3. Classical problems of the Standard Big Bang Model and an inflationary solution

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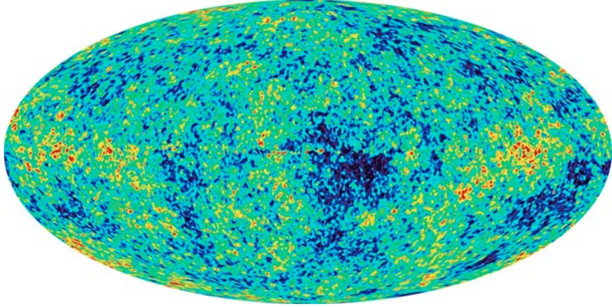
2020

The four classical problems of the BB model:

- Horizon problem:
  - Why is the CMB so smooth?
- The flatness problem:
  - Why is $\Omega \sim 1$? Why is the universe flat ?
- The initial fluctuation problem:
  - Where do the very initial fluctuations that seed the fluctuations we observe in the CMB come from?
- The monopole problem
  - Why aren't there magnetic monopoles (and why is there more matter than antimatter)?

The horizon problem

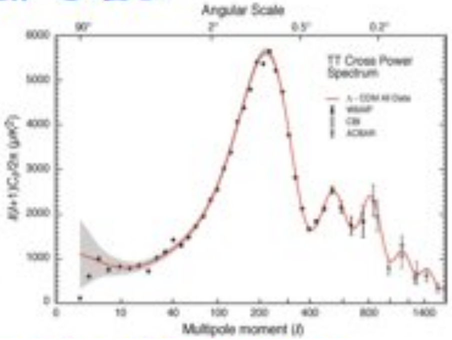


At the time of last scattering the Universe was matter dominated and the Hubble distance was:

$$\frac{c}{H(z_{ls})} = \frac{c}{H_0(z+1)^{3/2}} = \frac{3\times10^{8}\ m\,s^{-1}}{1.24\times10^{-18}\ s^{-1}(1101)^{3/2}} \approx 6.6\times10^{21}\ m \approx 0.2\ Mpc$$

Seen from Earth this has an angular size:

$$\theta_H = \frac{c/H(z_{ls})}{d_A} \approx \frac{0.2\,Mpc}{13\,Mpc} \approx 0.015\ rad \approx 1^o$$



Why is the CMB so smooth at scales > 2° if these regions were not causally connected? Why do they have the same T?

(From M. Georganopoulos' lecture lib)

The flatness problem

- **a multi-component universe satisfies**

$$1-\Omega(t) = -\frac{kc^2}{H(t)^2 a(t)^2} = \frac{H_0^2(1-\Omega_0)}{H(t)^2 a(t)^2}$$

and, neglecting Λ, $$\left(\frac{H(t)}{H_0}\right)^2 = \frac{\Omega_{r0}}{a^4} + \frac{\Omega_{m0}}{a^3}$$

- **therefore**
  - during radiation dominated era $|1 - \Omega(t)| \propto a^2$
  - during matter dominated era $|1 - \Omega(t)| \propto a$
  - if $|1 - \Omega_0| < 0.01$ (WMAP), then at CMB emission $|1 - \Omega| < 0.00001$
- **we have a fine tuning problem!**

Why is the Universe always so very close to flat?

$-0.0133 < \Omega_k < 0.0084$ at 95% CL (Komatsu et al. 2011)

(Adapted from S. Cartwright's lecture lib)

The flatness problem

SEVEN-YEAR WILKINSON MICROWAVE ANISOTROPY PROBE (*WMAP*[1]) OBSERVATIONS: COSMOLOGICAL INTERPRETATION

E. KOMATSU[2], K. M. SMITH[3], J. DUNKLEY[4], C. L. BENNETT[5], B. GOLD[5], G. HINSHAW[6], N. JAROSIK[7], D. LARSON[5], M. R. NOLTA[8], L. PAGE[7], D. N. SPERGEL[3,9], M. HALPERN[10], R. S. HILL[11], A. KOGUT[6], M. LIMON[12], S. S. MEYER[13], N. ODEGARD[11], G. S. TUCKER[14], J. L. WEILAND[11], E. WOLLACK[6], AND E. L. WRIGHT[15]

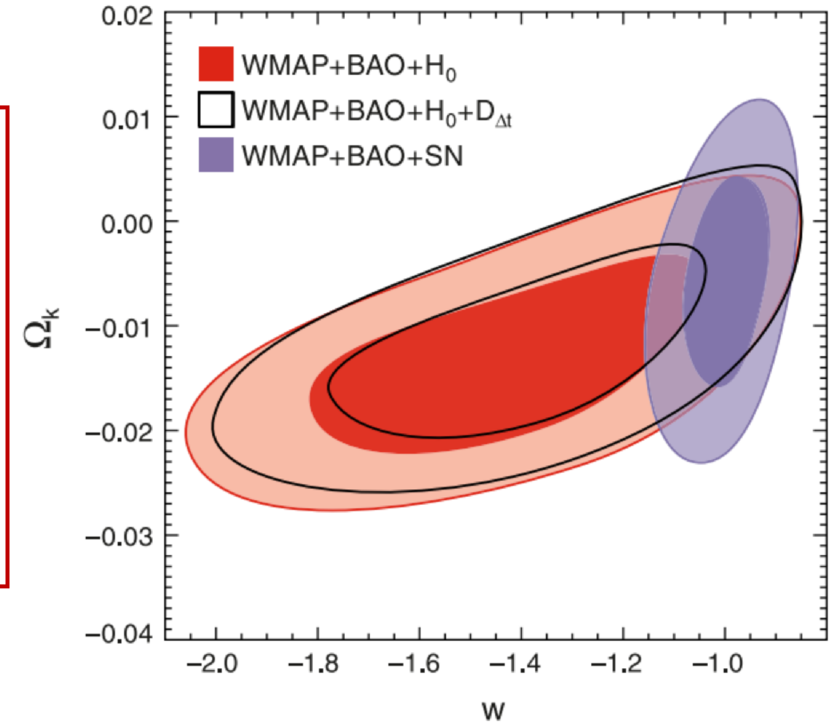
4.3. *Spatial Curvature*

While the *WMAP* data alone cannot constrain the spatial curvature parameter of the observable universe, Ω_k, very well, combining the *WMAP* data with other distance indicators such as H_0, BAO, or supernovae can constrain Ω_k (e.g., Spergel et al. 2007).

Assuming a ΛCDM model ($w = -1$), we find

$$-0.0133 < \Omega_k < 0.0084 \text{ (95\% CL)},$$

from *WMAP*+BAO+H_0.[22] However, the limit weakens



Why is the Universe always so very close to flat?

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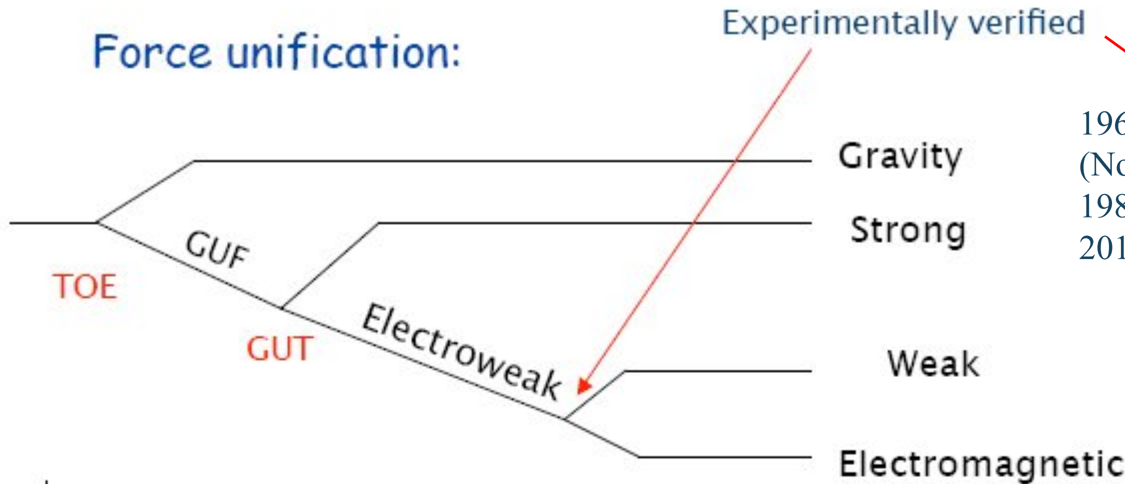
The monopole problem: unification

Force unification:



1968: Glashow, Salam, Weinberg (Nobel 1979)
1983 CERN (W & Z bosons) +
2012 LHC (Higgs boson)

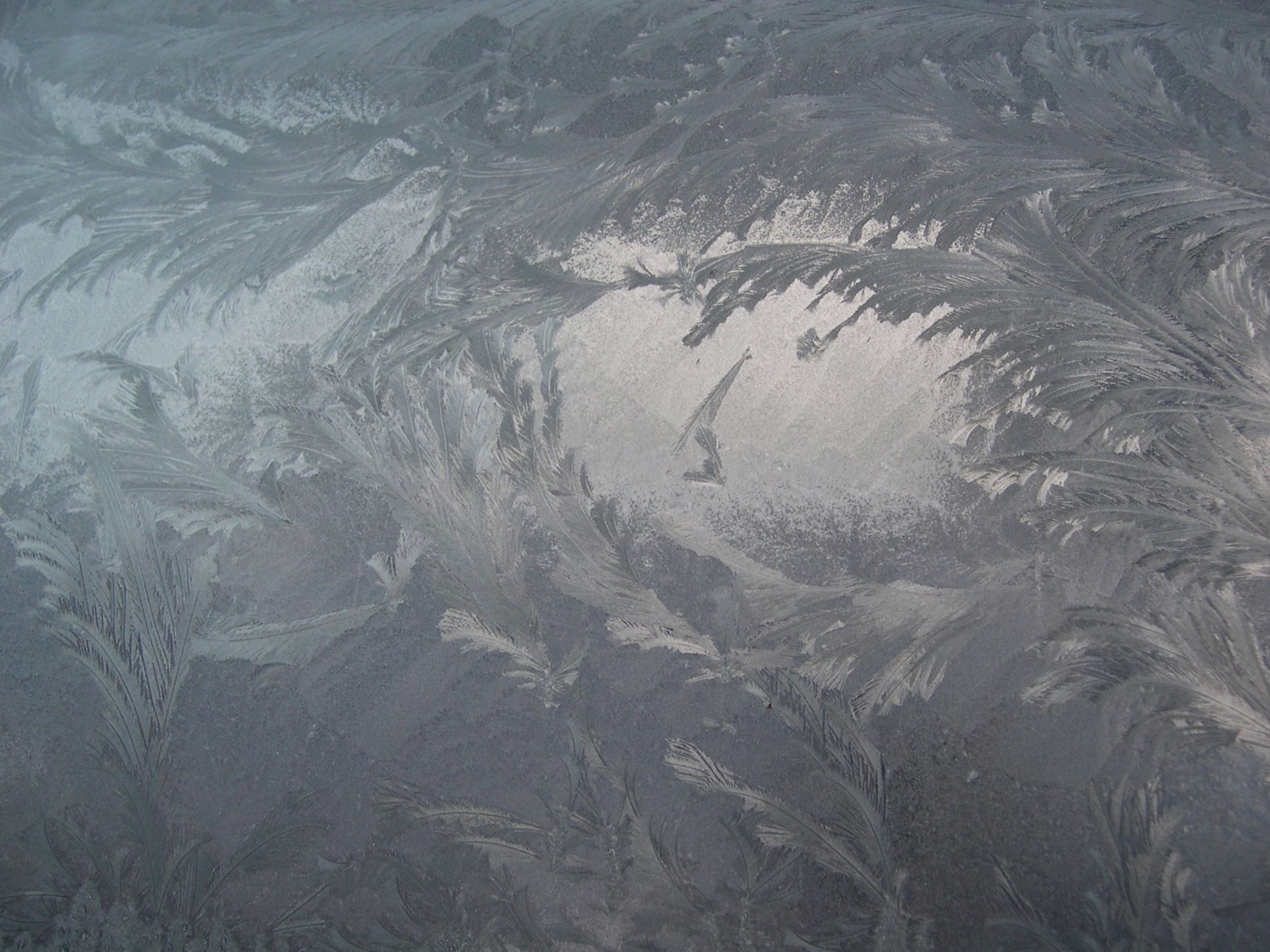
E (TeV)	10^{16}	10^{12}	1
T (K)	10^{32}	10^{28}	10^{16}
t (s)	10^{-43}	10^{-36}	10^{-12}

TOE: Theory of everything…
GUT: Grand Unified Theory
GUF: Grand Unified Force

Grand Unified Theories of particle physics: at high energies the strong, electromagnetic and weak forces are unified. But the symmetry between strong and electroweak forces 'breaks' at an energy of $\sim 10^{15}$ GeV ($T \sim 10^{28}$ K, $t \sim 10^{-36}$ s)

– this is a phase transition similar to freezing

– expect to form 'topological defects' (like defects in crystals)



The monopole problem

THE GUT phase transition gives rise to point-like topological defects that act as **magnetic monopoles**. Their rest mass energy is predicted to be $m_M \sim E_{GUT} \sim 10^{12}$ TeV. This is HUGE for a single elementary particle, it is ~ the mass of a bacterium!

When the GUT transition took place, areas equal to the horizon volume back then came to contanct with each other, forming ~ one topological defect per horizon volume:

$$n_M(t_{GUT}) \sim \frac{1}{(2ct_{GUT})^3} \sim 10^{82}\, m^{-3}, \quad \varepsilon_M(t_{GUT}) \sim m_M c^2 n_M \sim 10^{94}\, TeV\ m^{-3}$$

Size of the Horizon

This is much smaller than the energy density of radiation:

$$\varepsilon_\gamma(t_{GUT}) \sim aT_{GUT}^4 \sim 10^{104}\, TeV\ m^{-3}$$

Radiation dominated at GUT phase transition

(From M. Georganopoulos' lecture lib) (A. Guth, H. Tye, John Preskill)

The monopole problem

Although at GUT the Universe was radiation dominated,
because $\rho_{rad} \propto 1/a^4 \quad \rho_M \propto 1/a^3$,
by the time the scale factor has increased by ~10^{10}
($T \sim 10^{-10}\, T_{GUT} \sim 10^{18}$ K, $t \sim t_{GUT}\, 10^{20} \sim 10^{-16}$ s)
the Universe would be dominated by magnetic monopoles.

But we haven't seen a single magnetic monopole. Where are they?

From all three problems with the Universe, this is the most "model dependent", because it is based on GUT, an unproven theory.

Even if you are willing to put this one aside,
The flatness and horizon problems should make you loose your sleep

(From M. Georganopoulos' lecture lib)

Inflation: GUT comes to the rescue

Developed primarily by Alan Guth (Henry Tye), Andrei Linde and Alexei Starobinsky in the 80's as a consequence of GUTs. Here we will explore these consequences, not the mechanism that produces inflation.

Postulate: there was a period in the early Universe when the expansion of the Universe was accelerating.

$$INFLATION \Leftrightarrow \ddot{a}(t) > 0$$

Recall the acceleration equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)$$

Accelerated expansion requires a negative pressure:

$$p < -\frac{\rho c^2}{3}$$

(From M. Georganopoulos' lecture lib)

Inflation: exponential growth

We have already studied a Universe possessing a cosmological Constant, with $p=-\rho c^2$. Friedmann's equation in this case is:

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda_i}{3}$$

Expansion reduces the first two terms very quickly, so we are left with the Λ-term only:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda_i}{3} \Rightarrow \dot{a} = a\sqrt{\frac{\Lambda_i}{3}} \Rightarrow a(t) \propto \exp\left(t\sqrt{\frac{\Lambda_i}{3}}\right)$$

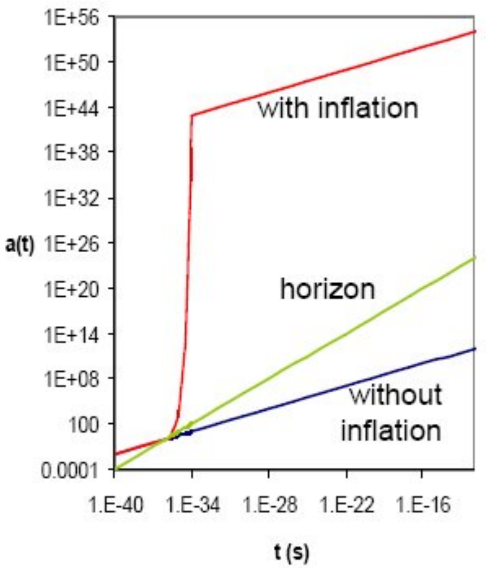
The Hubble parameter is constant,
the Universe is expanding exponentially

(From M. Georganopoulos' lecture lib)

Inflation: exponential growth

Assume that the inflationary period started at $t=t_i$ and finished at $t=t_f$, and that during this time $H=H_i$.

$$a(t) = \begin{cases} a_i (t/t_i)^{1/2} & t < t_i \\ a_i \exp[H_i(t-t_i)] & t_i < t < t_f \\ a_i \exp[H_i(t_f - t_i)](t/t_f)^{1/2} & t > t_f \end{cases}$$



The scale factor increases by a factor: $\frac{a(t_f)}{a(t_i)} = \exp(N), \quad N = H_i(t_f - t_i)$

If the duration of inflation was long compared to the Hubble time $1/H_i$, N was large and the increase of the scale factor huge.

<u>Example:</u> suppose that inflation started around the GUT time, $t_i=t_{GUT}=10^{-36}$ s => $H_i=1/t_{GUT}=10^{36}$ s^{-1} and stopped at $t_f=10^{-34}$ s.

Then

$$\frac{a(t_f)}{a(t_i)} = \exp[H_i(t_f - t_i)] = \exp(99) \approx 10^{43} \quad !!!$$

(From M. Georganopoulos' lecture lib)

Inflation versus Dark Energy

How does the energy density of the cosmological constant during inflation compares to that of the current cosmological constant?

Recall that both follow:

$$\rho_\Lambda = \frac{c^2}{8\pi G}\Lambda = \frac{3c^2}{8\pi G}H^2$$

Since now $H \sim 10^{-18}\ s^{-1}$, while at inflation $H \sim 10^{36}\ s^{-1}$, the cosmological constant density at inflation was 108 orders of magnitude higher!!!

Obviously, the inflationary Λ comes from a very different mechanism than the Λ we experience now.

(From M. Georganopoulos' lecture lib)

Inflation and the flatness problem

Recall from last time: $$\left|\Omega_{tot}(t) - 1\right| = \frac{|k|}{a^2 H^2}$$

What happens to Ω_{tot} during inflation?

$$\ddot{a} > 0 \Rightarrow \frac{d}{dt}(\dot{a}) > 0 \Rightarrow \frac{d}{dt}(aH) > 0$$

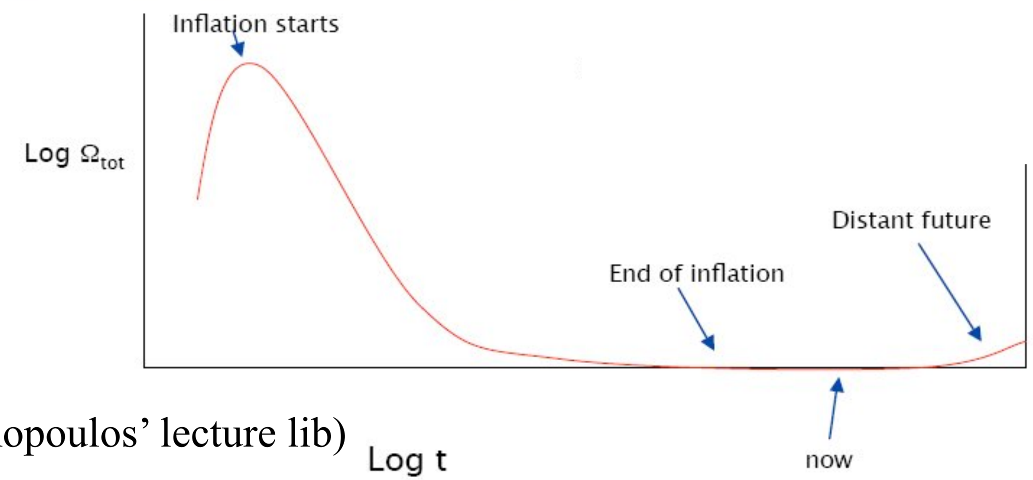
$$H^2 = \Lambda_i c^2 / 3$$

Ω_{tot} gets closer to 1 !!!
In the special case of exponential expansion:

$$\left|\Omega_{tot}(t) - 1\right| \propto \exp\left(-t\sqrt{\frac{4\Lambda_i}{3}}\right) = \exp(-2H_i t)$$

In our previous example $$2H_i(t_f - t_i) \approx 200$$

Ω_{tot} gets extremely-extremely-extremely close to 1!



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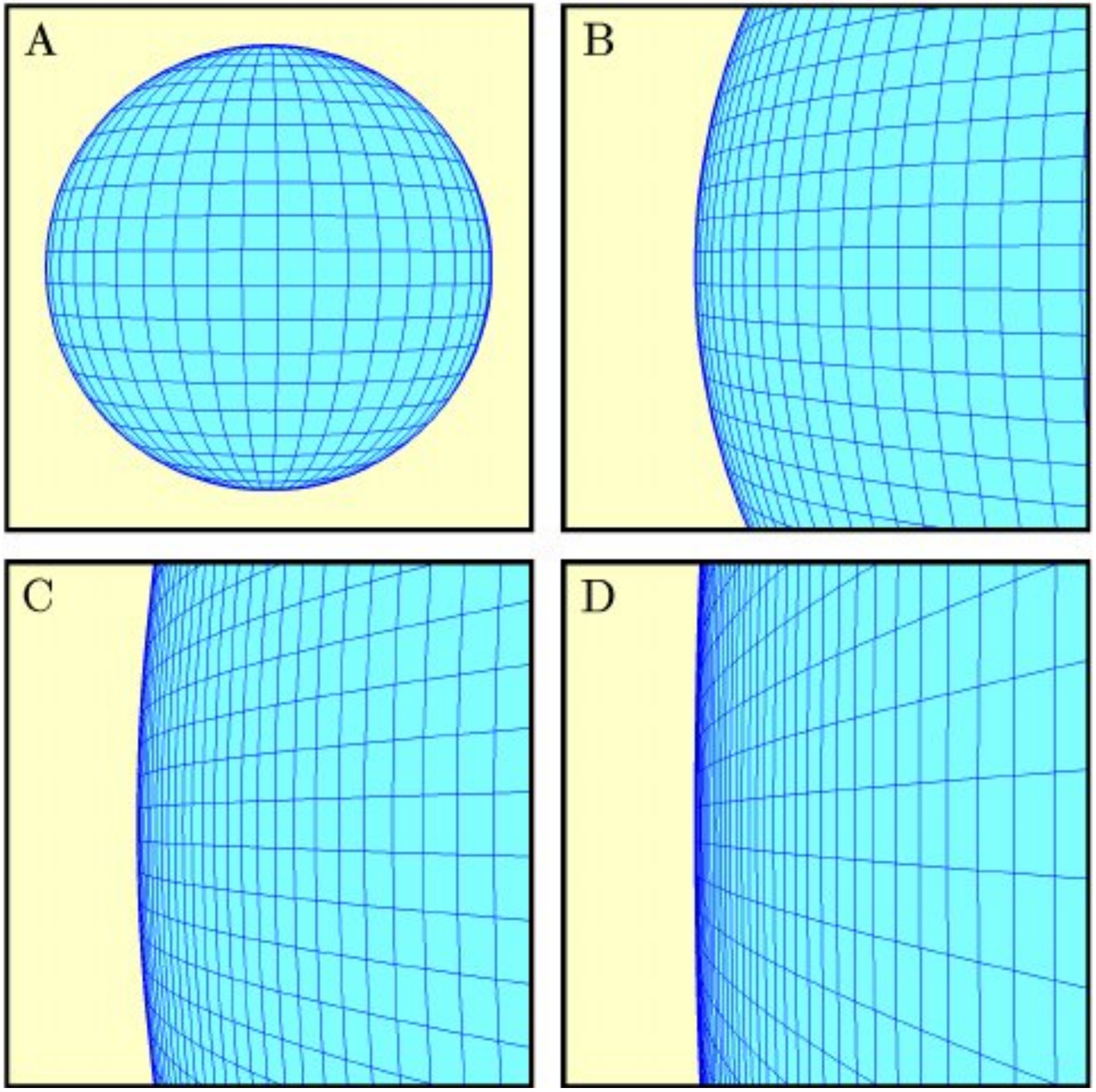
Inflation and the flatness problem



Inflation and the horizon problem

At the beginning of inflation:

$$d_{hor}(t_i) = a_i \int_0^{t_i} \frac{c\ dt'}{a_i (t/t_i)^{1/2}} = 2ct_i$$

At the end of inflation:

$$d_{hor}(t_f) = a_i\, e^N c \left[\int_0^{t_i} \frac{dt'}{a_i (t'/t_i)^{1/2}} + \int_{t_i}^{t_f} \frac{dt'}{a_i \exp[H_i (t' - t_i)]} \right] = e^N c\,(2t_i + 1/H_i) = 3c\, t_i\, e^N$$

For $t_i = 10^{-36}$ s, N=100, $d_{hor}(t_i) = 6\times10^{-28}$ m, $d_{hor}(t_f) = 2\times10^{16}$ m~0.8 pc

Inflation increases the post-inflation horizon distance by e^N relative to the value it would have without inflation.

The horizon distance at last scattering is not 0.4 Mpc, as we found it to be without inflation. With inflation it is larger by a factor $e^{100} = 10^{43}$, which is more than enough for the whole last scattering surface to be in causal contact.

(From M. Georganopoulos' lecture lib)

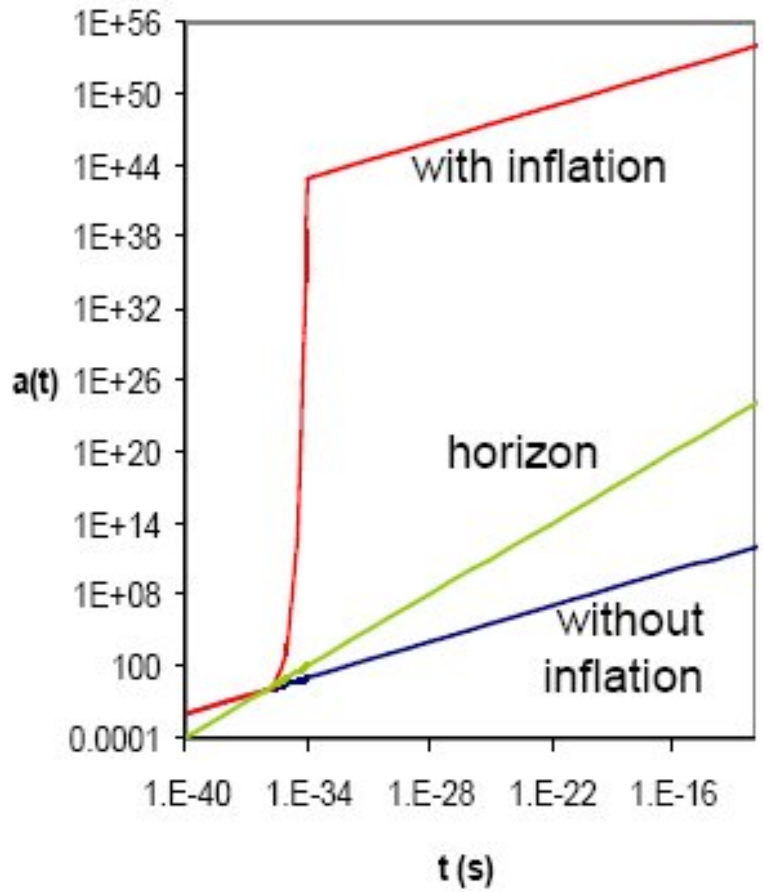
Inflation and the horizon problem

At the begi | $)^{1/2}$ = $2ct_i$
of inflatio

$d_{hor}(t_f) = a_i\, e^N c[\int_0^{t_i} \frac{}{a_i}$ | $+1/H_i) = 3c\, t_i\, e^N$

For $t_i = 10^{-36}$ s, | 2×10^{16} m~0.8 pc

Inflation incre | tance by e^N
relative | inflation.
The horizon dist | pc, as we found
it to be withou | er by a factor
$e^{100} = 10^{43}$, which | e last scattering



(From M. Georganopoulos' lecture lib)

Inflation and the horizon problem

Current proper distance of last scattering: $d_p(t_0) \approx 1.4 \times 10^4\ Mpc$

At the end of inflation $t_f \sim 10^{-34}$ s -> $a_f \sim 2 \times 10^{-27}$

$$d_p(t_f) = a_f d_p(t_0) \sim 3 \times 10^{-23}\ Mpc \sim 0.9\ m$$

At the end of inflation all the Universe we see today was packed in a sphere the size of a man.

Even more amazing, before inflation all the visible Universe was packed in a sphere or radius:

$$d_p(t_i) = e^{-N} d_p(t_f) \sim 3 \times 10^{-44}\ m$$

This is much smaller than the horizon distance at t_i, $d_{hor}(t_i) \sim 6 \times 10^{-28}$ m, so there was plenty of time to reach thermal uniformity.

(From M. Georganopoulos' lecture lib)

Inflation and the monopole problem

Assuming that magnetic monopoles were created before or during inflation, their number density was diluted below detectability because of the exponential expansion.

Start with a number density at GUT: $n_M(t_{GUT}) \sim 10^{82}\ m^{-3}$

By the end of inflation: $n_M(t_f) \sim e^{-300}\ 10^{82}\ m^{-3} \sim 5 \times 10^{-49}\ m^{-3} \sim 15\ pc^{-3}$

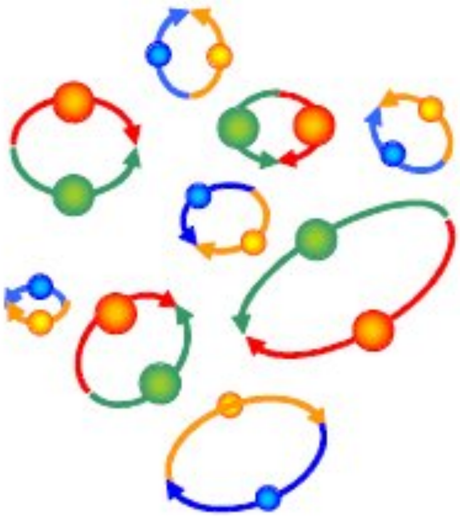
Today, after an extra expansion of $\sim 10^{27}$, $n_M(t_0) \sim 10^{-61}\ Mpc^{-3}$!!!

Not a single one inside our last scattering surface!
So, it's normal that we do not see them at all.

(From M. Georganopoulos' lecture lib)

Inflation and the source of structure

- Uncertainty Principle means that in quantum mechanics vacuum constantly produces temporary particle-antiparticle pairs
  - minute density fluctuations
  - inflation blows these up to macroscopic size
  - seeds for structure formation
- Expect spectrum of fluctuations to be approximately scale invariant
  - possible test of inflation idea?



(From S. Cartwright's lecture lib)

The driver behind Inflation

Inflation is thought to be related to a phase transition controlled by a form of matter known as a scalar field. Scalar fields can have negative pressure, satisfying the inflationary condition. After the phase transition is over, the scalar field decays away and inflation stops. There are many variants of inflation.

- Universe in the state of false vacuum
- energy of Universe dominated by vacuum energy
- Universe expands exponentially
- In some models, when it transits to true vacuum matter/antimatter is created and inflation ends.