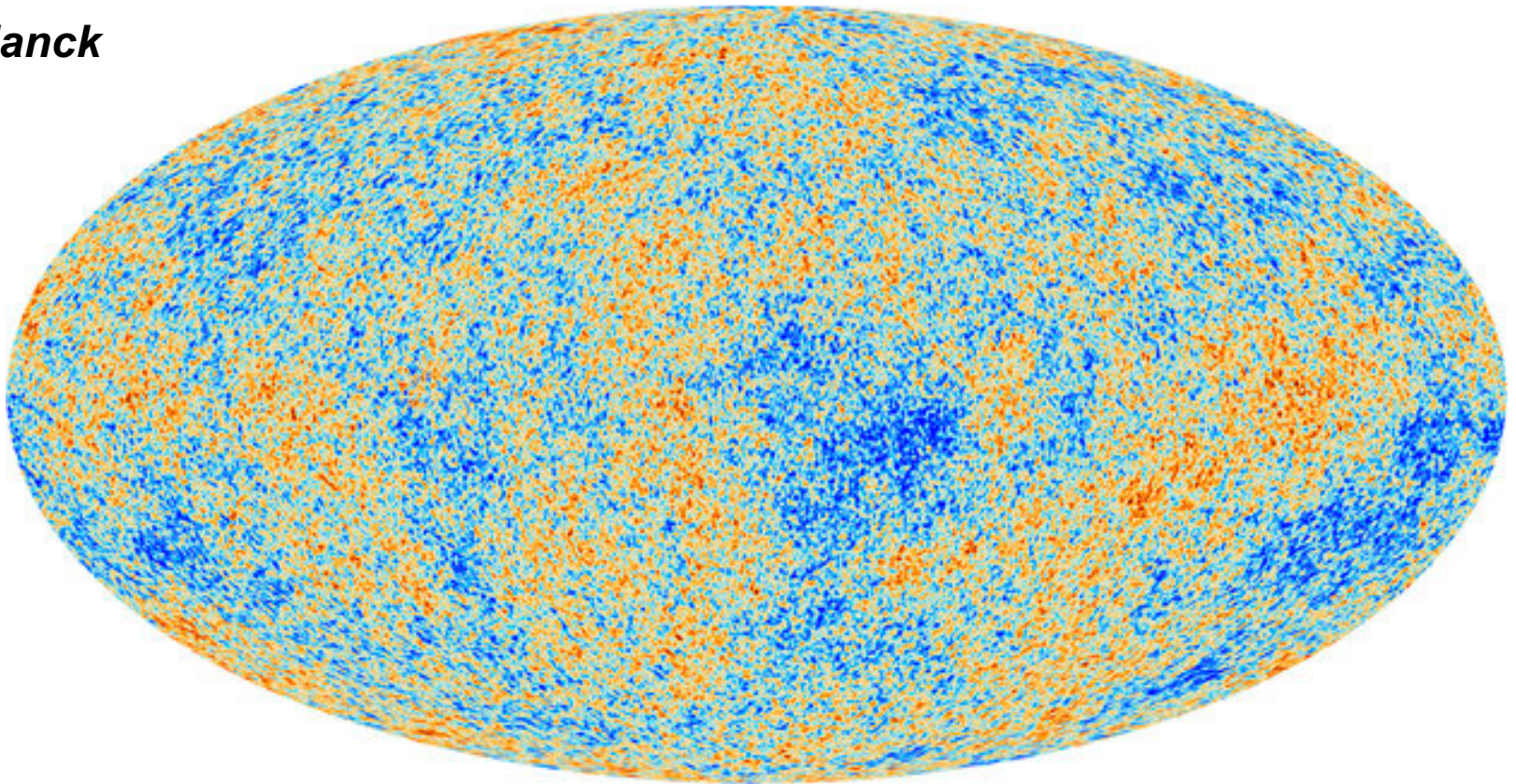
WMAP: Wilkinson Microwave
Anisotropy Probe



Band	FWHM
K-band (23 GHz)	52.8'
Ka-band (33 GHz)	39.6'
Q-band (41 GHz)	30.6'
V-band (61 GHz)	21'
W-band (94 GHz)	13.2'

CMB: observations

Planck



9 Bands with FWHMs = $33'$ – $5'$

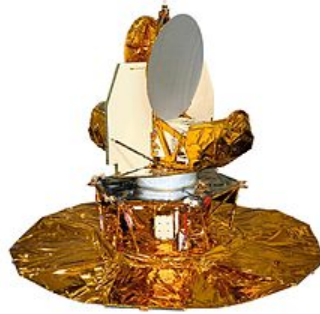
Low Frequency Instrument (LFI)

High Frequency Instrument (HFI)

30 – 70 GHz receivers

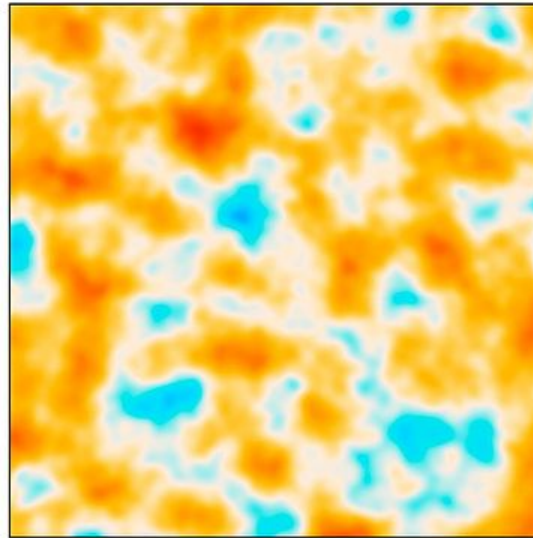
100–857 GHz receivers

CMB: experiments



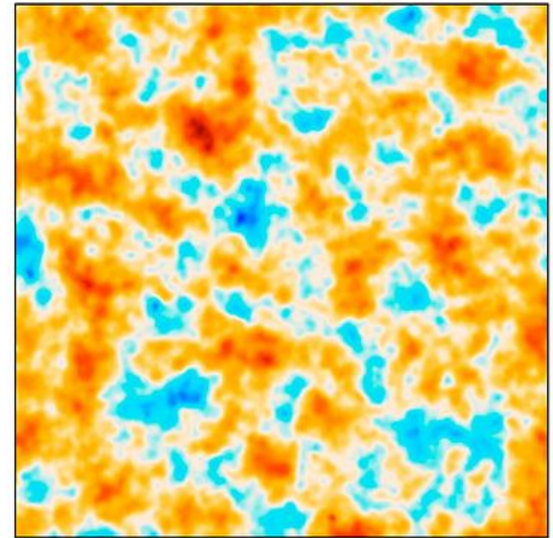
COBE

FWHM $\sim 7^\circ$



WMAP

FWHMs = $53' - 13'$

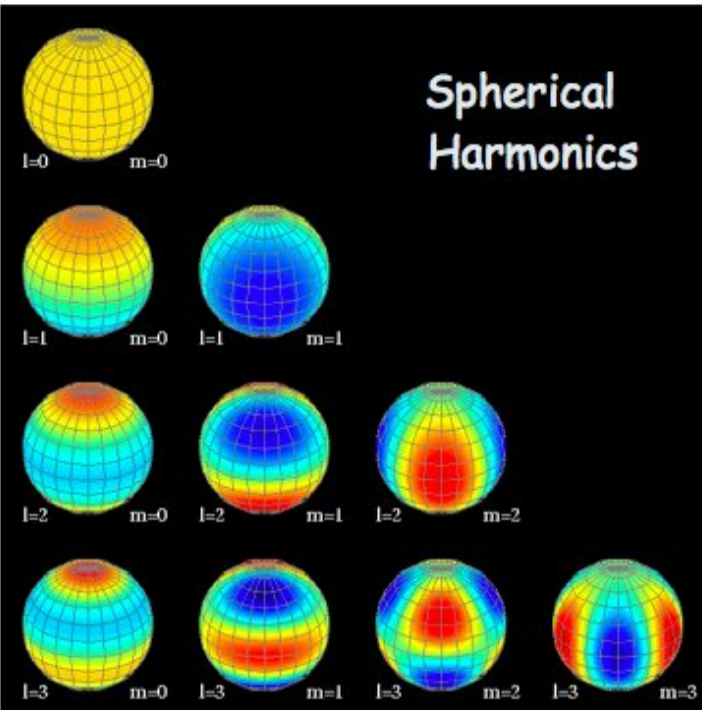


Planck

FWHMs = $33' - 5'$

CMB spectrum: statistical properties

$T(l, b)$ can be fully specified by either the angular correlation function $C(\theta)$ or its Legendre transformation, the angular power spectrum C_l .

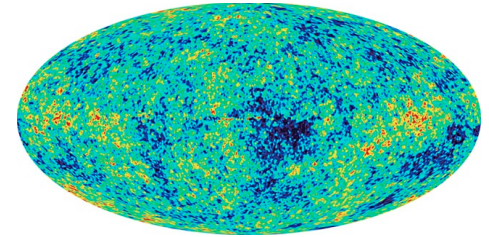


Write the temperature fluctuations as a series of spherical harmonics:

$$\frac{\delta T(\theta, \phi)}{T} = \sum_{l=0}^{\infty} \sum_{m=-l}^l a_{lm} Y_{lm}(\theta, \phi)$$

The correlation function, a CMB map derived function:

$$C(\theta) = \left\langle \frac{\delta T(\hat{n})}{T} \frac{\delta T(\hat{n}')}{T} \right\rangle_{\hat{n}\hat{n}'=\theta}$$



$$a_{lm} = \int \frac{\Delta T(\theta, \phi)}{T_{\text{CMB}}} Y_{lm}(\theta, \phi) d\Omega$$

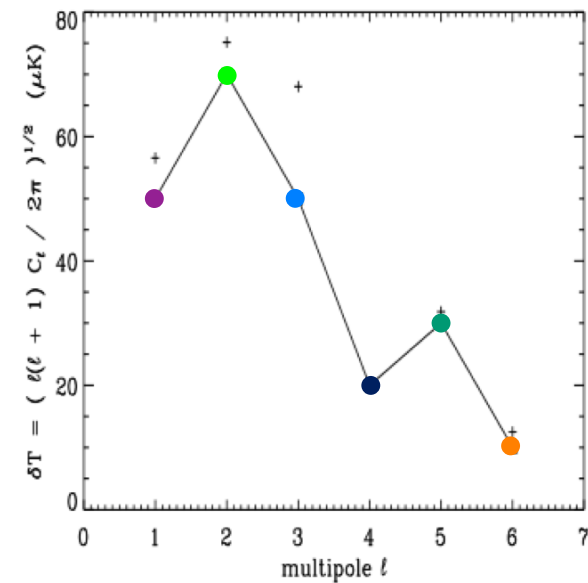
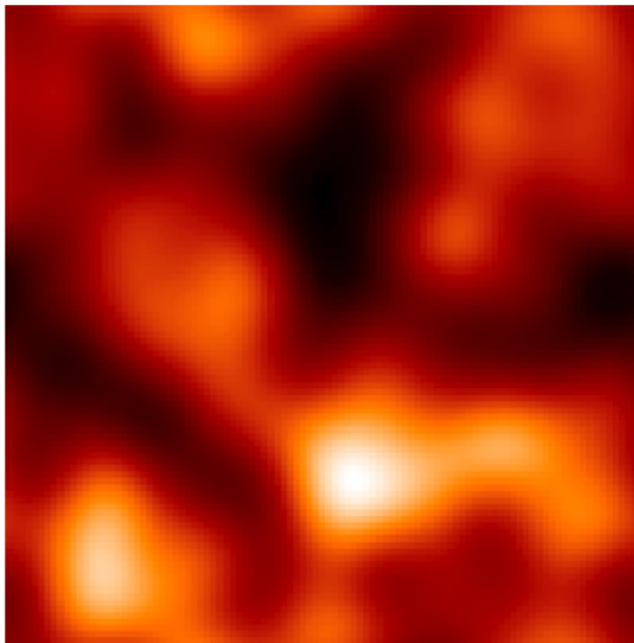
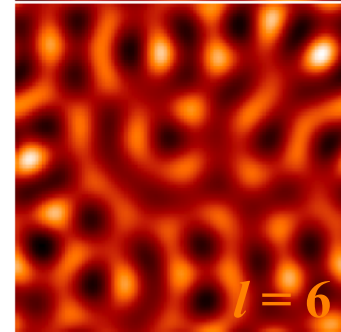
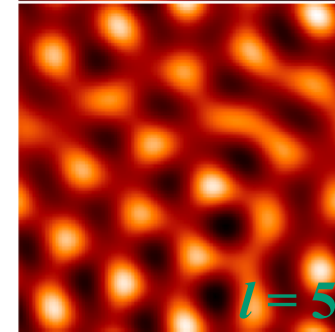
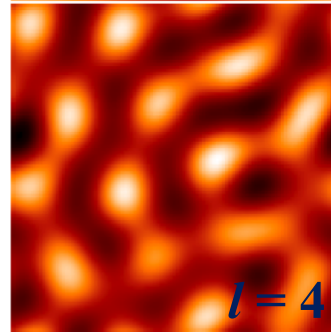
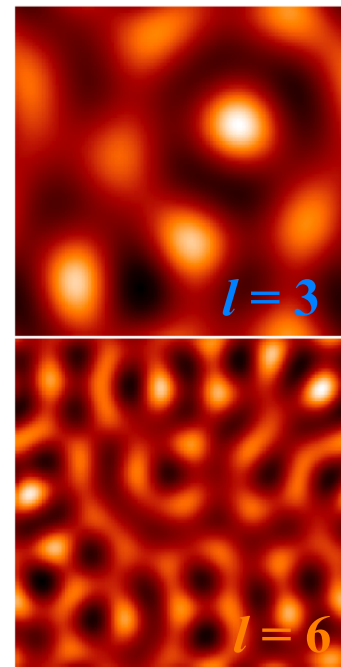
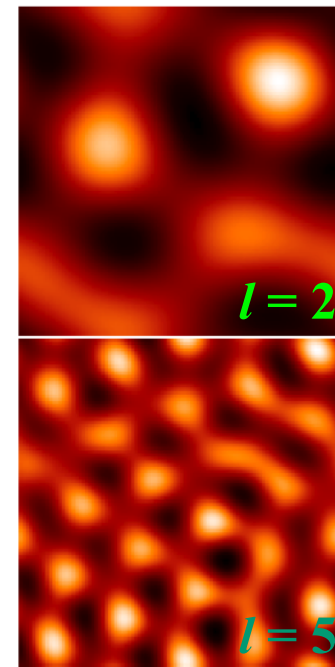
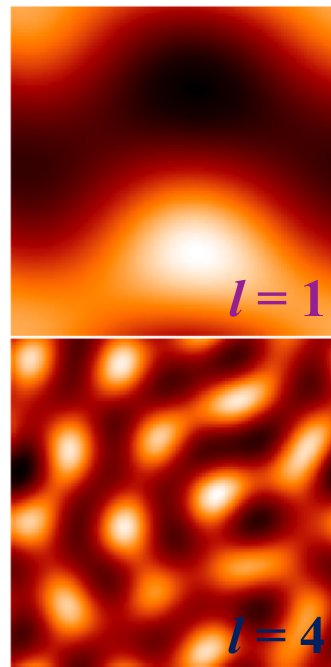
$$C_\ell = \langle |a_{\ell m}|^2 \rangle$$

Which are related by $C(\theta) = \frac{1}{4\pi} \sum_l (2l+1) C_l P_l(\cos\theta)$ where P_l are Legendre polynomials of order l .

A term C_l is a measure of angular fluctuations on the angular scale $\theta \sim 180^\circ/l$. If the sky had equal power on all scales C_l should be a constant.

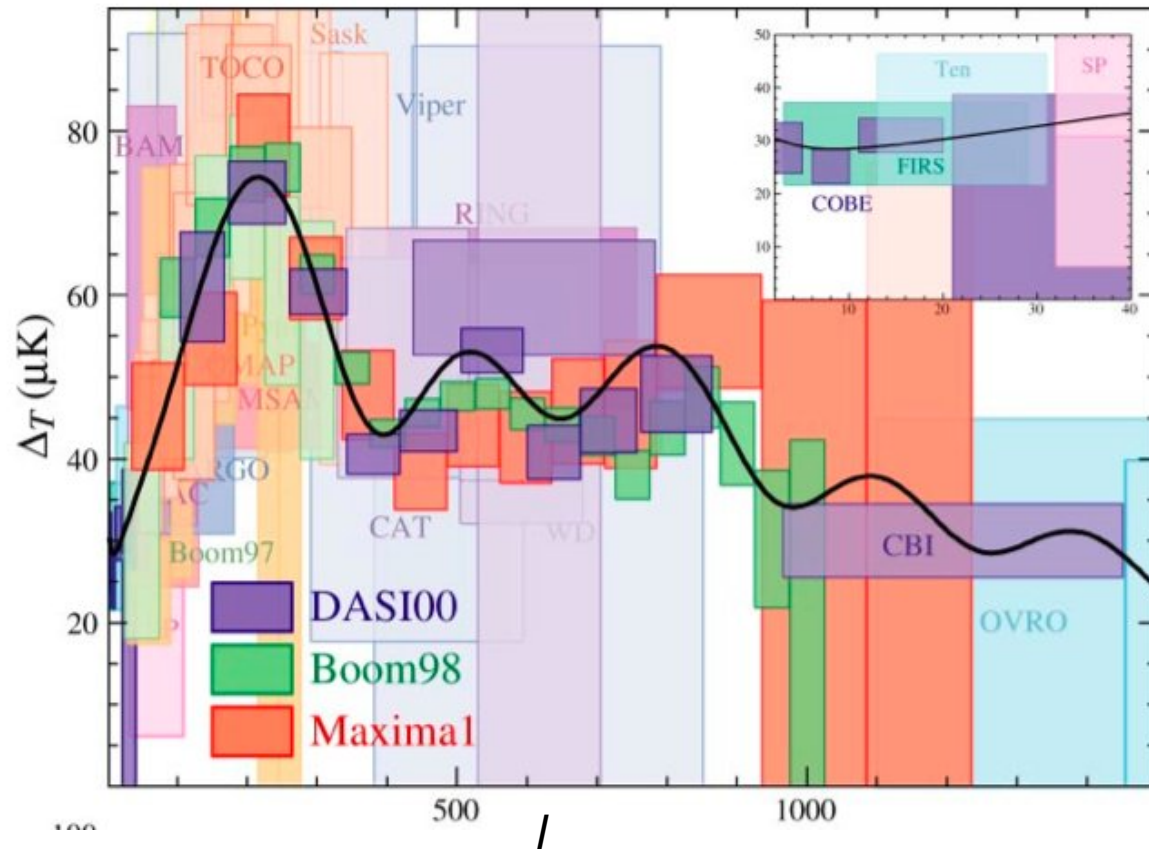
CMB spectrum: statistical properties

Spherical harmonics
decomposition
(small angle approx.)



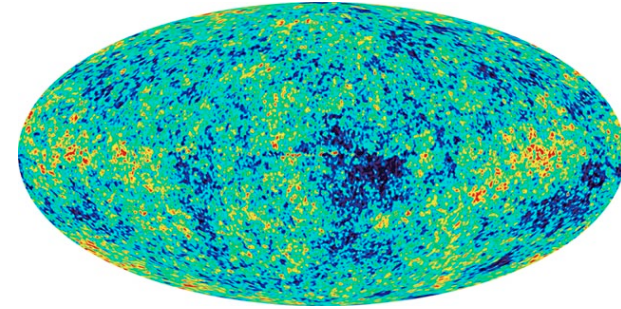
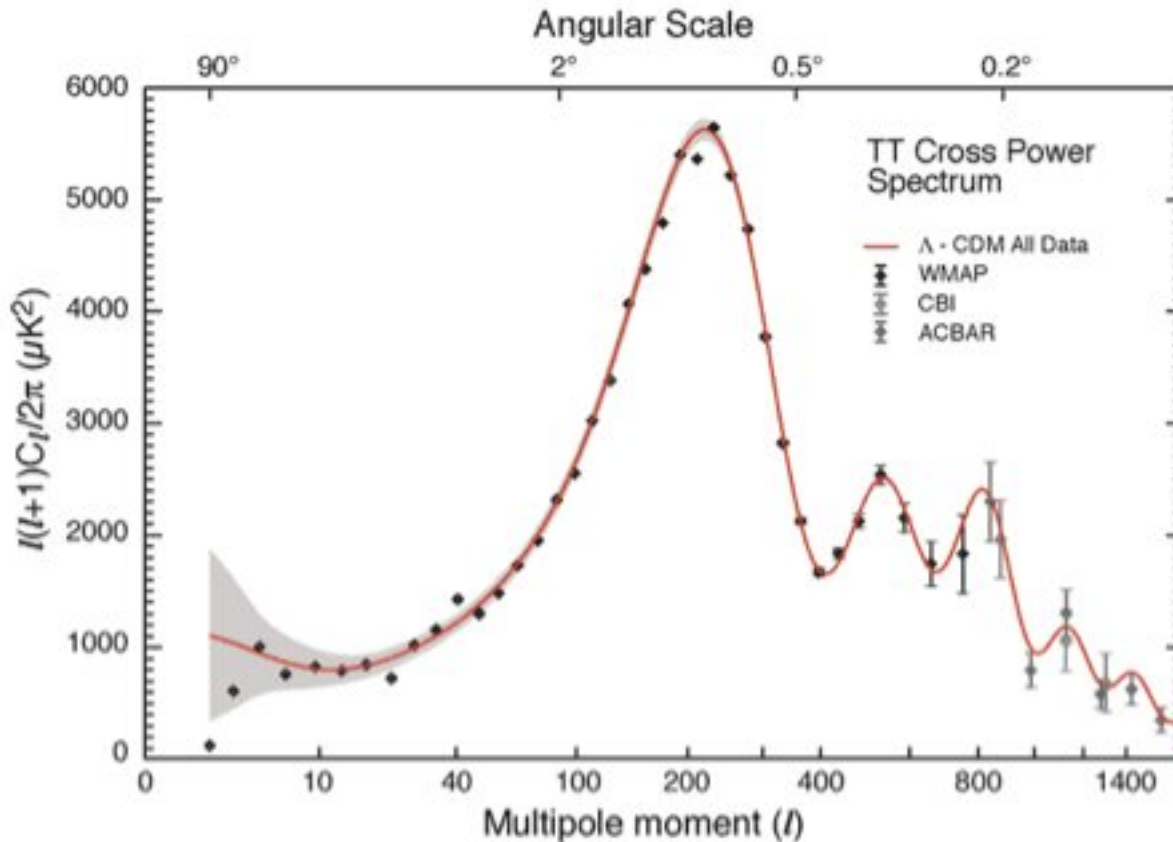
CMB spectrum: power spectrum

$$(C_l l(l+1))^{1/2} \sim$$



First peak had already been constrained by an array of 1992-2000 missions, and sampled in its full amplitude by Boomerang (1998) & Maxima (2000)

CMB spectrum: power spectrum



Why is there a major peak at $l \sim 200$ ($\theta \sim 1^\circ$)?
And what are the secondary peaks?

CMB spectrum: power spectrum

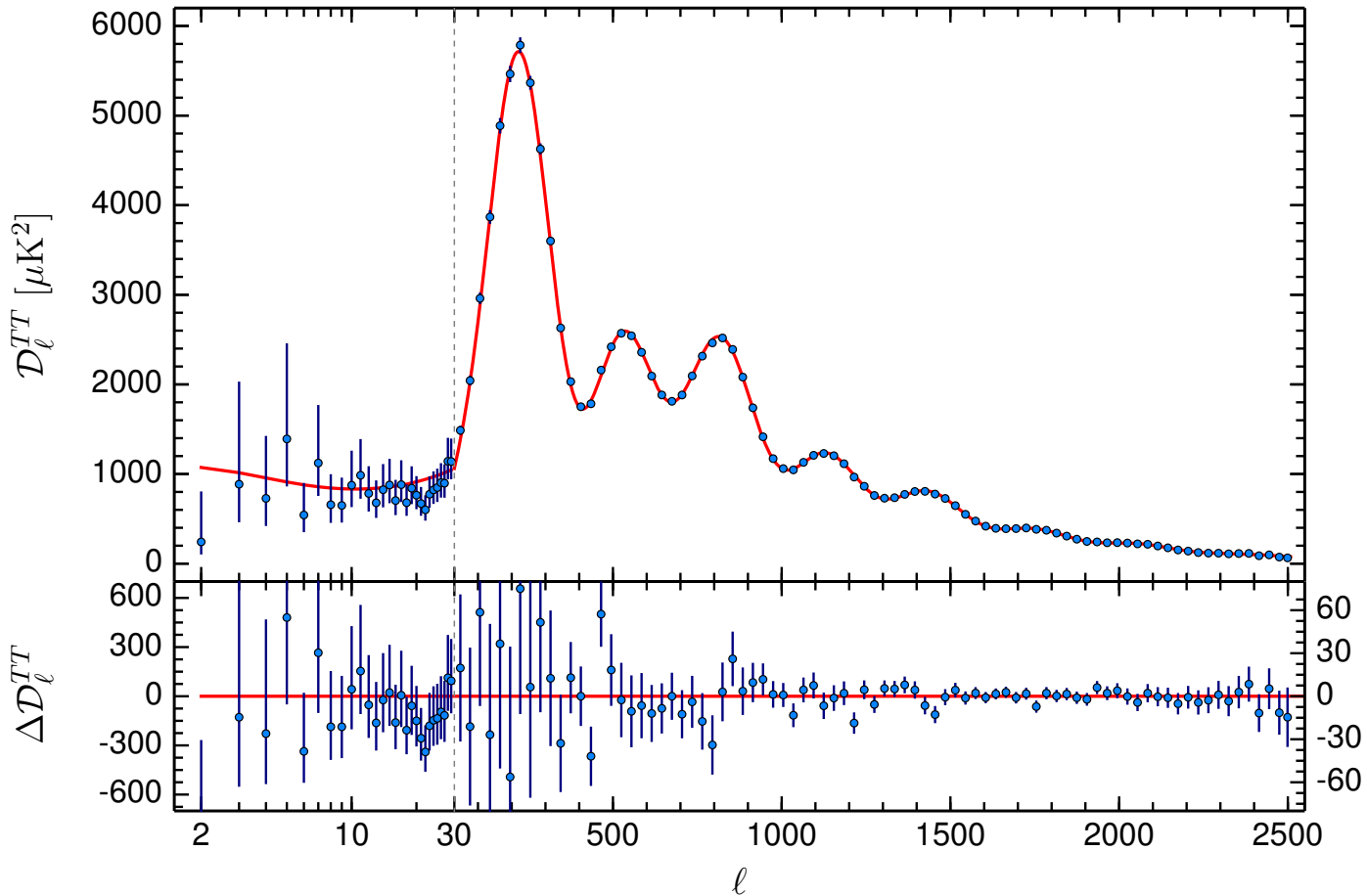


Fig. 1. *Planck* 2015 temperature power spectrum. At multipoles $\ell \geq 30$ we show the maximum likelihood frequency-averaged temperature spectrum computed from the Plik cross-half-mission likelihood, with foreground and other nuisance parameters determined from the MCMC analysis of the base ΛCDM cosmology. In the multipole range $2 \leq \ell \leq 29$, we plot the power spectrum estimates from the Commander component-separation algorithm, computed over 94 % of the sky. The best-fit base ΛCDM theoretical spectrum fitted to the *Planck* TT+lowP likelihood is plotted in the upper panel. Residuals with respect to this model are shown in the lower panel. The error bars show $\pm 1\sigma$ uncertainties.

CMB spectrum: sources of fluctuations

Sources of Fluctuations in the CMB		
Primary	Gravity	
	Doppler	
	Density fluctuations	
	Damping	
	Defects	Strings Textures
Secondary	Gravity	Early ISW Late ISW Rees-Sciama Lensing
		Thermal Sunyaev Zeldovich effect Kinetic Sunyaev Zeldovich effect
	Global reionization	Suppression New Doppler Vishniac
	Extragalactic	Radio point sources IR point sources
		Dust Free-free Synchrotron
Tertiary	Galactic	Solar system Atmosphere Noise, etc.
	Local	

CMB spectrum: primary fluctuations

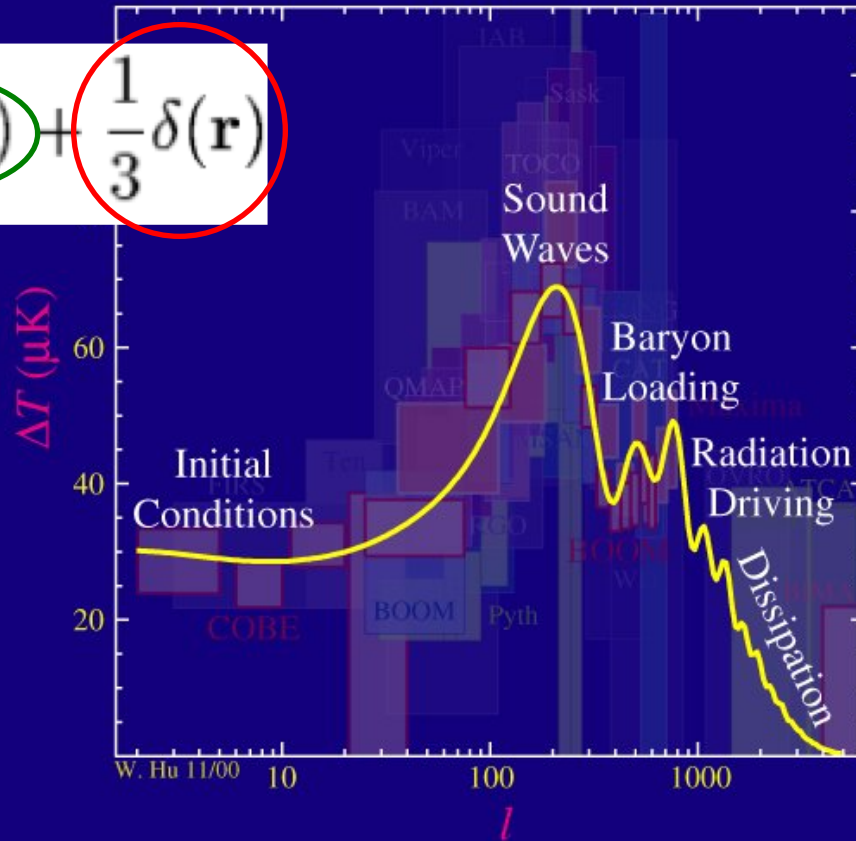
$$\delta T(\mathbf{r}) = \underbrace{\Psi(\mathbf{r})}_{\text{Gravitational potential}} - \underbrace{\mathbf{r} \cdot \mathbf{v}(\mathbf{r})}_{\text{Doppler effect}} + \underbrace{\frac{1}{3}\delta(\mathbf{r})}_{\text{Density fluctuations = Temperature anisotropies}}$$

Gravitational potential

Doppler effect

Density fluctuations =
Temperature anisotropies

Jeans Analysis
Relativistic Multifluid
Dark Matter
Neutrinos



Soundscape: COBE measured the temperature fluctuations ΔT on the largest angular scales which correspond to multipoles as roughly $l=100/\text{angle (degrees)} \sim 2-20$. The current generation of experiments are measuring multipoles $l>100$ where the acoustic peaks are expected to dominate the scene (yellow curve). The physical landscape described in these pages begins with sound waves and proceeds through baryon loading, radiation driving, and dissipation by diffusion damping. In the background, are the measurements as of January 2001.

CMB spectrum: Horizon at LSS

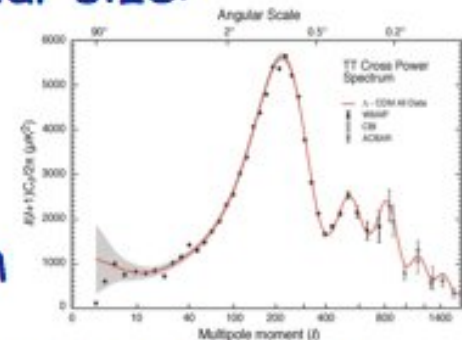
At the time of last scattering the Universe was matter dominated and the Hubble distance was:

$$\frac{c}{H(z_{ls})} = \frac{c}{H_0(z+1)^{3/2}} = \frac{3 \times 10^8 \text{ m s}^{-1}}{1.24 \times 10^{-18} \text{ s}^{-1} (1101)^{3/2}} \approx 6.6 \times 10^{21} \text{ m} \approx 0.2 \text{ Mpc}$$

Seen from Earth this has an angular size:

$$\theta_H = \frac{c/H(z_{ls})}{d_A} \approx \frac{0.2 \text{ Mpc}}{13 \text{ Mpc}} \approx 0.015 \text{ rad} \approx 1^\circ$$

This is the first peak of the power spectrum



The Hubble distance at the time of decoupling is \sim the maximum length information (photons, matter) could have traveled. The fluctuations at $\theta > \theta_H$ and $\theta < \theta_H$ must reveal different things.

CMB spectrum: gravitational effects

At the time of last scattering the Universe, the non-baryonic dark matter dominates:

$$\Omega_{dm} : \Omega_{\gamma} : \Omega_b = 6.4 : 1.4 : 1$$

If the dark matter density distribution at the time of last scattering has a spatially varying component $\delta\rho$, then there is a spatially varying gravitational potential $\delta\Phi$ given by Poisson's equation:

$$\nabla^2(\delta\Phi) = 4\pi G \delta\rho$$

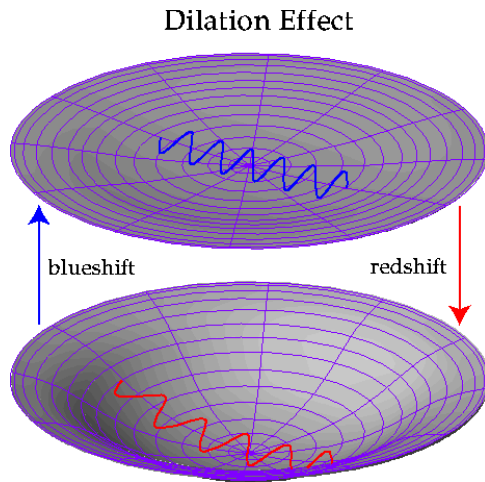
CMB spectrum: power spectrum $\theta > \theta_H$

If at the time of last scattering a CMB photon is at a local minimum (in a “potential well”), it will spend energy to climb out of the well and it will be redshifted.

If it is at a local maximum, it will roll down the maximum and gain energy (it will be blueshifted). In general, Sachs and Wolfe (1967) showed that:

$$\frac{\delta T}{T} = \frac{1}{3} \frac{\delta \Phi}{c^2}$$

All these are valid for angular scales $\theta > \theta_H$, because the photon-baryon fluid has no time to move further than θ_H . At smaller scales, we need to consider the fact that the photon-baryon fluid has time to move substantially under the influence of the gravity of dark matter.



CMB spectrum: power spectrum $\theta < \theta_H$

The photon-baryon fluid moves under the influence of the non baryonic matter that dominates. When the fluid finds itself in a potential of dark matter, it flows toward the bottom of the well.

As it falls its pressure rises. As the pressure rises, it slows down, stops and inverts the fluid fall. The fluid expands, and a series of oscillations takes place, called "acoustic oscillations".

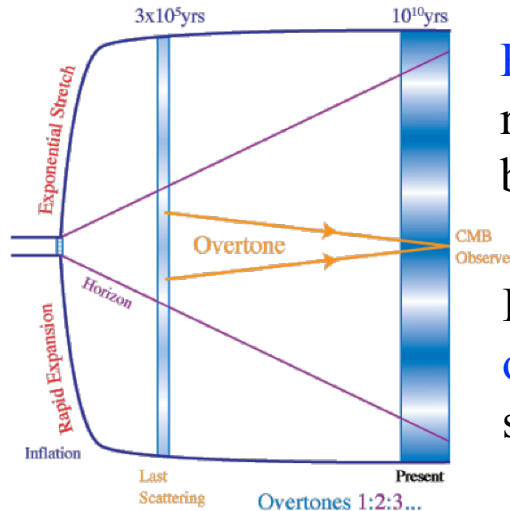
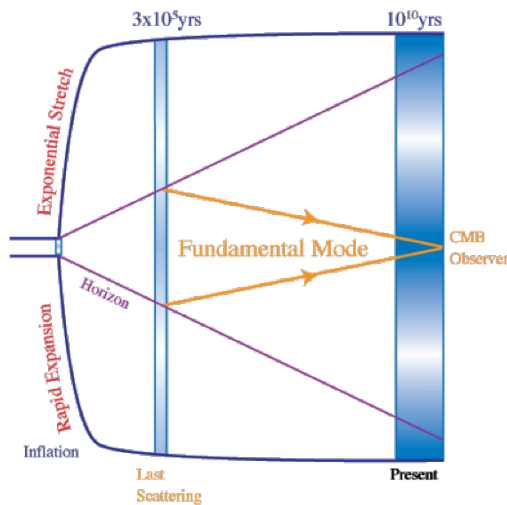
The oscillations stop at recombination, when photons decouple from baryons. How many oscillations take place? The larger the well the fewer the oscillations. For $\theta \sim \theta_H$ there is time for ~ 1 oscillation.

If the photon-baryon fluid is compressed in a well at the time of recombination, the decoupled photons will be hotter than average.

If the photon-baryon fluid is expanded at the time of recombination, the decoupled photons will be cooler than average.

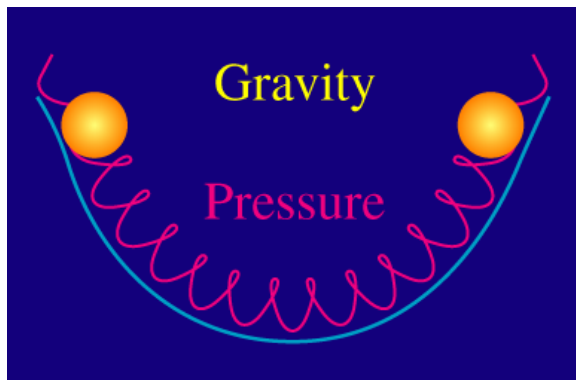
The highest peak at $\theta \sim \theta_H$ ($l \sim 200$) represents the wells where the photon-baryon fluid had just reach maximum compression at the recombination time.

CMB spectrum: power spectrum $\theta < \theta_H$

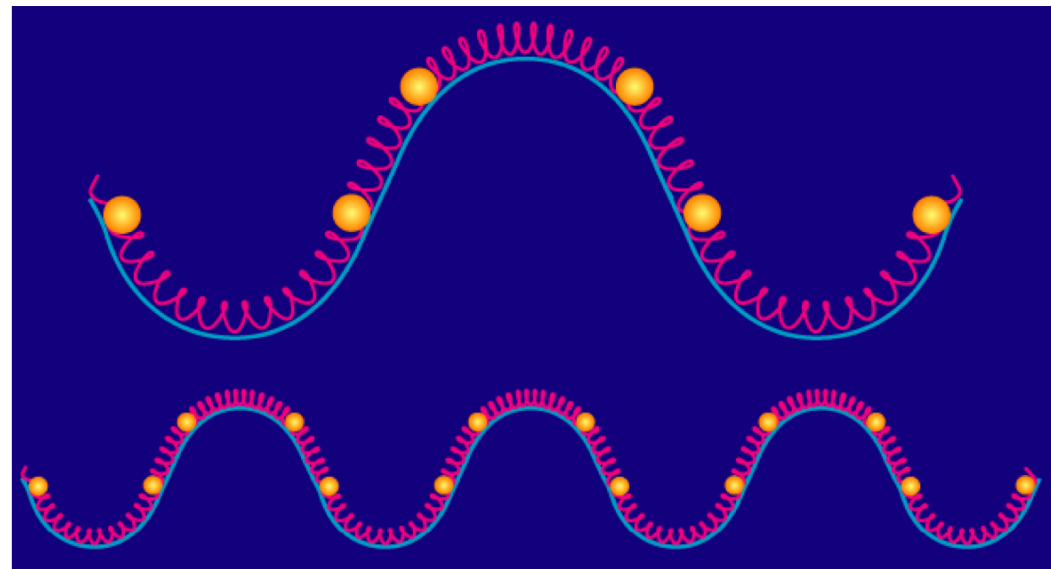


Fundamental mode of sound waves is related to the size: in the microwave background context, the horizon.

Inflation's signatures are that the **overtones** follow a pure harmonic series with frequency ratios of 1:2:3..



- Potential fluctuations on all scales
- Each mode oscillates independently
- Modes that are half as long oscillate twice as fast



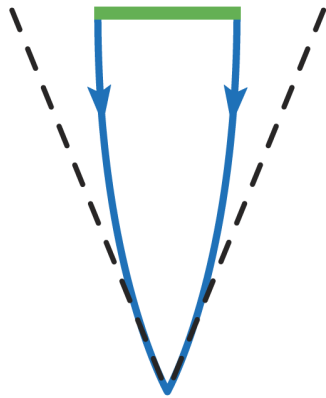
CMB spectrum: 1st peak of power spectrum

In a negatively curved Universe, the angular size of an object of known intrinsic size at a given redshift is smaller than it is in a positively curved Universe.

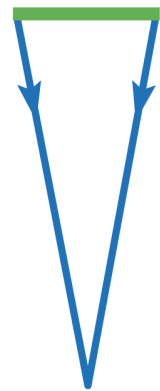
If the Universe were negatively curved, the first peak would be seen at $l > 180$ or $\theta < 1^\circ$.

If the Universe were positively curved, the first peak would be seen at $l < 180$ or $\theta > 1^\circ$.

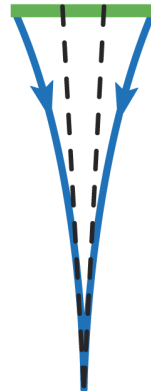
The observed peak at $l \sim 200$ is consistent with $k=0$ or $\Omega_0=1$



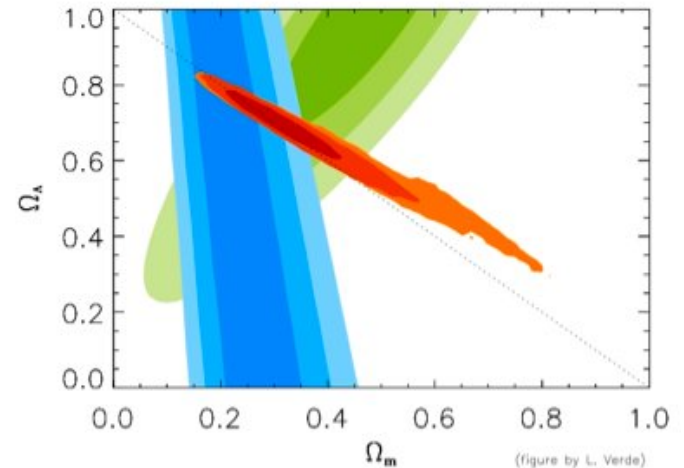
Spherical space



Flat space



Hyperbolic space



Supernovae, large scale
structure, CMB

(From M. Georganopoulos' lecture lib)

CMB spectrum: 1st peak of power spectrum

The amplitude of the first peak depends on the sound speed of the photon baryon fluid:

$$c_s = c \sqrt{w_{pb}}$$

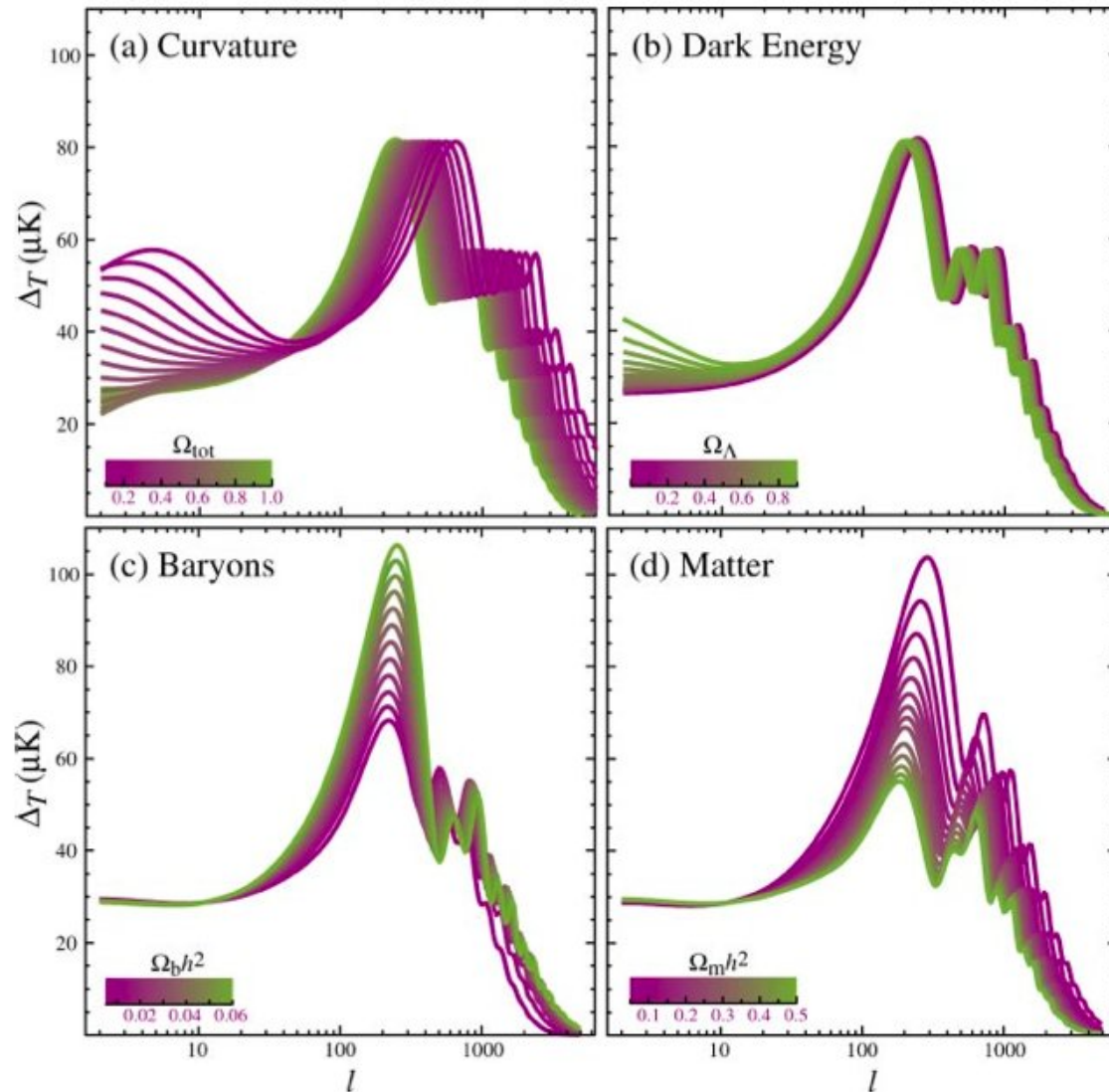
The equation of state, $p_{pw} = w_{pb} \rho$ depends of the baryon to photon ratio. To reproduce the power spectrum we need

$$\Omega_{bary,0} = 0.04 \pm 0.02.$$

How nice! This is in good agreement with the nucleosynthesis result!

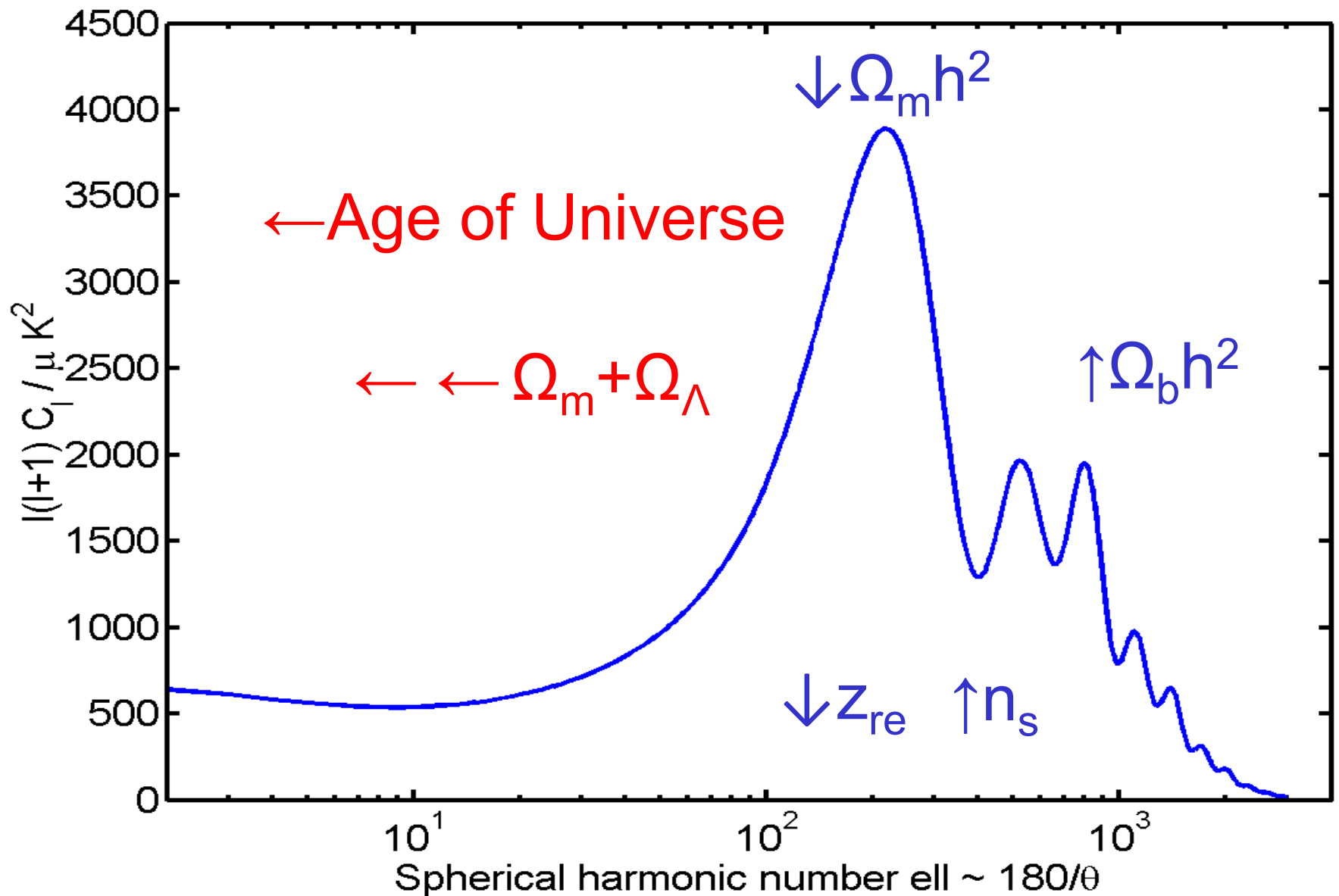
There is much more to the CMB power spectrum.

CMB spectrum: cosmic dependences



Varied around a fiducial model $\Omega_{\text{tot}}=1$, $\Omega_{\Lambda}=0.65$, $\Omega_B=0.02h^2$, $\Omega_m=0.147h^2$, $n=1$

CMB spectrum: cosmic dependences

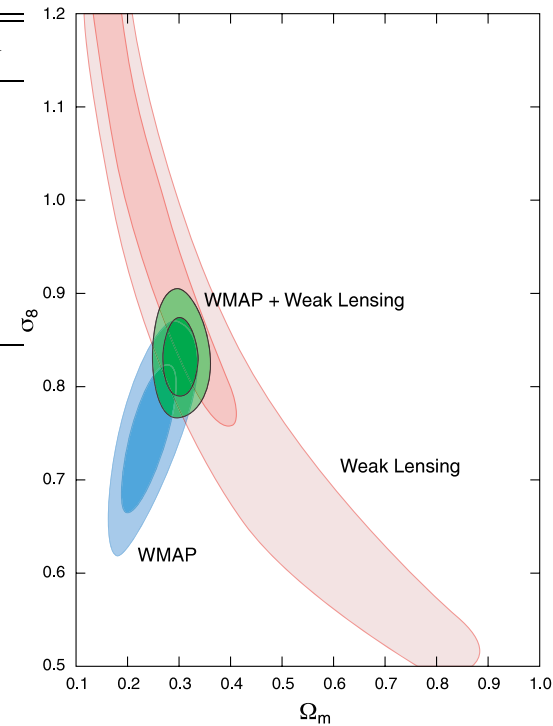


WMAP spectrum: precision cosmology

Parameter	WMAP Only	WMAP + CBI + VSA	WMAP + ACBAR + BOOMERANG
$100\Omega_b h^2$	$2.230^{+0.075}_{-0.073}$	2.208 ± 0.071	2.232 ± 0.074
$\Omega_m h^2$	$0.1265^{+0.0081}_{-0.0080}$	$0.1233^{+0.0075}_{-0.0074}$	0.1260 ± 0.0081
h	0.735 ± 0.032	0.742 ± 0.031	$0.739^{+0.033}_{-0.032}$
τ	$0.088^{+0.029}_{-0.030}$	0.087 ± 0.029	$0.088^{+0.031}_{-0.032}$
n_s	0.951 ± 0.016	0.947 ± 0.015	0.951 ± 0.016
σ_8	0.742 ± 0.051	$0.721^{+0.047}_{-0.046}$	$0.739^{+0.050}_{-0.051}$
Ω_m	0.237 ± 0.034	0.226 ± 0.031	$0.233^{+0.033}_{-0.034}$

JOINT DATA SET CONSTRAINTS ON GEOMETRY AND VACUUM ENERGY

Data Set	Ω_K	Ω_Λ
WMAP + $h = 0.72 \pm 0.08$	-0.014 ± 0.017	0.716 ± 0.055
WMAP + SDSS	$-0.0053^{+0.0068}_{-0.0060}$	0.707 ± 0.041
WMAP + 2dFGRS	$-0.0093^{+0.0098}_{-0.0092}$	$0.745^{+0.025}_{-0.024}$
WMAP + SDSS LRG	-0.012 ± 0.010	0.728 ± 0.021
WMAP + SNLS	-0.011 ± 0.012	0.738 ± 0.030
WMAP + SNGold	-0.023 ± 0.014	0.700 ± 0.031



Parameter	WMAP + SDSS	WMAP + LRG	WMAP + SNLS	WMAP + SNGold	WMAP + CFHTLS
$100\Omega_b h^2$	$2.230^{+0.071}_{-0.070}$	$2.242^{+0.069}_{-0.070}$	$2.234^{+0.075}_{-0.074}$	$2.230^{+0.069}_{-0.072}$	2.255 ± 0.067
$\Omega_m h^2$	$0.1327^{+0.0063}_{-0.0064}$	0.1336 ± 0.0049	0.1293 ± 0.0059	$0.1349^{+0.0061}_{-0.0060}$	0.1409 ± 0.0038
h	0.710 ± 0.026	$0.709^{+0.019}_{-0.018}$	0.724 ± 0.023	0.701 ± 0.021	0.687 ± 0.018
τ	$0.080^{+0.029}_{-0.030}$	0.082 ± 0.029	0.085 ± 0.030	$0.079^{+0.030}_{-0.029}$	$0.088^{+0.028}_{-0.027}$
n_s	$0.948^{+0.016}_{-0.015}$	0.951 ± 0.016	$0.950^{+0.016}_{-0.017}$	0.946 ± 0.016	0.953 ± 0.016
σ_8	$0.772^{+0.040}_{-0.041}$	0.780 ± 0.036	0.758 ± 0.041	$0.784^{+0.042}_{-0.041}$	$0.827^{+0.026}_{-0.025}$
Ω_m	0.265 ± 0.030	$0.266^{+0.020}_{-0.021}$	$0.248^{+0.024}_{-0.025}$	0.276 ± 0.026	0.300 ± 0.021

Planck spectrum: precision cosmology

Parameter	<i>Planck</i>		<i>Planck+lensing</i>		<i>Planck+WP</i>	
	Best fit	68% limits	Best fit	68% limits	Best fit	68% limits
$\Omega_b h^2$	0.022068	0.02207 ± 0.00033	0.022242	0.02217 ± 0.00033	0.022032	0.02205 ± 0.00028
$\Omega_c h^2$	0.12029	0.1196 ± 0.0031	0.11805	0.1186 ± 0.0031	0.12038	0.1199 ± 0.0027
$100\theta_{MC}$	1.04122	1.04132 ± 0.00068	1.04150	1.04141 ± 0.00067	1.04119	1.04131 ± 0.00063
τ	0.0925	0.097 ± 0.038	0.0949	0.089 ± 0.032	0.0925	$0.089^{+0.012}_{-0.014}$
n_s	0.9624	0.9616 ± 0.0094	0.9675	0.9635 ± 0.0094	0.9619	0.9603 ± 0.0073
$\ln(10^{10} A_s)$	3.098	3.103 ± 0.072	3.098	3.085 ± 0.057	3.0980	$3.089^{+0.024}_{-0.027}$
Ω_Λ	0.6825	0.686 ± 0.020	0.6964	0.693 ± 0.019	0.6817	$0.685^{+0.018}_{-0.016}$
Ω_m	0.3175	0.314 ± 0.020	0.3036	0.307 ± 0.019	0.3183	$0.315^{+0.016}_{-0.018}$
σ_8	0.8344	0.834 ± 0.027	0.8285	0.823 ± 0.018	0.8347	0.829 ± 0.012
z_{re}	11.35	$11.4^{+4.0}_{-2.8}$	11.45	$10.8^{+3.1}_{-2.5}$	11.37	11.1 ± 1.1
H_0	67.11	67.4 ± 1.4	68.14	67.9 ± 1.5	67.04	67.3 ± 1.2
$10^9 A_s$	2.215	2.23 ± 0.16	2.215	$2.19^{+0.12}_{-0.14}$	2.215	$2.196^{+0.051}_{-0.060}$
$\Omega_m h^2$	0.14300	0.1423 ± 0.0029	0.14094	0.1414 ± 0.0029	0.14305	0.1426 ± 0.0025
$\Omega_m h^3$	0.09597	0.09590 ± 0.00059	0.09603	0.09593 ± 0.00058	0.09591	0.09589 ± 0.00057
Y_p	0.247710	0.24771 ± 0.00014	0.247785	0.24775 ± 0.00014	0.247695	0.24770 ± 0.00012
Age/Gyr	13.819	13.813 ± 0.058	13.784	13.796 ± 0.058	13.8242	13.817 ± 0.048
z_*	1090.43	1090.37 ± 0.65	1090.01	1090.16 ± 0.65	1090.48	1090.43 ± 0.54
r_*	144.58	144.75 ± 0.66	145.02	144.96 ± 0.66	144.58	144.71 ± 0.60
$100\theta_*$	1.04139	1.04148 ± 0.00066	1.04164	1.04156 ± 0.00066	1.04136	1.04147 ± 0.00062
z_{drag}	1059.32	1059.29 ± 0.65	1059.59	1059.43 ± 0.64	1059.25	1059.25 ± 0.58
r_{drag}	147.34	147.53 ± 0.64	147.74	147.70 ± 0.63	147.36	147.49 ± 0.59
k_D	0.14026	0.14007 ± 0.00064	0.13998	0.13996 ± 0.00062	0.14022	0.14009 ± 0.00063
$100\theta_D$	0.161332	0.16137 ± 0.00037	0.161196	0.16129 ± 0.00036	0.161375	0.16140 ± 0.00034
z_{eq}	3402	3386 ± 69	3352	3362 ± 69	3403	3391 ± 60
$100\theta_{eq}$	0.8128	0.816 ± 0.013	0.8224	0.821 ± 0.013	0.8125	0.815 ± 0.011
$r_{drag}/D_V(0.57)$	0.07130	0.0716 ± 0.0011	0.07207	0.0719 ± 0.0011	0.07126	0.07147 ± 0.00091

Planck spectrum: precision cosmology

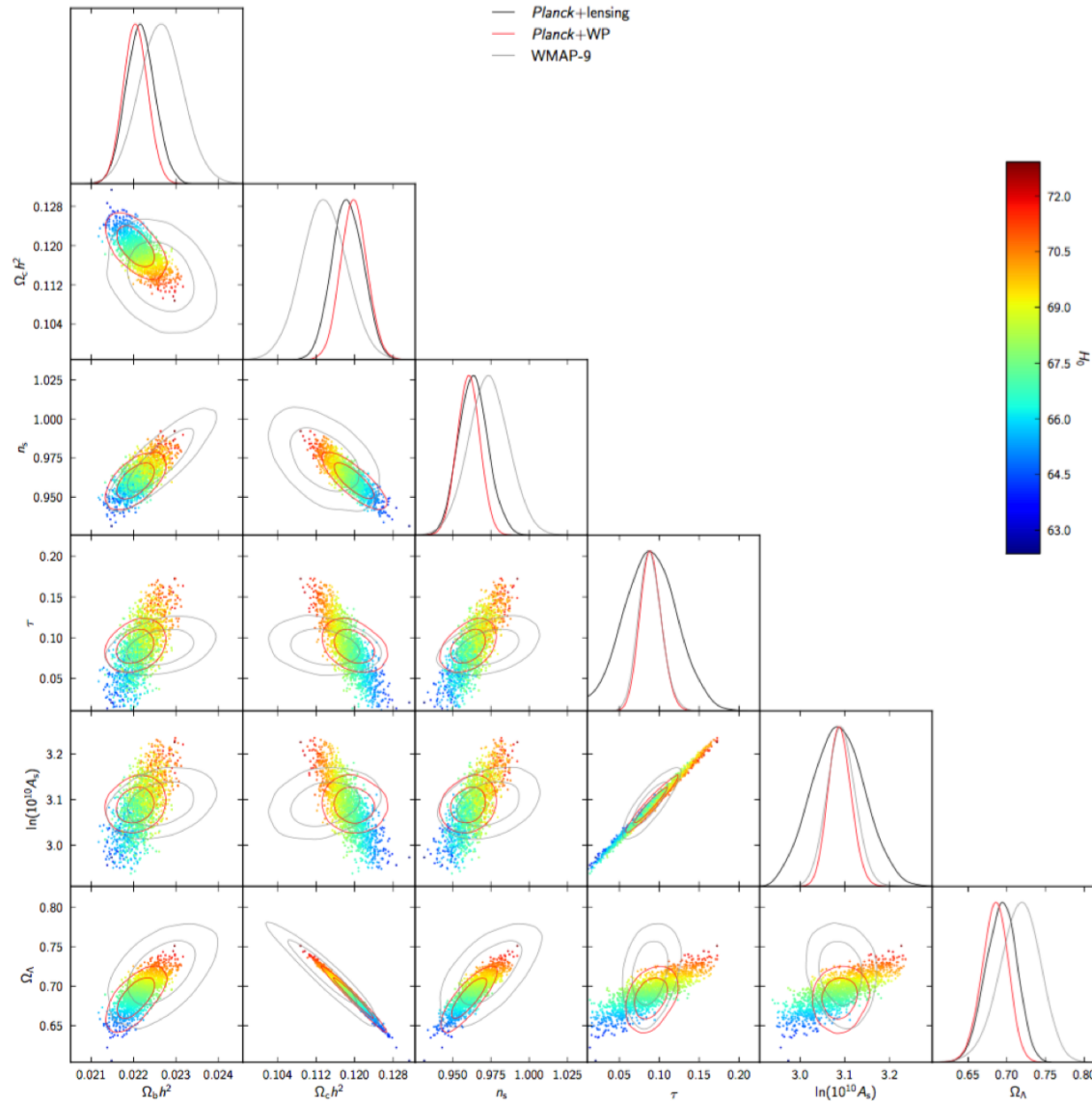


Fig. 2. Comparison of the base Λ CDM model parameters for *Planck*+lensing only (colour-coded samples), and the 68% and 95% constraint contours adding WMAP low- ℓ polarization (WP; red contours), compared to WMAP-9 (Bennett et al. 2013; grey contours).

Planck spectrum: precision cosmology

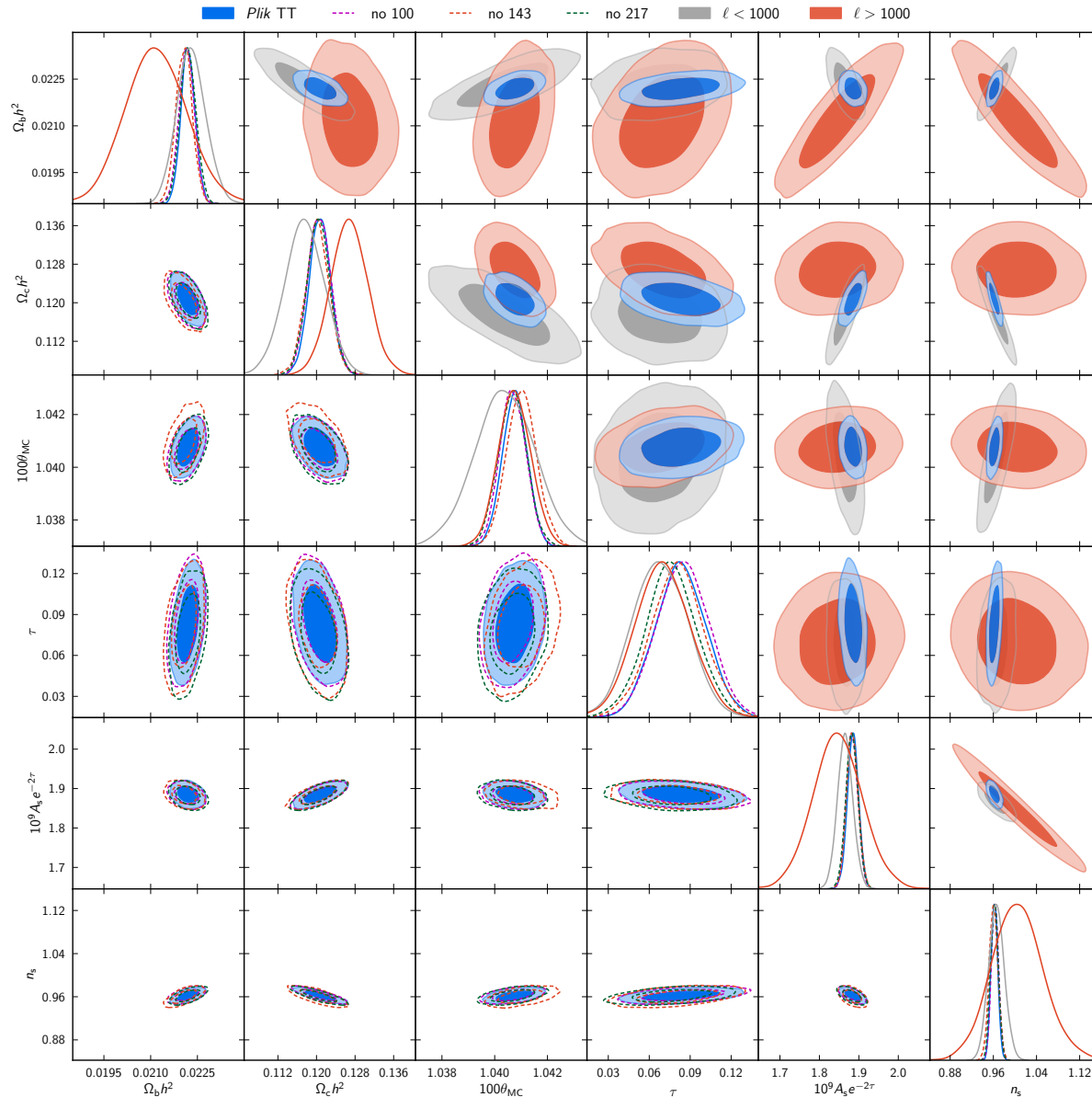


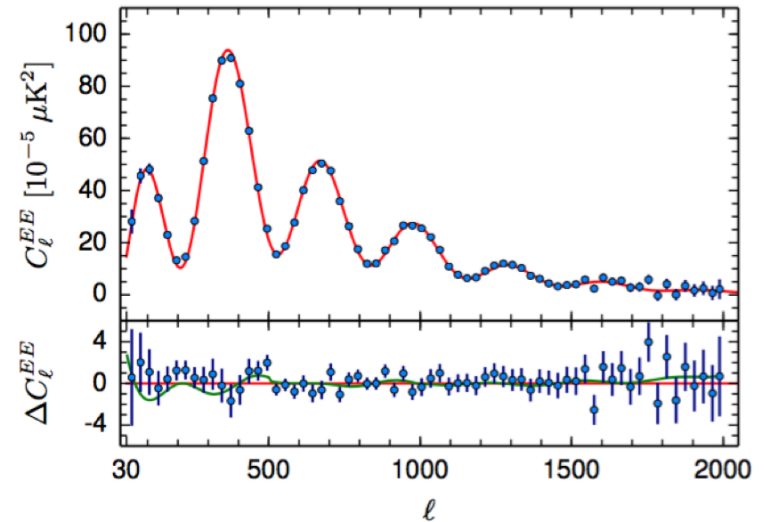
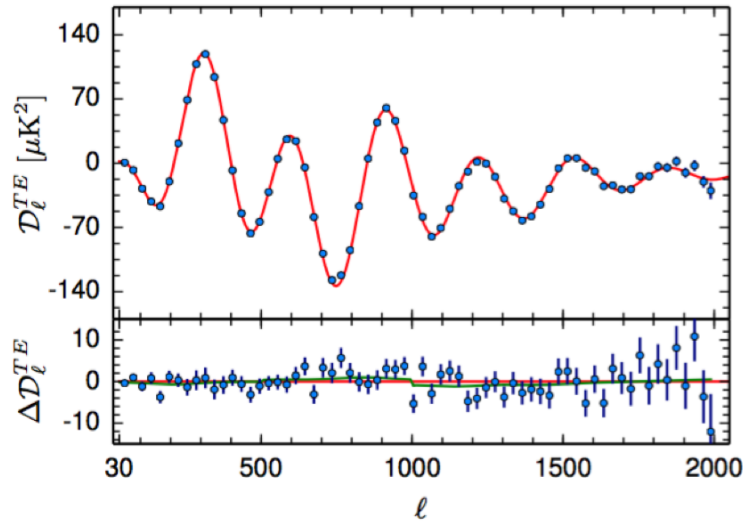
Fig. 33. Λ CDM parameters posterior distribution for PlikTT+tauprior. The lower left triangle of the matrix displays how the constraints are modified when the information from one of the frequency channels is dropped. The upper right triangle displays how the constraints are modified when the information from multipoles ℓ greater or less than 1000 is dropped. All the results shown in this figure were obtained using the CAMB code.

CMB Polarization: precision cosmology

Table 3. Parameters of the base Λ CDM cosmology computed from the 2015 baseline *Planck* likelihoods, illustrating the consistency of parameters determined from the temperature and polarization spectra at high multipoles. Column [1] uses the *TT* spectra at low and high multipoles and is the same as column [6] of Table 1. Columns [2] and [3] use only the *TE* and *EE* spectra at high multipoles, and only polarization at low multipoles. Column [4] uses the full likelihood. The last column lists the deviations of the cosmological parameters determined from the *Planck* TT+lowP and *Planck* TT,TE,EE+lowP likelihoods.

Parameter	[1] <i>Planck</i> TT+lowP	[2] <i>Planck</i> TE+lowP	[3] <i>Planck</i> EE+lowP	[4] <i>Planck</i> TT,TE,EE+lowP	([1] – [4])/ $\sigma_{[1]}$
$\Omega_b h^2$	0.02222 ± 0.00023	0.02228 ± 0.00025	0.0240 ± 0.0013	0.02225 ± 0.00016	–0.1
$\Omega_c h^2$	0.1197 ± 0.0022	0.1187 ± 0.0021	$0.1150^{+0.0048}_{-0.0055}$	0.1198 ± 0.0015	0.0
$100\theta_{\text{MC}}$	1.04085 ± 0.00047	1.04094 ± 0.00051	1.03988 ± 0.00094	1.04077 ± 0.00032	0.2
τ	0.078 ± 0.019	0.053 ± 0.019	$0.059^{+0.022}_{-0.019}$	0.079 ± 0.017	–0.1
$\ln(10^{10} A_s)$	3.089 ± 0.036	3.031 ± 0.041	$3.066^{+0.046}_{-0.041}$	3.094 ± 0.034	–0.1
n_s	0.9655 ± 0.0062	0.965 ± 0.012	0.973 ± 0.016	0.9645 ± 0.0049	0.2
H_0	67.31 ± 0.96	67.73 ± 0.92	70.2 ± 3.0	67.27 ± 0.66	0.0
Ω_m	0.315 ± 0.013	0.300 ± 0.012	$0.286^{+0.027}_{-0.038}$	0.3156 ± 0.0091	0.0
σ_8	0.829 ± 0.014	0.802 ± 0.018	0.796 ± 0.024	0.831 ± 0.013	0.0
$10^9 A_s e^{-2\tau}$	1.880 ± 0.014	1.865 ± 0.019	1.907 ± 0.027	1.882 ± 0.012	–0.1

TT is ~ x20 higher!



(Planck Collaboration 2015)



**The End ...
of the Beginning**