

# Mathematical characterization of galaxies using IFS data: II - adding spatial information

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# Outline

- 1 Binning
  - Voronoi
  - HII explorer
  - BaTMA
  
- 2 Morphological characterization

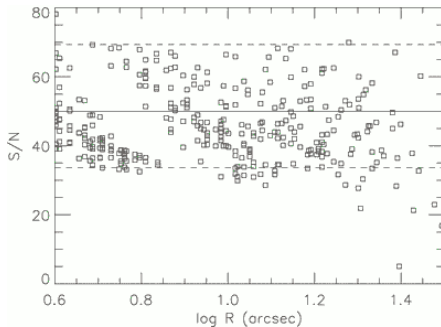
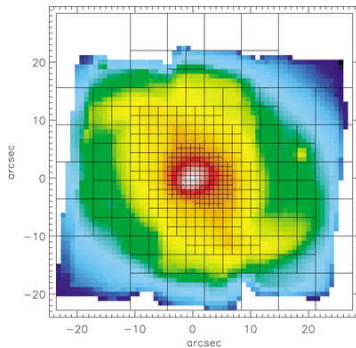
# Voronoi binning

[\(Cappellari & Copin 2003\)](#)

# Motivation

Constant signal-to-noise

$$(S/N)_b \simeq (S/N)_0$$



# Motivation

Constant signal-to-noise

$$(S/N)_b \simeq (S/N)_0$$

Desired properties

- Topological tessellation
- Morphological compactness
- Uniformity

# Algorithm

## Weighted average

$$(S/N)_{A+B} = \frac{w_A S_A + w_B S_B}{\sqrt{w_A^2 N_A^2 + w_B^2 N_B^2}}$$

## "Optimal" weighting

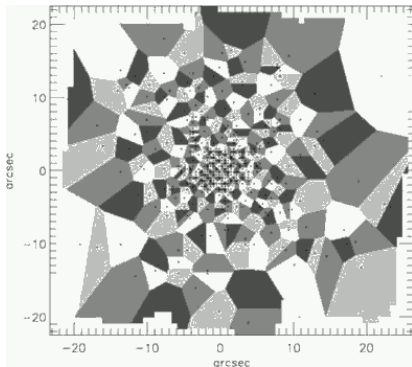
$$w_i \propto S_i / N_i^2$$

$$(S/N)_{A+B} = \sqrt{(S/N)_A^2 + (S/N)_B^2}$$

## (S/N) "density"

$$\rho(\vec{x}) \equiv (S/N)^2(\vec{x})$$

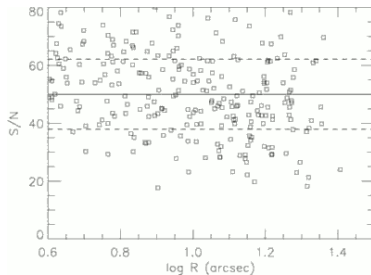
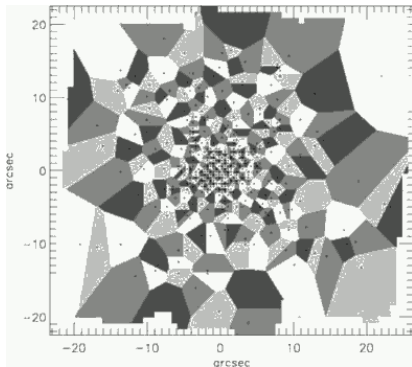
# Algorithm



## Modified k-means

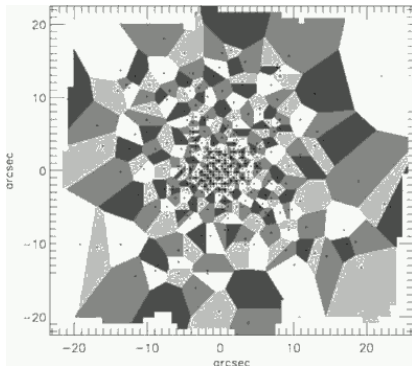
- 1 Select random seed
- 2 Tessellate
- 3 
$$\vec{x}_b = \frac{\int_b \vec{x} \rho^2(\vec{x}) d\vec{x}}{\int_b \rho^2(\vec{x}) d\vec{x}}$$
- 4 Iterate to convergence

# Algorithm





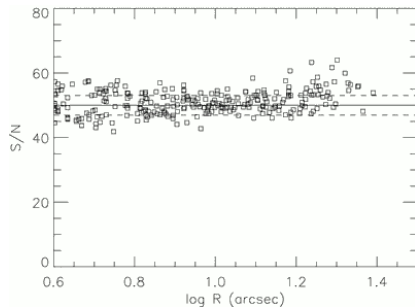
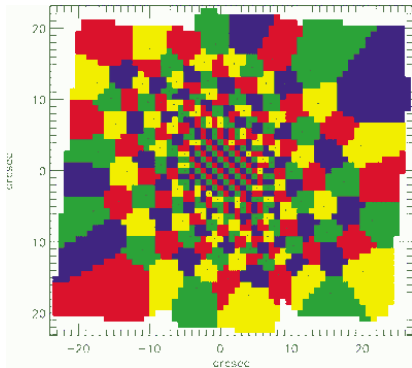
# Algorithm



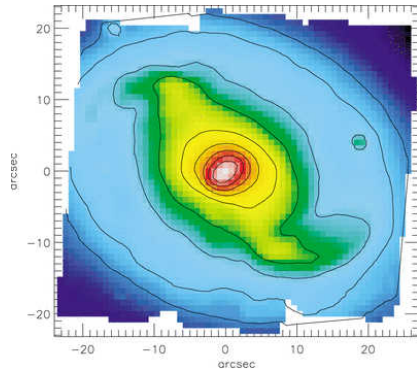
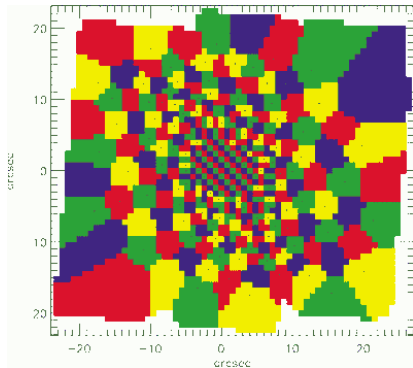
## Initial seeds

- 1 Start with highest S/N
- 2 Accrete nearest pixel while
  - adjacent
  - “roundness”  
$$R = \frac{r_{max}}{r_{eff}} - 1 < 0.3$$
  - $(S/N)_b$  approaches  $(S/N)_0$
- 3 Mark as unsuccessful if  $(S/N)_b < 0.8(S/N)_0$
- 4 Start new bin with nearest pixel

# Results



# Results



# HII explorer

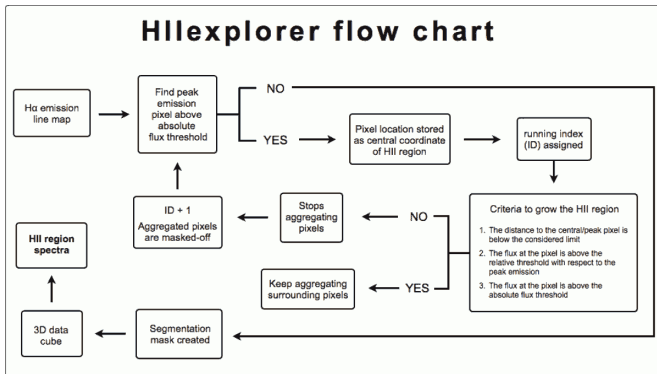
[\(Sánchez et al. 2012\)](#)

# Algorithm

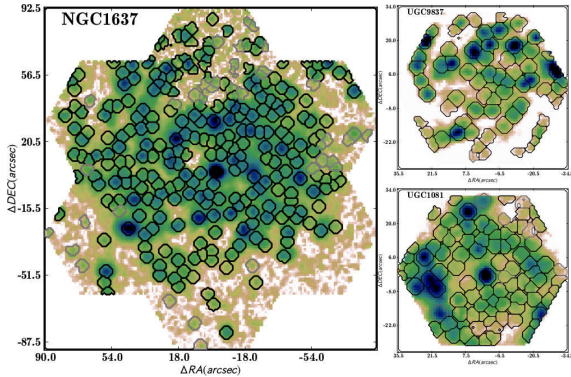
## Input

- Emission-line map ( $H\alpha$ )
- Minimum peak intensity
- Intensity contrast
- Maximum radius
- Minimum flux

# Algorithm



# Results



# Bayeseian **T**echnique for **M**ulti-image **A**nalysis

(Casado et al., in prep.)



# A long-standing battle

## Frequentist approach

- Likelihood
- Confidence intervals

### Likelihood

$\mathcal{L}_m(\vec{x}|\vec{\theta}_m) d\vec{x}$ : probability of observing the datum  $\vec{x}$ , given model  $m$  with parameters  $\vec{\theta}_m$

### Prior probability

$\pi_m(\vec{\theta}_m) d\vec{\theta}_m$ : expected probability distribution for  $\vec{\theta}_m$

## Bayesian approach

- Likelihood
- Prior prob.
- Model evidence
- Posterior prob.

### Model evidence

$E_m(\vec{x}) = \int \pi(\vec{\theta}_m) \mathcal{L}_m(\vec{x}|\vec{\theta}_m) d\vec{\theta}_m$ : probability of  $\vec{x}$ , given  $m$

### Posterior probability

$p(\vec{\theta}_m|\vec{x}) = \frac{\pi(\vec{\theta}_m) \mathcal{L}_m(\vec{x}|\vec{\theta}_m)}{E_m(\vec{x})}$ : probability of  $\vec{\theta}_m$ , given  $m$  and  $\vec{x}$

# Dear Sir or Madam?

## Data

The subject gave birth to twins

## Likelihood

Probability of a woman having twins  $\sim 10^{-5}$

Probability of an amoeba having twins  $\sim 1$

## Frequentist approach

The hypothesis that the subject is a woman can be ruled out at the 99.999% level

## Bayesian approach

- Prior:  $\pi_m = \pi_w = 0.5$
- $E = \pi_m \mathcal{L}_m + \pi_w \mathcal{L}_w = 5 \times 10^{-6}$
- Posterior:  $p_m = 0; p_w = 1$

# Model comparison

## Data

$N$  measurements with errors:  $\{x_i, \sigma_i\}_{i=1,N}$

## Model 1

- One object with actual value  $x$
- $\pi(x) dx = \frac{dx}{L}$  if  $0 < x < L$
- $\mathcal{L}_1(\{x_i\}|x) = \prod_{i=1}^N e^{-\frac{(x_i-x)^2}{2\sigma_i^2}} / \sqrt{2\pi\sigma_i^2}$

$$E_1(\{x_i\}) = \int \prod_{i=1}^N e^{-\frac{(x_i-x)^2}{2\sigma_i^2}} \frac{dx}{L}$$

## Model 2

- Two objects, with  $x_A$  and  $x_B$
- $\pi(x_A) = \pi(x_B) = \pi(x)$
- "labels"  $\{l_i\}_{i=1,N} = A|B$
- $\mathcal{L}(\{x_i\}|x) = \prod_{i=1}^N e^{-\frac{(x_i-x_{l_i})^2}{2\sigma_i^2}} / \sqrt{2\pi\sigma_i^2}$

$$E_2(\{x_i\}) = \int \int \prod_{i=1}^N e^{-\frac{(x_i-x_{l_i})^2}{2\sigma_i^2}} \frac{dx_A}{L} \frac{dx_B}{L}$$

# BaTMAn

## Formulation of the problem

- Data:  $N$  measurements (pixels) with
  - intensity  $\{x_i\}_{i=1,N}$
  - errors  $\{\sigma_i\}_{i=1,N}$
- Model:  $B$  regions (bins) where  $x = x_b$ 
  - $\pi(x_b) dx_b \sim \frac{dx_b}{\max(x_i) - \min(x_i)}$
  - "labels"  $\{b_i\}_{i=1,N}$

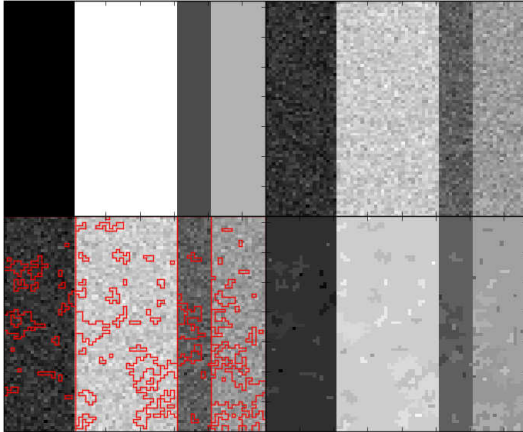
$$\bullet \mathcal{L}(\{x_i\}_{i=1,N} | \{x_b\}_{b=1,B}) = \prod_{i=1}^N e^{-\frac{(x_i - x_{b_i})^2}{2\sigma_i^2}} \frac{1}{\sqrt{2\pi\sigma_i^2}}$$

# BaTMAn

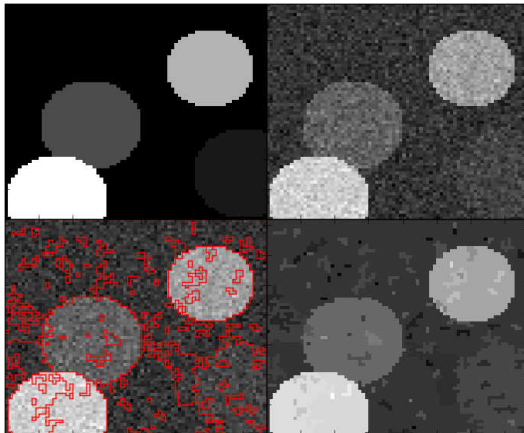
## Algorithm

- 1 Start with  $B = N$
- 2 While  $E_{B-1} > E_B$ :
  - merge the two adjacent regions that yield the highest  $E_{B-1}$

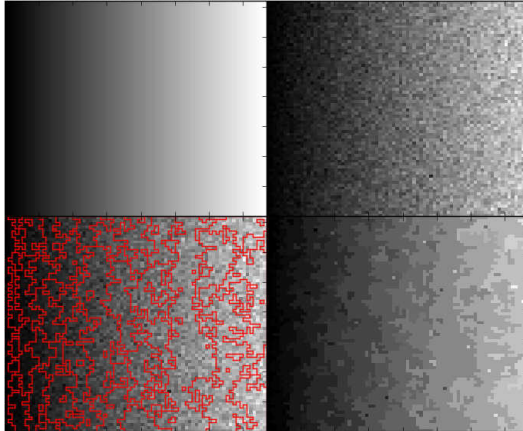
# Results



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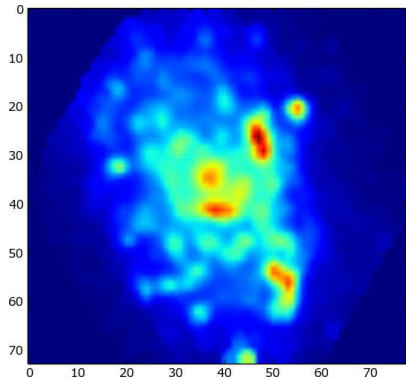
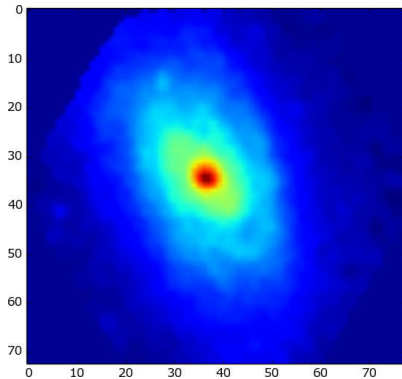


# Results



# Morphological characterization

# Multi-wavelength morphology



# Multi-wavelength morphology

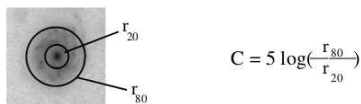
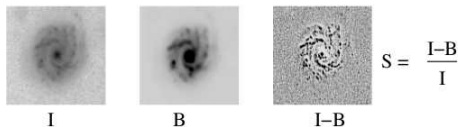
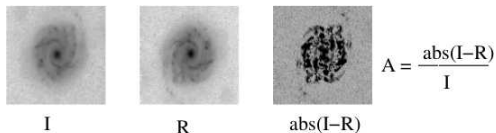
## What for?

- 1 Classification and comparison
- 2 Statistics of the galaxy population
- 3 Input for chemical evolution models
- 4 Low signal-to-noise kinematics

## How?

- 1 Statistical descriptors
- 2 Parametric models
- 3 Non-parametric models

# Statistical descriptors



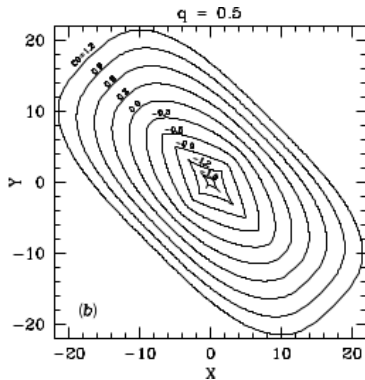
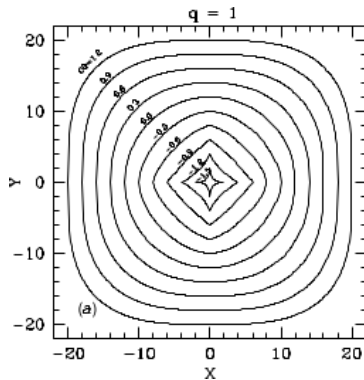
[Conselice \(2003\)](#)

# Model-based decomposition



[de Souza et al. \(2004\)](#)

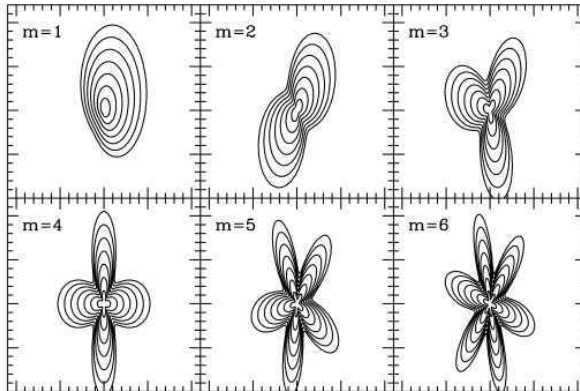
# Generalized ellipsoids



[Peng et al. \(2010\)](#)

# Fourier modes

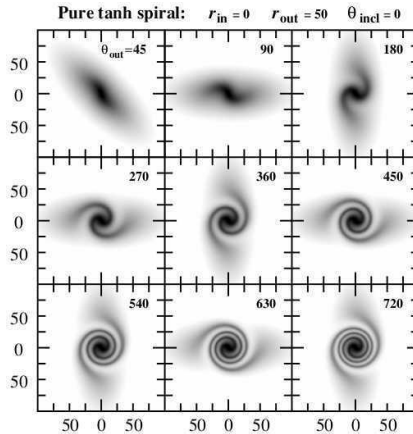
$$q = 0.5, a_m = 0.5, \phi_m = -45$$



[Peng et al. \(2010\)](#)

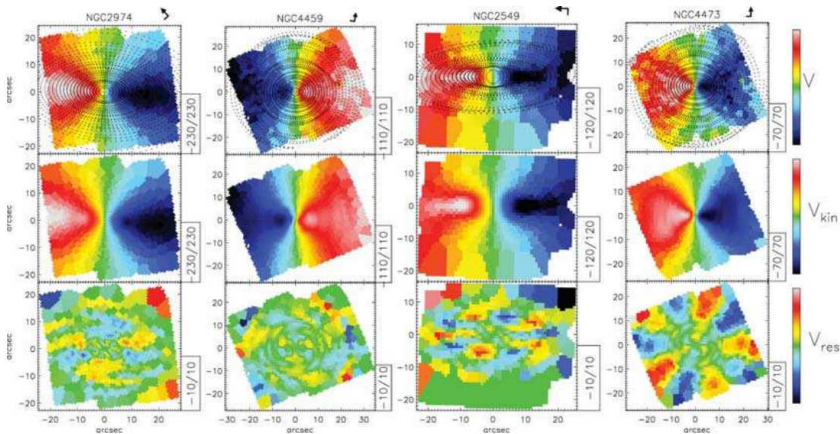


# Spiral modes



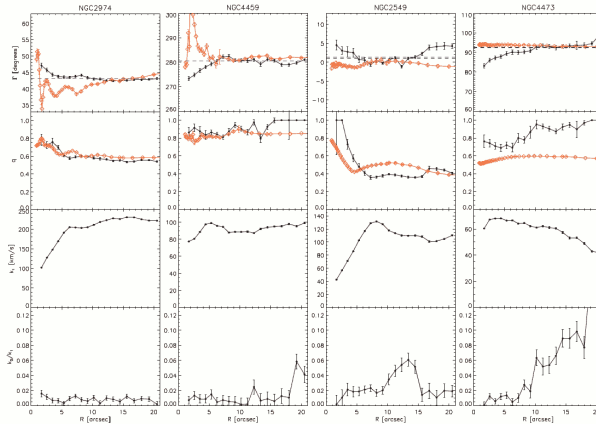
[Peng et al. \(2010\)](#)

# Kinematics



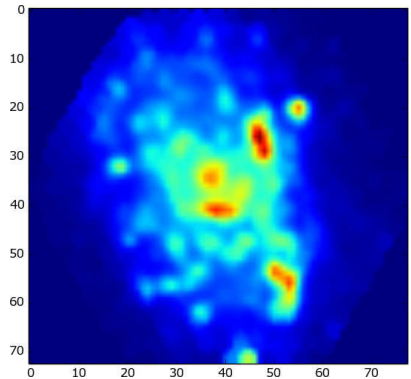
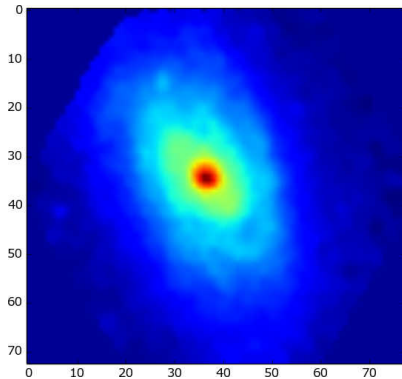
[Krajnović et al. \(2006\)](#)

# Kinematics



[Krajnović et al. \(2006\)](#)

# What next?



# Summary

Binning

Bayes' theorem

Morphological characterization

There is no need to re-invent the wheel...  
...but there is plenty of information to recover!