

# A BAYESIAN PHOTOMETRIC REDSHIFT TECHNIQUE FOR MM-SELECTED GALAXIES

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**Abstract** We present a Bayesian technique, which uses a luminosity function prior, to calculate photometric redshifts for optically-obscured starburst galaxies. An example is compared with previous Monte Carlo simulations.

Multi-wavelength measurements of the SEDs of local galaxies have led to the development of radio-(sub)mm-FIR colour indices in order to derive photometric redshifts for the population of dust-enshrouded starburst galaxies identified in the blank-field SCUBA and MAMBO surveys (Hughes et al. 1998, 2002; Carilli & Yun 2000; Yun & Carilli 2002; Wiklind 2003; Aretxaga et al. 2003). The bland spectral features in the radio-FIR regime lead to a degeneracy between the colours and redshift (e.g. Blain et al. 2003). The inclusion of luminosity information breaks some of that degeneracy, and provides robust redshift estimates with uncertainties of  $\Delta z \simeq \pm 0.5$  determined from Monte Carlo simulations. We present a continuation of our work (see also Hughes - these proceedings) which, instead of using simulated catalogues, provides an analytical expression for the redshift probability distribution, given observed data, and prior information - the luminosity function of mm galaxies. These calculations produce a higher resolution redshift probability distribution than the Monte Carlo technique in a fraction of the computation time.

Given a set of flux density measurements  $S = \{s_i\}$ , and some prior information  $I$  the redshift probability density function  $p(z|S, I)$  is factored using a general form of Bayes' theorem in three variables:

$$p(z|S, I) = p(S|z, I)p(z|I)/p(S|I) \quad (1)$$

\*Partial funding for this work was provided by NSERC grant PGSB-253705-2002, CONACyT grants 39953-F and 39548-F, INAOE, and the conference local organizing committee.

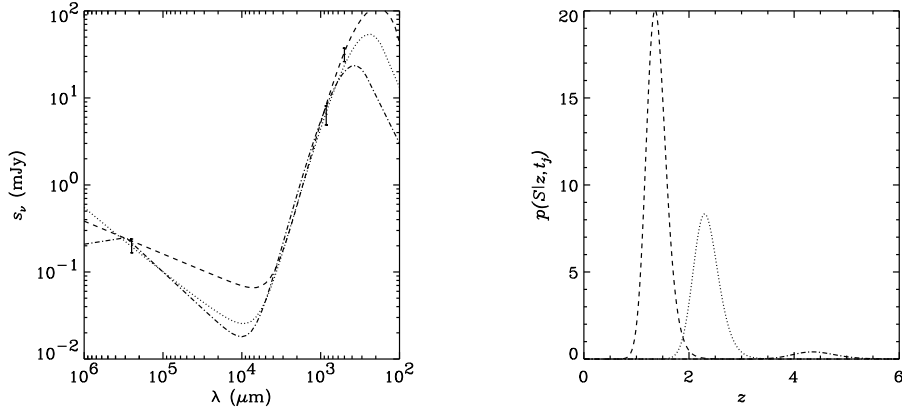


Figure 1. *left panel:* Simulated data at 1.2GHz, 850 $\mu$ m and 500 $\mu$ m with 10% errors, for a galaxy with the same colours as M82 lying at  $z=2.5$ , scaled to an observed 850 $\mu$ m flux density of 8 mJy. Observed SED models (*dashed:* Arp220, *dotted:* M82, and *dot-dashed:* Mrk231) are over-plotted using the maximum likelihood models. *right panel:* Likelihood functions integrated over  $a$  for the three models. For this realization of the noise, the data is more consistent with the incorrect model (an Arp220-like SED at  $z \simeq 1.5$ ) than the true model (an M82-like SED at  $z \simeq 2.5$ ).

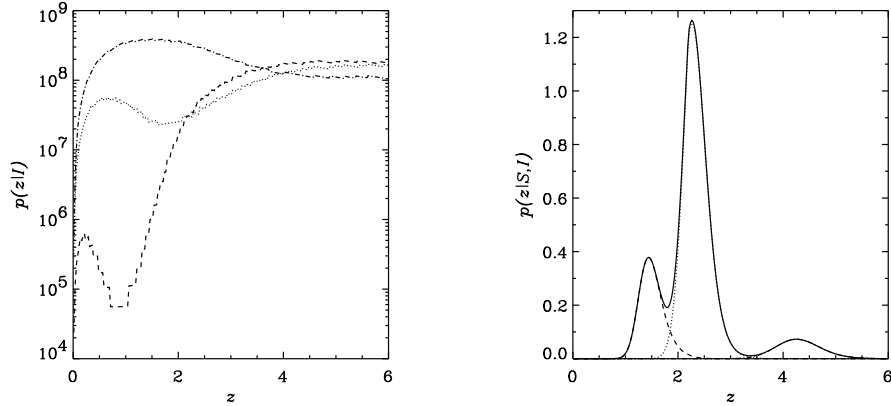


Figure 2. *left panel:* The redshift prior evaluated at the maximum likelihood values of  $a$  for each model, as a function of  $z$ , illustrating the relative weighting to be applied to the likelihood functions. *right panel:* The Bayesian redshift probability distribution calculated from the product of the prior with the sum of the three likelihood functions, and then integrated over  $a$  (solid line). The weighted likelihood functions are plotted beneath the total distribution. Weighting the Arp220 model at lower redshift by the much weaker prior probability with respect to the M82 model correctly assigns the maximum probability density to the correct redshift of  $z \simeq 2.5$ .

The first term in the numerator is the *likelihood function*. In the absence of prior information, the likelihood is the probability of measuring  $S$  given an object at redshift  $z$ . Maximizing this function through model fitting yields the *maximum likelihood* solution. This solution does not however identify the best model from the set of all models with which the data are consistent. If some other information is known beforehand, the second *prior* term in the numerator can include an appropriate weight to the likelihood function to find the most probable solution. Ignoring the denominator in Eq. 1 which serves only as a normalization constant, a model for the observed SED is required to calculate the likelihood function, and a prior is constructed from the probability of observing an object with the luminosity of that model.

The observed SED for mm galaxies is modeled by redshifting a rest-frame SED template  $t_j(\lambda)$  and multiplying it by a scale factor  $a$ ,  $\tilde{s}_i = at_j(\lambda_i/(1+z))$ . Since  $\tilde{s}_i$  is determined completely from the model parameters, a standard likelihood function is constructed assuming independent Gaussian errors in the data, independently of the prior, of the form  $p(S|z, t_j, a)$ . Using simulated data at three FIR-radio wavelengths, Figure 1 illustrates this procedure for three different SED templates.

The prior is constructed from the  $60\mu\text{m}$  luminosity function (Saunders et al. 1990) which undergoes pure luminosity evolution as  $(1+z)^{3.2}$  for  $0 < z < 2.2$ , and constant evolution for  $2.2 < z < 10$ . Taking  $a$  as the  $60\mu\text{m}$  flux density of the model galaxy in the rest-frame:

$$p(z, a|I) = (N(L(a, z), z)/N_T)(dL(a, z)/da) \quad (2)$$

where  $N(L, z)$  is the number density of objects as a function of  $60\mu\text{m}$  luminosity and redshift (the product of the evolving luminosity function and the differential volume element),  $N_T$  is the total number of objects in the Universe, and  $L(a, z)$  is the  $60\mu\text{m}$  luminosity corresponding to the flux density  $a$  at redshift  $z$ .

The Bayesian redshift probability is a function of  $t_j$  and  $a$ . Assuming that the colours of *any* observed galaxy are represented by a unique template  $t_j$  from a discrete SED library, the Bayesian redshift distribution may be written as the sum over all the marginal probabilities that the observed galaxy is simultaneously at redshift  $z$  and belongs to each template class  $t_j$ , and then integrate over  $a$  (see also Benitez 2000):

$$p(z|S, I) = (1/p(S|I)) \int \sum_j p(S|z, t_j, a)p(z, a|I)da \quad (3)$$

In Figure 2 the *best-fit* model derived from the maximum likelihood analysis has been rejected in favor of the *most probable* model according

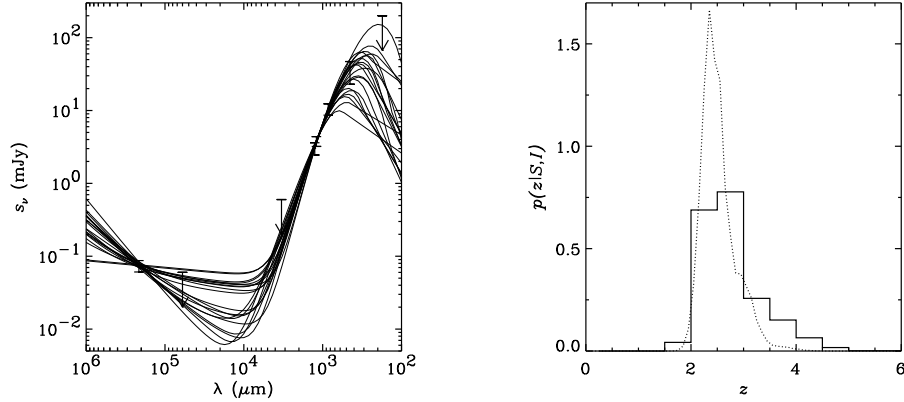


Figure 3. *left panel:* Observed data for the submm source Lockman Hole 850.1 and the most likely SED models using a library of 20 templates (eliminating models that exceed  $3\text{-}\sigma$  upper limits). *right panel:* Comparison between a Bayesian photometric redshift distribution (dotted line) and a Monte Carlo simulation, using the same SED templates and luminosity function prior (Aretxaga et al. 2003). Both methods give qualitatively similar distributions centered about a peak probability of  $z \simeq 2.5$ .

to Bayesian inference, to obtain the redshift of the simulated galaxy. Although the lower redshift model is intrinsically dimmer for the same observed flux density (and would therefore seem favorable), the chance of observing it is much smaller than that of the brighter model at higher redshift, given the strong evolution of the luminosity function, and the greater differential volume sampled at that distance.

A photometric redshift has been calculated for the blank-field submm source Lockman Hole 850.1 using a Monte Carlo simulation (Aretxaga et al. 2003). A direct comparison with a Bayesian prediction is provided in Figure 3. The clear advantage of the Bayesian probability distribution is that it may be evaluated with arbitrary redshift precision, and furthermore it can be calculated in a few seconds, whereas the Monte Carlo simulation required several minutes. Clearly this order-of-magnitude improvement in execution time, and resolution, make it the technique of choice for analyzing future surveys with thousands of (sub)mm sources.

## References

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