



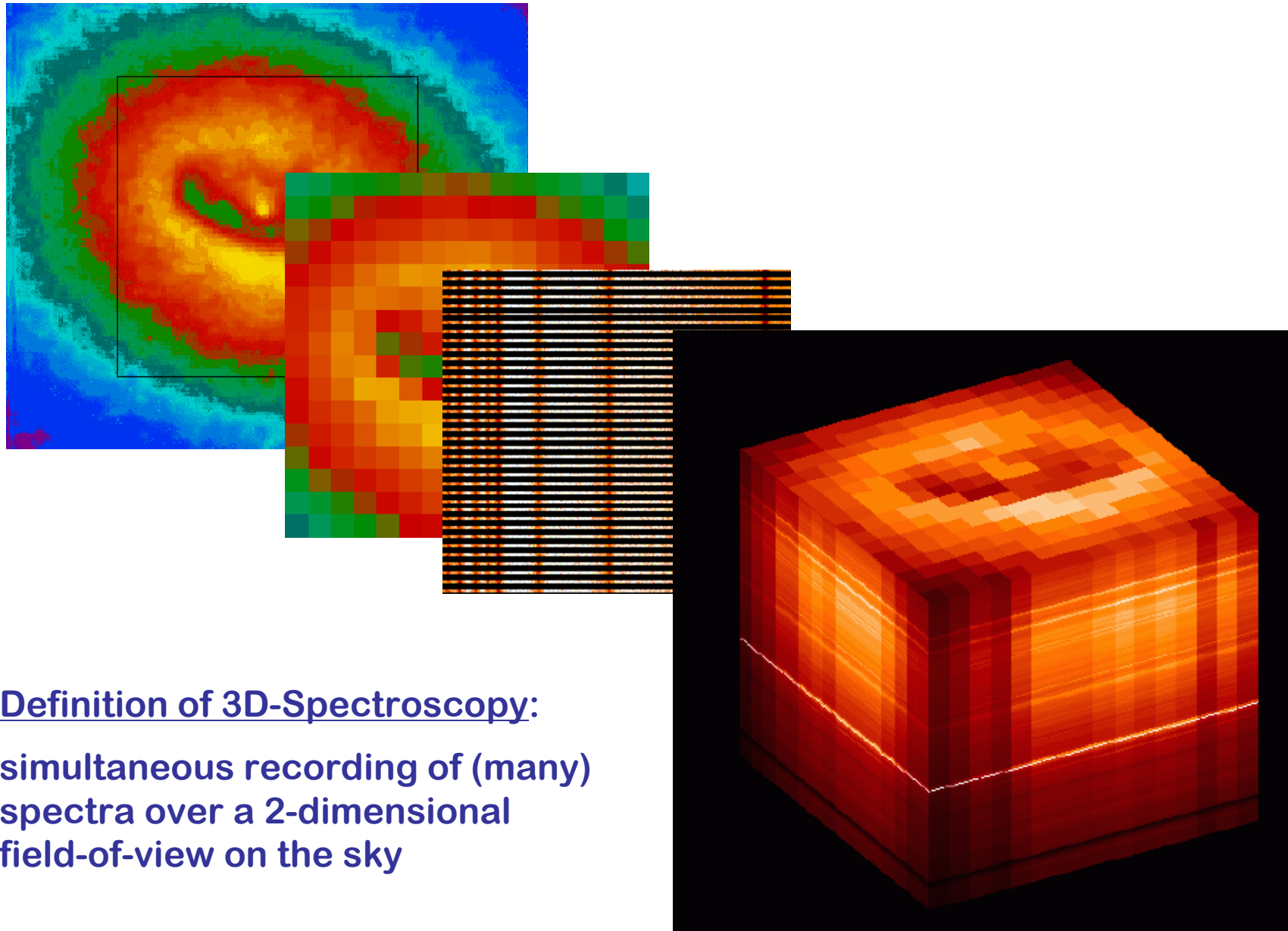
IFS technique and instrumentation, present and future

Martin M. Roth
innoFSPEC Potsdam
Leibniz-Institut für Astrophysik Potsdam

Overview

- (1) Definition, terminology, principles of operation
- (2) Spatial sampling, binning
- (3) Instrumentation
- (4) Data Reduction
- (5) Observation

(1) Definition, terminology, principles of operation



Definition of 3D-Spectroscopy:

simultaneous recording of (many) spectra over a 2-dimensional field-of-view on the sky

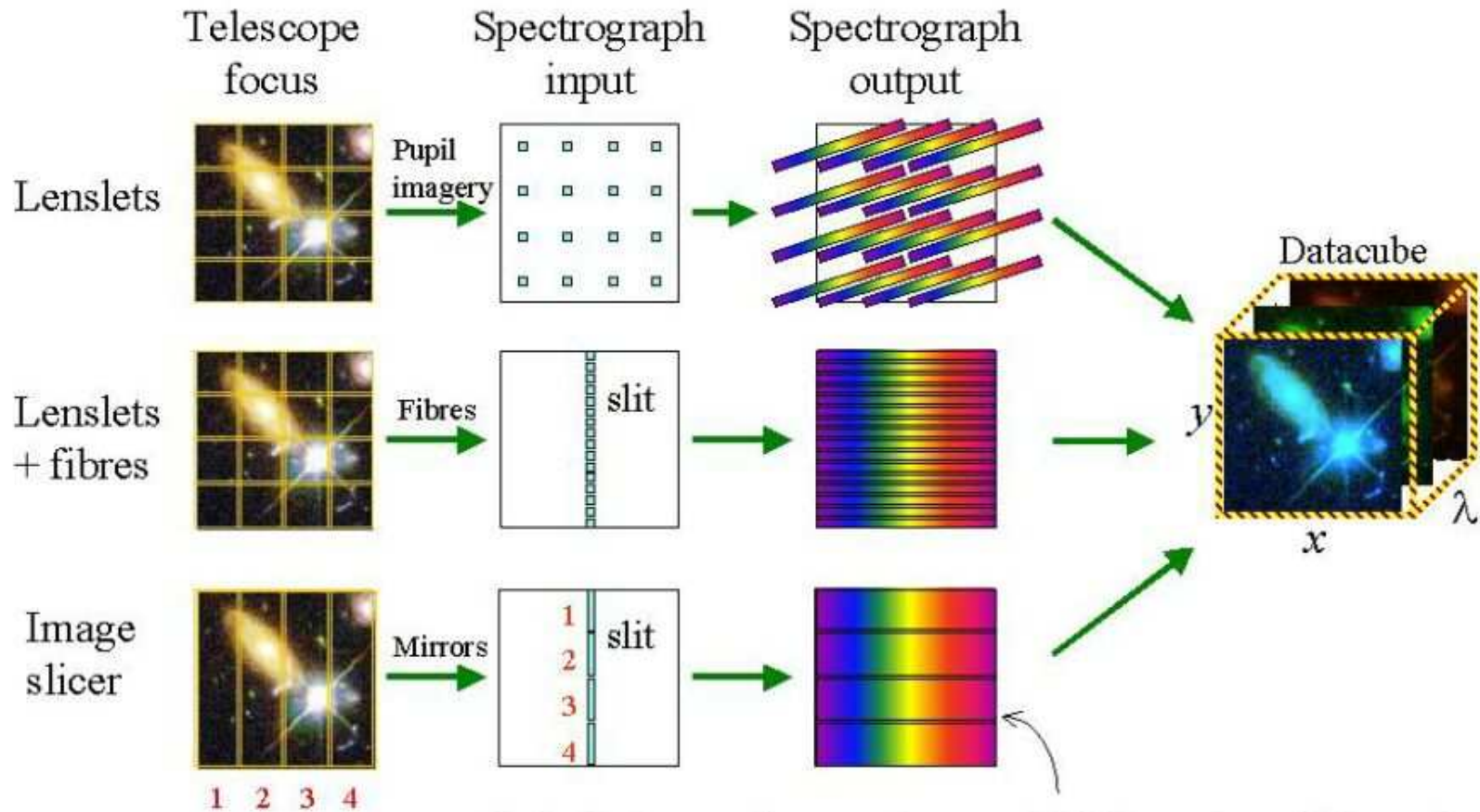
Terminology

- integral field spectroscopy (IFS)
- = 3D spectroscopy
- = tridimensional spectroscopy

- (2D spectroscopy)
- (bidimensional spectroscopy)

- integral field unit (IFU)
- spaxel
- data cube
- row-stacked spectra (RSS)
- pixel table

Principle of Operation

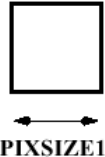


(Credit: J. Allington-Smith)

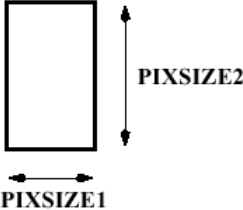
(2) Spatial sampling, binning

Spatial sampling, binning

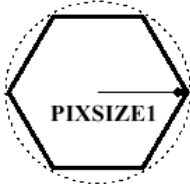
SQUARE



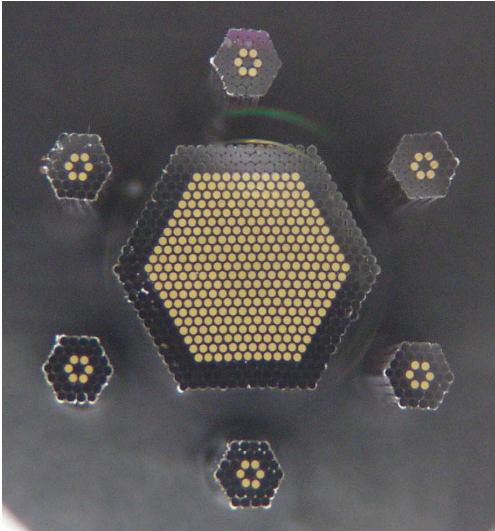
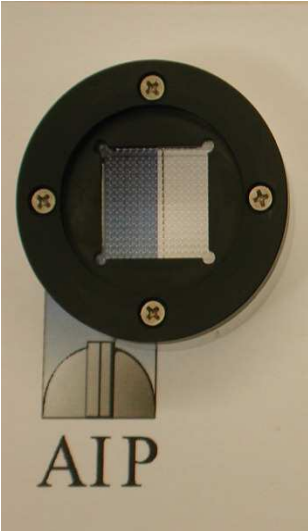
RECTANG



HEXAGON

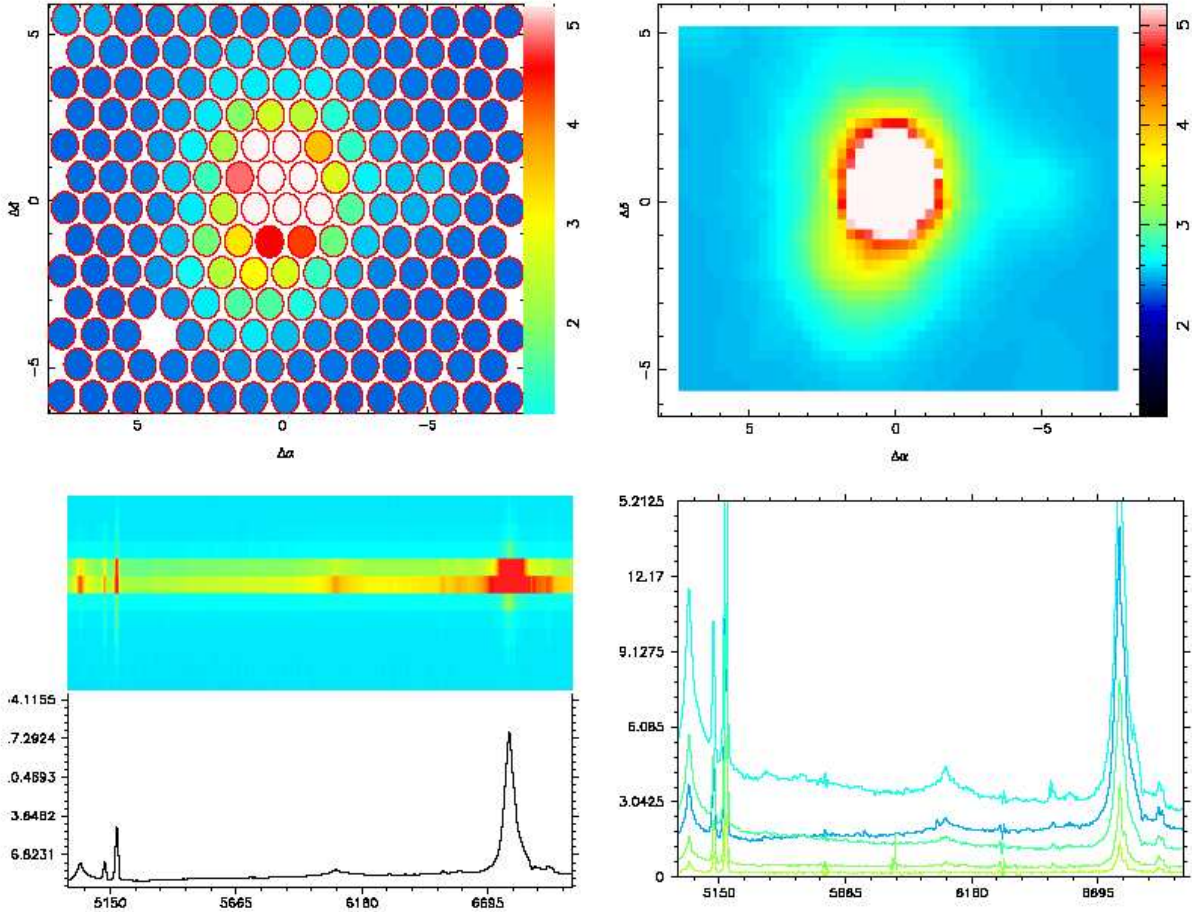


CIRCLE



Spatial sampling, binning

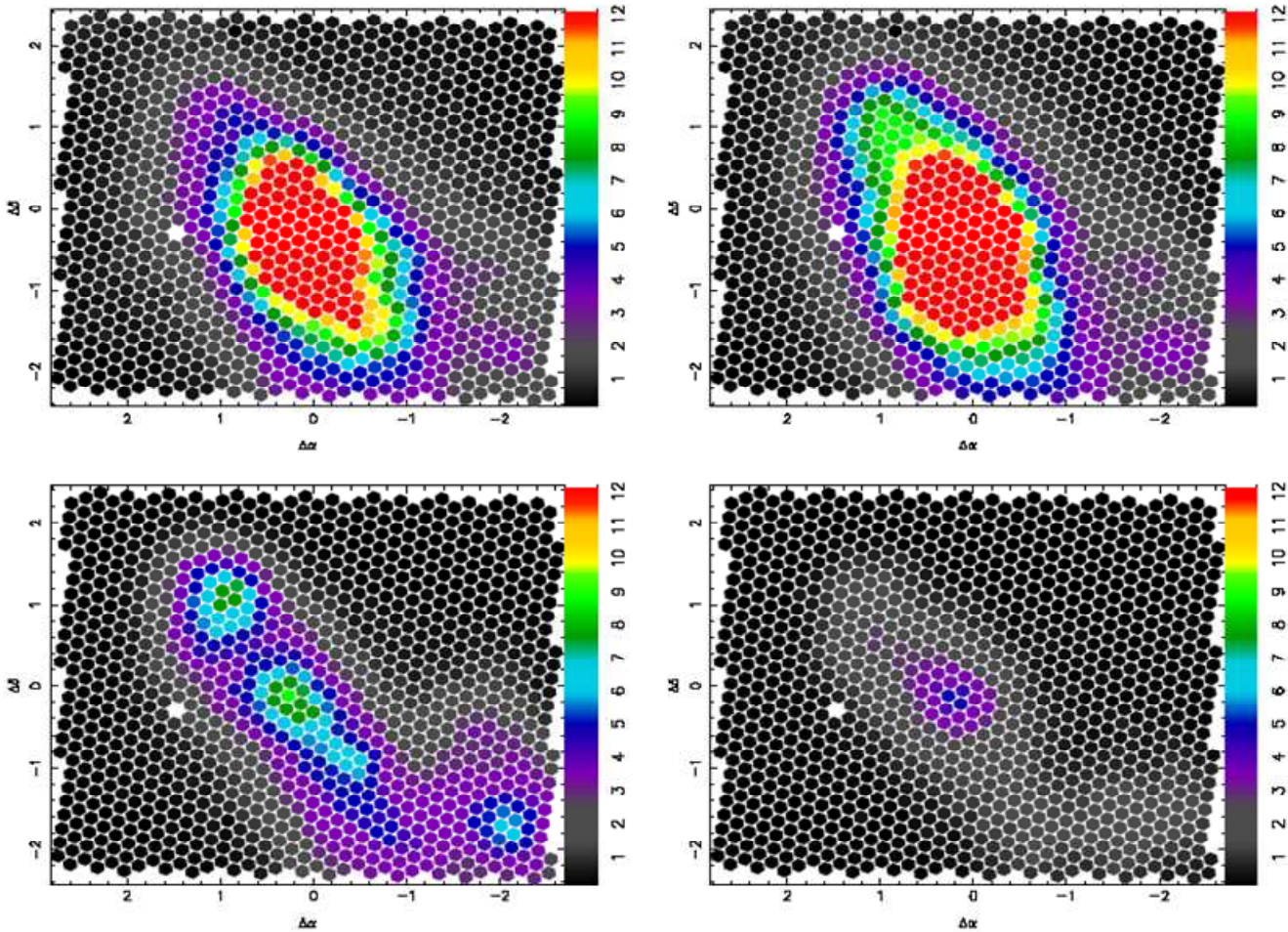
INTEGRAL: 3C 120



(courtesy S. Sánchez)

Spatial sampling, binning

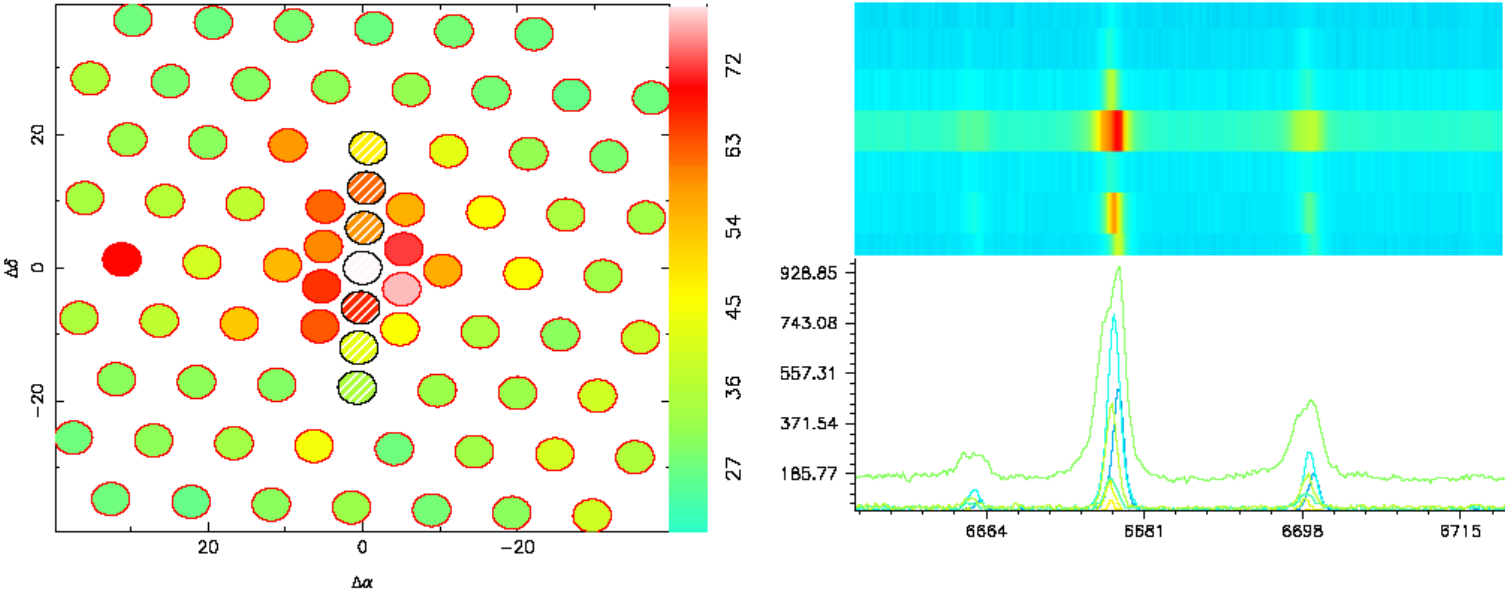
SAURON: NGC1068



(courtesy S. Sánchez)

Spatial sampling, binning

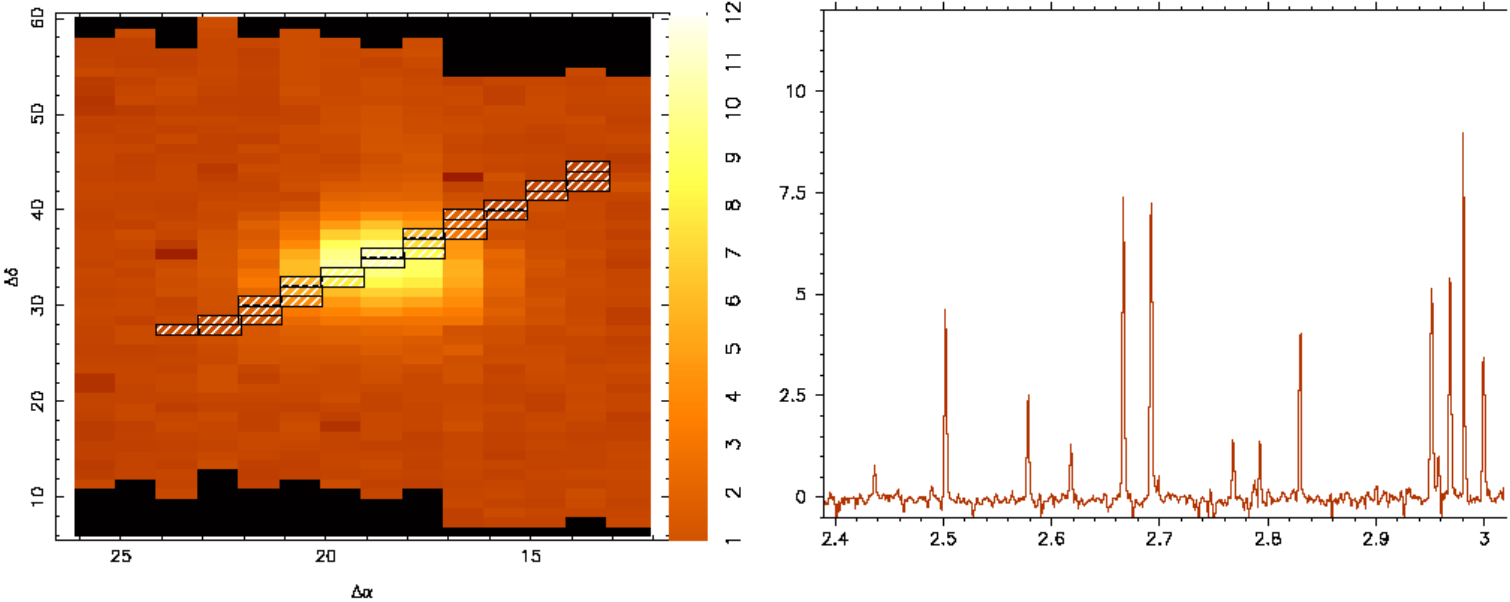
SparsePAK : U4256



(courtesy S. Sánchez)

Spatial sampling, binning

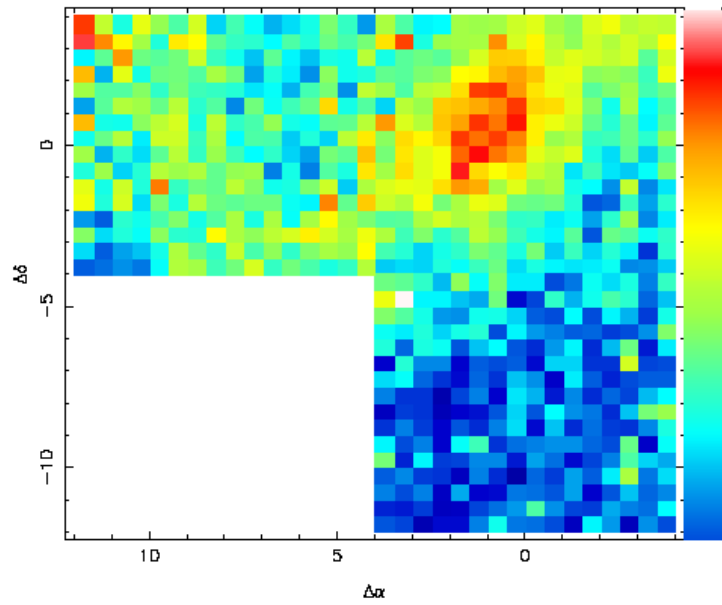
UIST: Herbig Haro



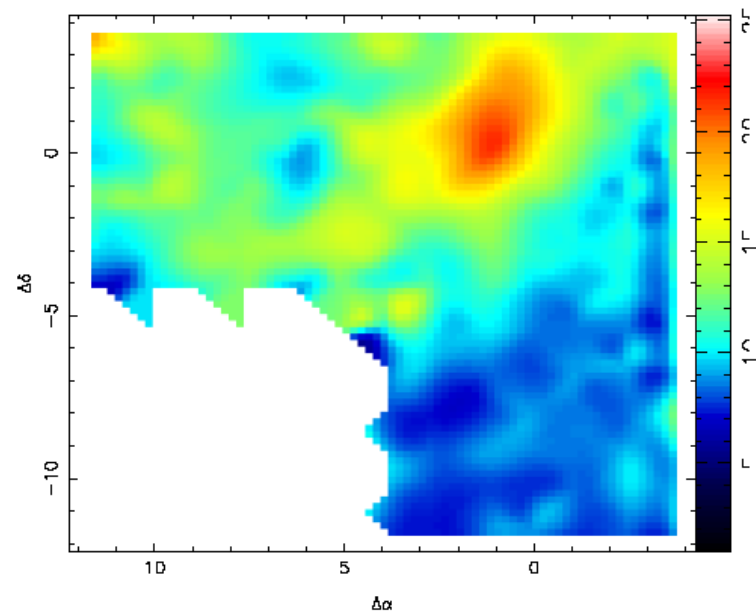
(courtesy S. Sánchez)

Spatial sampling, binning

PMAS Mosaic: IRAS 12110+1624



original data

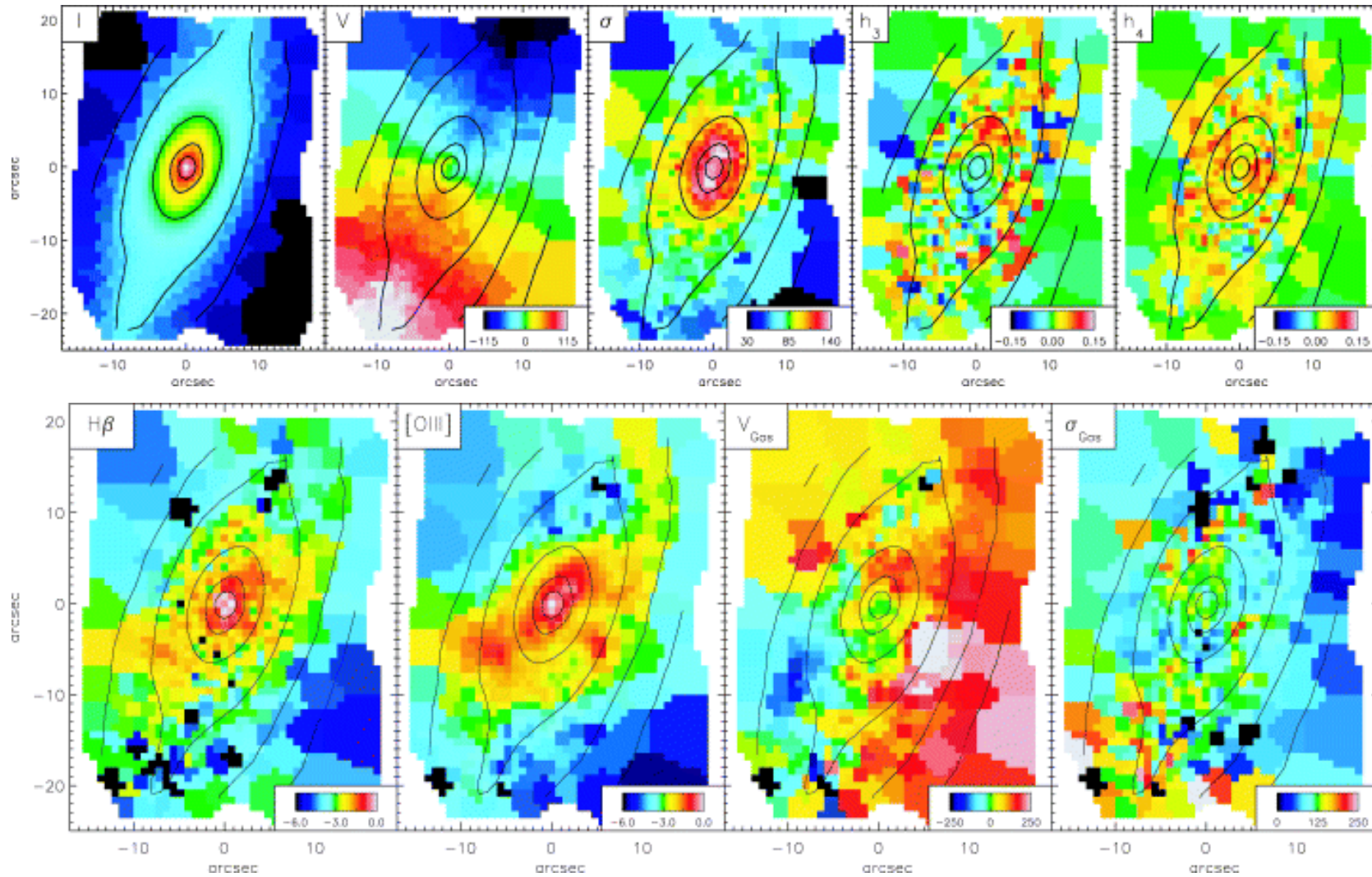


rebinned (interpolated)

(courtesy S. Sánchez)

Spatial sampling, binning

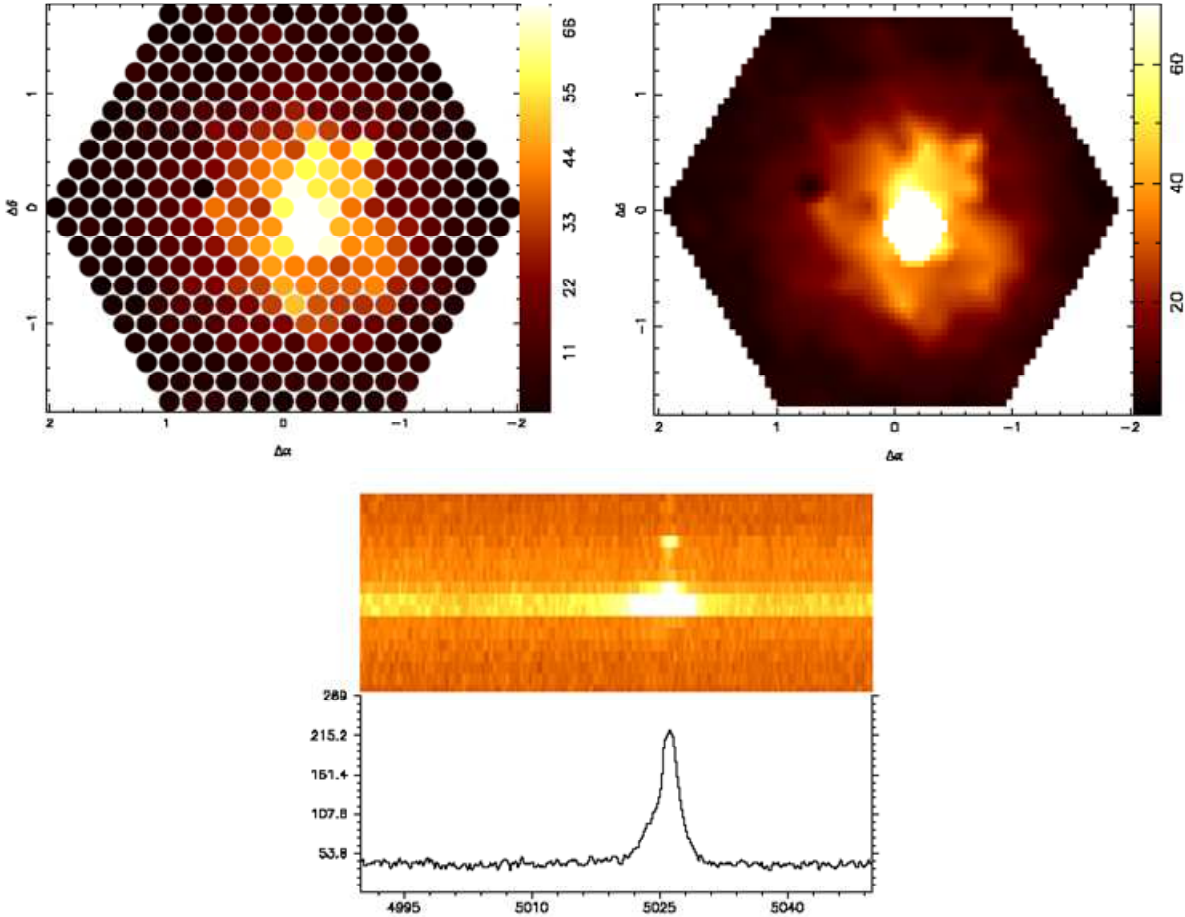
→ *adaptive binning*
Capellari & Copin 2003
MN 342, 345



Falcon-Barroso et al. 2004, MNRAS 350, 35

Spatial sampling, binning

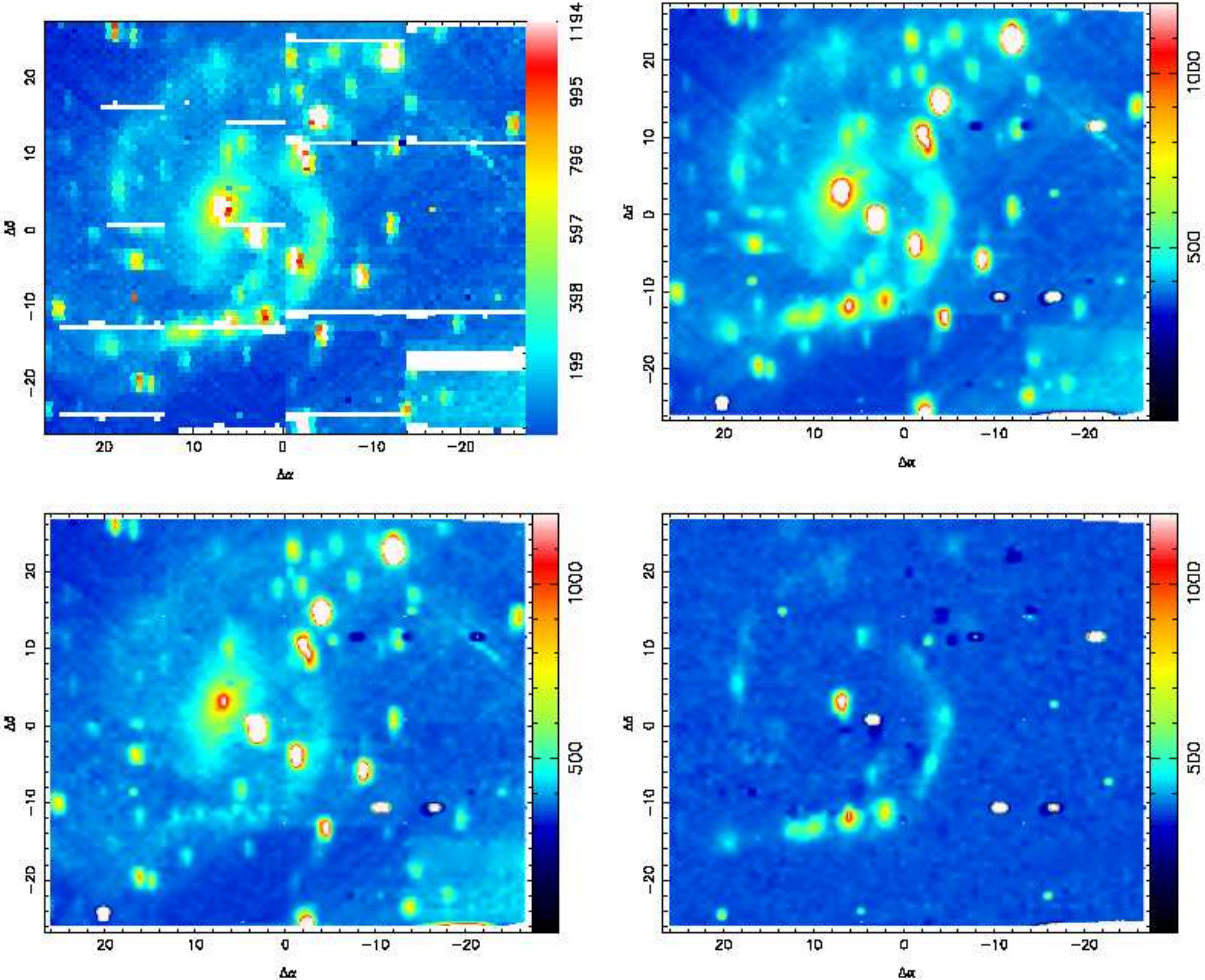
PMAS + PPAk-IFU: face-on disk galaxy



(courtesy S. Sánchez)

Spatial sampling, binning

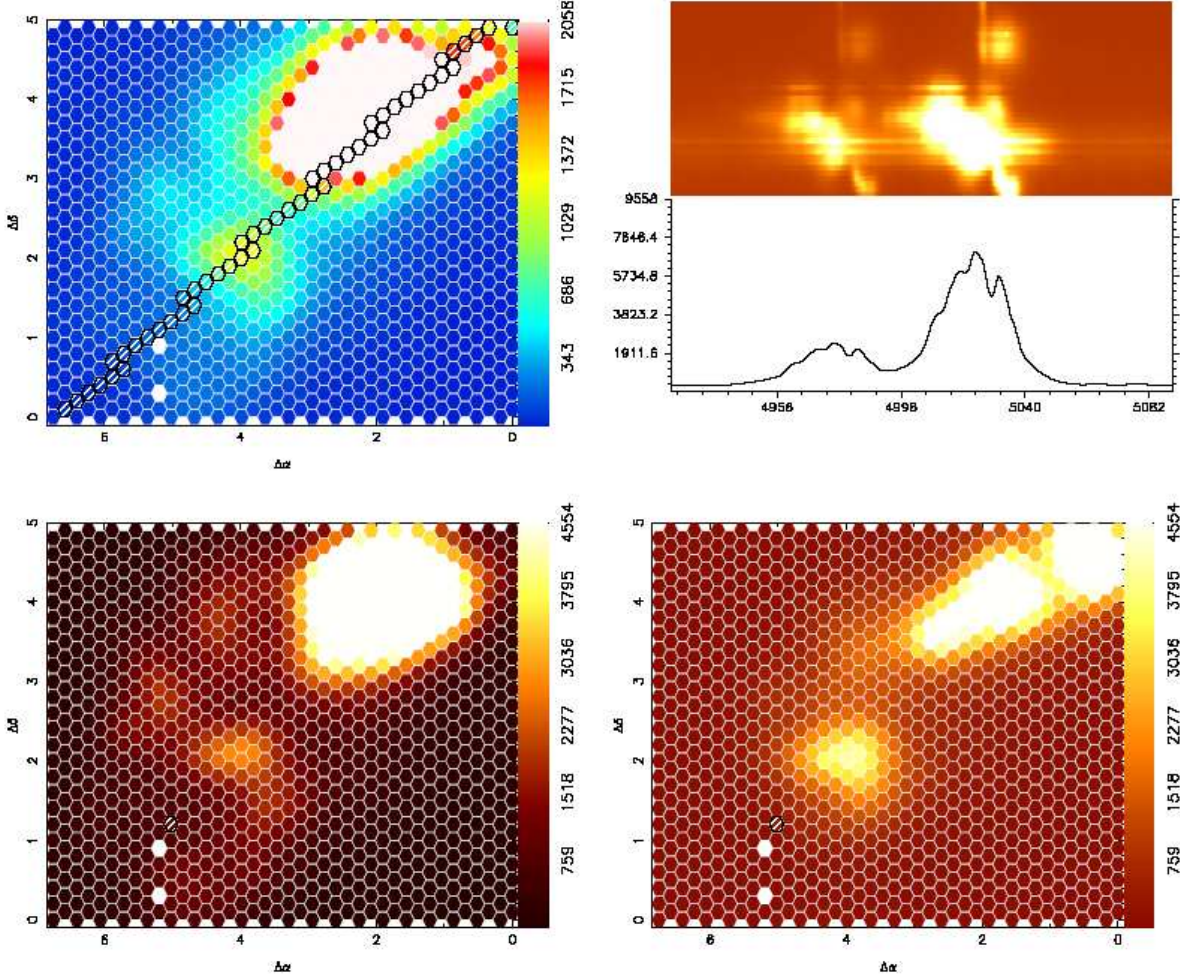
VMOS (LRr): IRAS 13031-571



(courtesy S. Sánchez)

Spatial sampling, binning

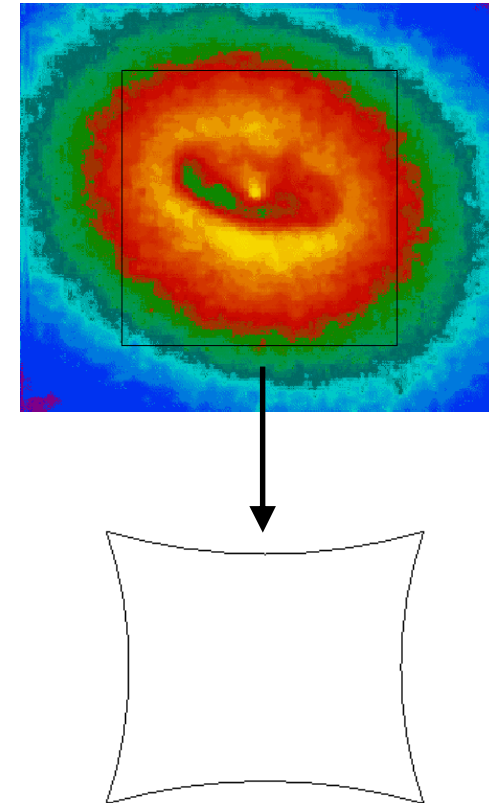
GMOS: NGC1068



(courtesy S. Sánchez)

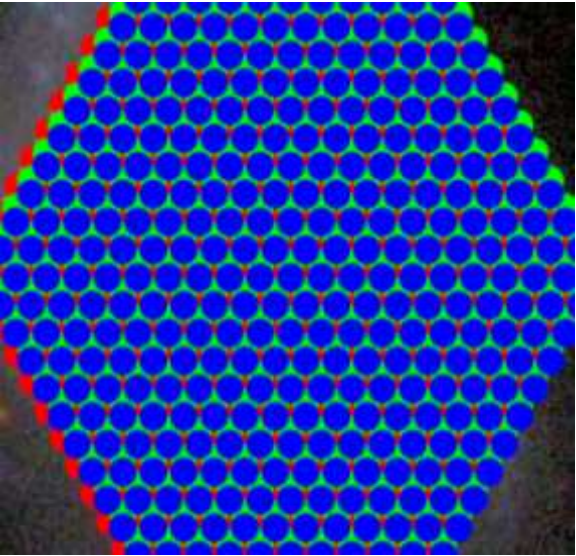
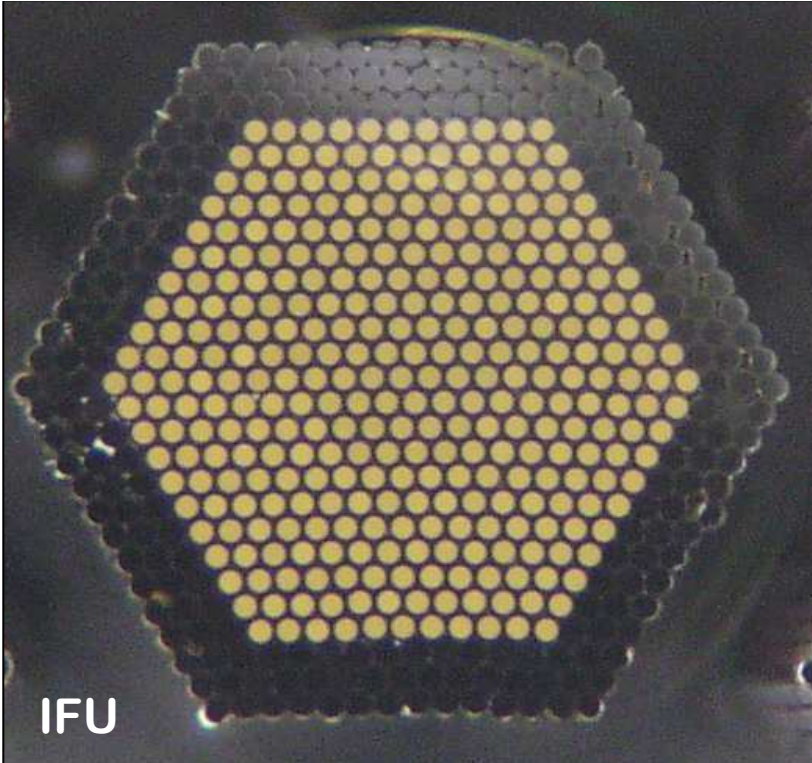
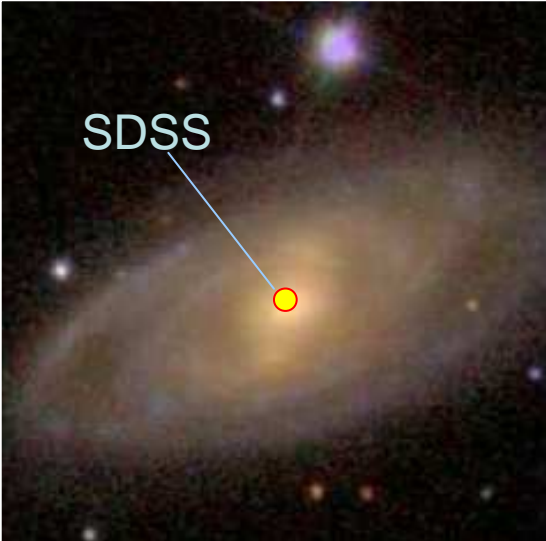
Spatial sampling, binning

- spatial sampling element „SPAXEL“
to be distinguished from *pixel* (on detector)
- geometry of spaxel
(not necessarily invariant for given IFU)
- geometry of sampling grid
 - orthogonal / non-orthogonal
 - irregular
 - field distortion
- geometry: fill factor (continuous / non-continuous sampling)
- numerical analysis: imaging theory (convolution of object intensity distribution with transfer function of the optical system)



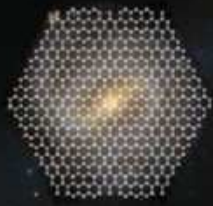
Note: interpolation makes „nice“ images, however does not add information

Spatial sampling, binning



3-point-dithering

Sanchez et al. 2012
A&A 538, 8



CALIFA SURVEY

Calar Alto Legacy Integral field spectroscopy Area survey

public access:
www.caha.es/CALIFA

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 - SDSS Poststamps: Obs. Sample
 - SDSS poststamps: Full sample
- ▶ CALIFA Meetings
 - 6th Busy Week
 - 5th Busy Week
 - 4th Busy Week
 - 3rd Busy Week
 - 2nd Busy Week

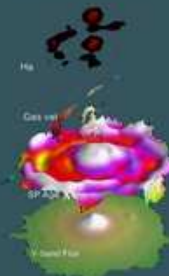
CALIFA 1st DATA RELEASE



NGC 6125



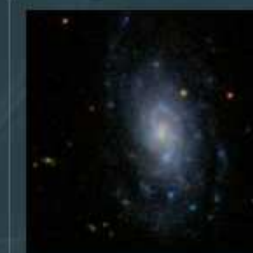
NGC 1349



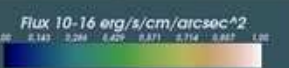
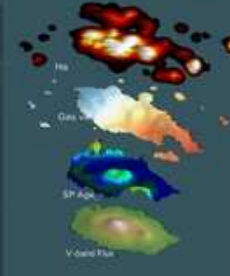
NGC 4003



NGC 5408

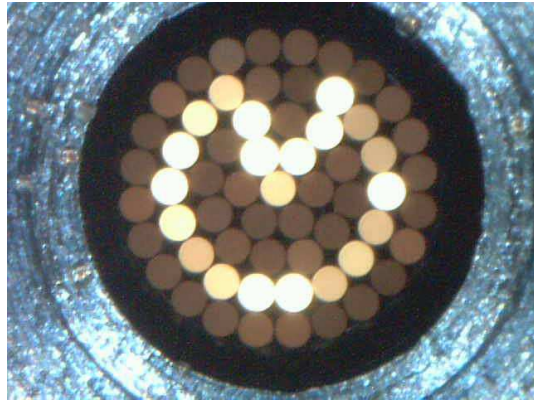


UGC 07012

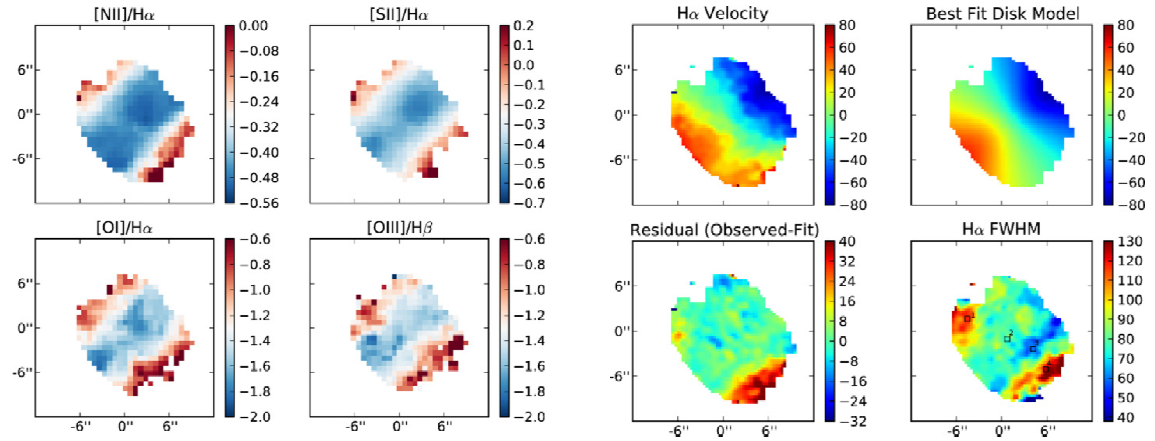


Spatial sampling, binning

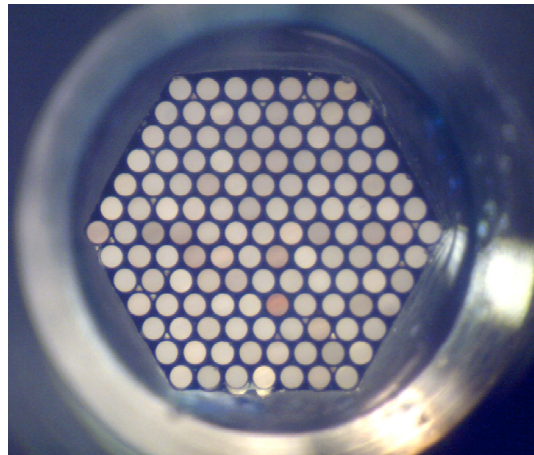
SAMI (AAO)



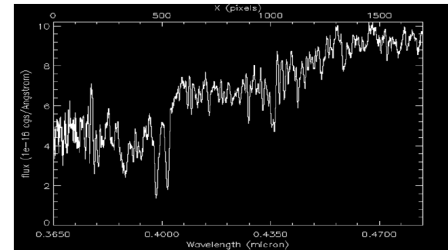
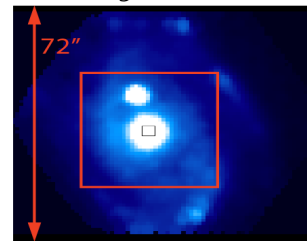
Fogarty et al. 2012



MaNGA (SDSS-4)

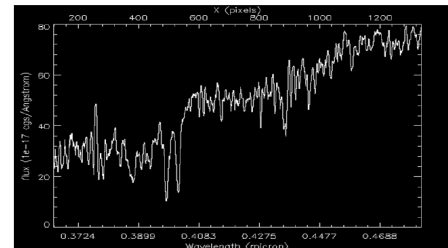
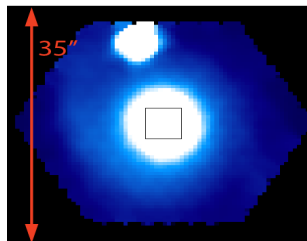


CALIFA High-Resolution Data



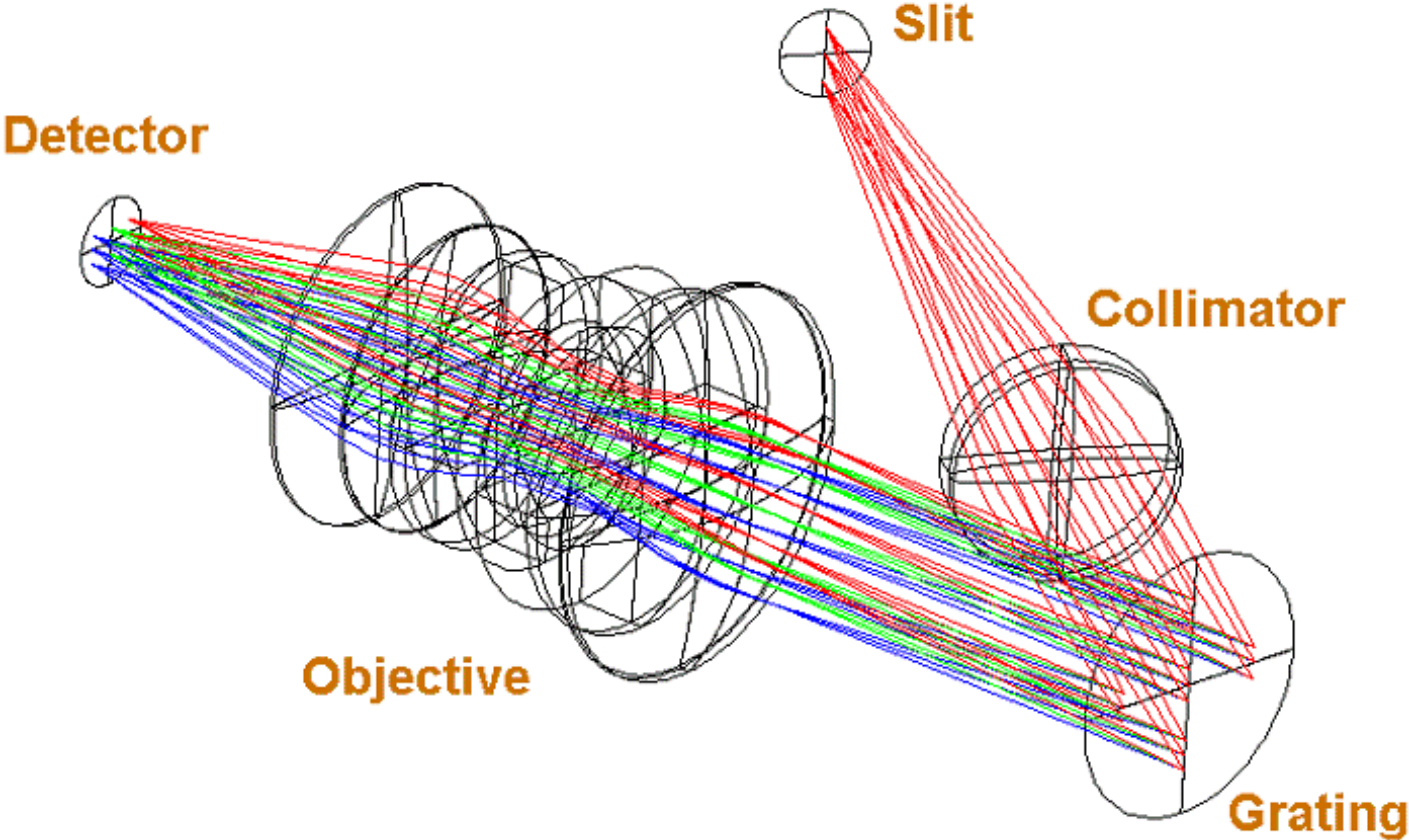
NGC 2916

MaNGA Test-Run Data



(3) Instrumentation

Spectrograph

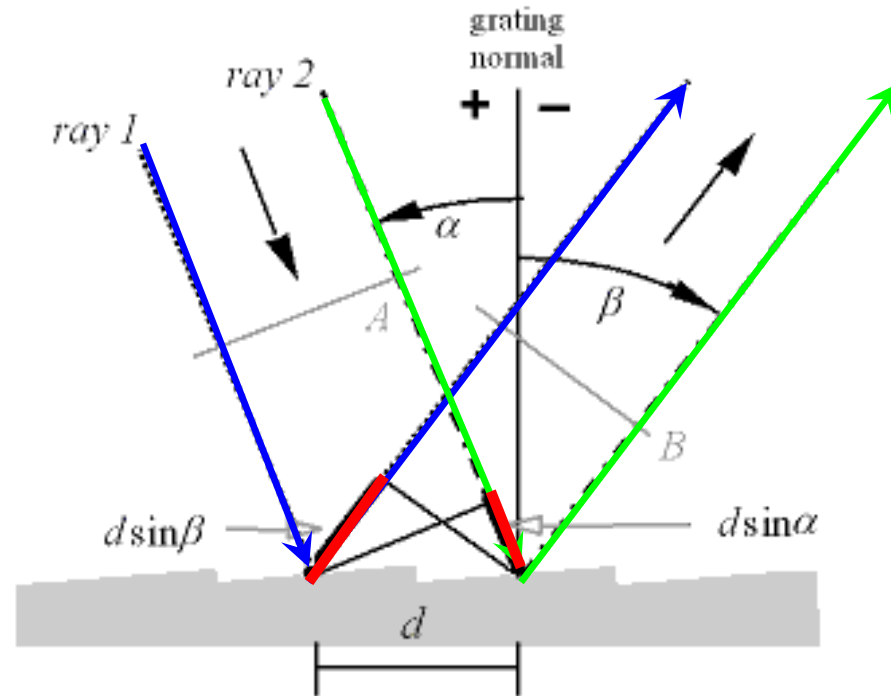


Grating equation

for plane wavefronts:

$$m \lambda = d \sin(\alpha) + d \sin(\beta)$$

optical path length difference must be m times λ for constructive interference



Dispersion

dispersion equation,
angular dispersion

$$\frac{d\theta'}{d\lambda} = \frac{m}{a \cos \theta'}$$

→ grows with m

linear dispersion

$$\frac{dx}{d\lambda} = f \frac{d\theta'}{d\lambda} = \frac{m f}{a \cos \theta'}$$

f : focal length of spectrograph camera

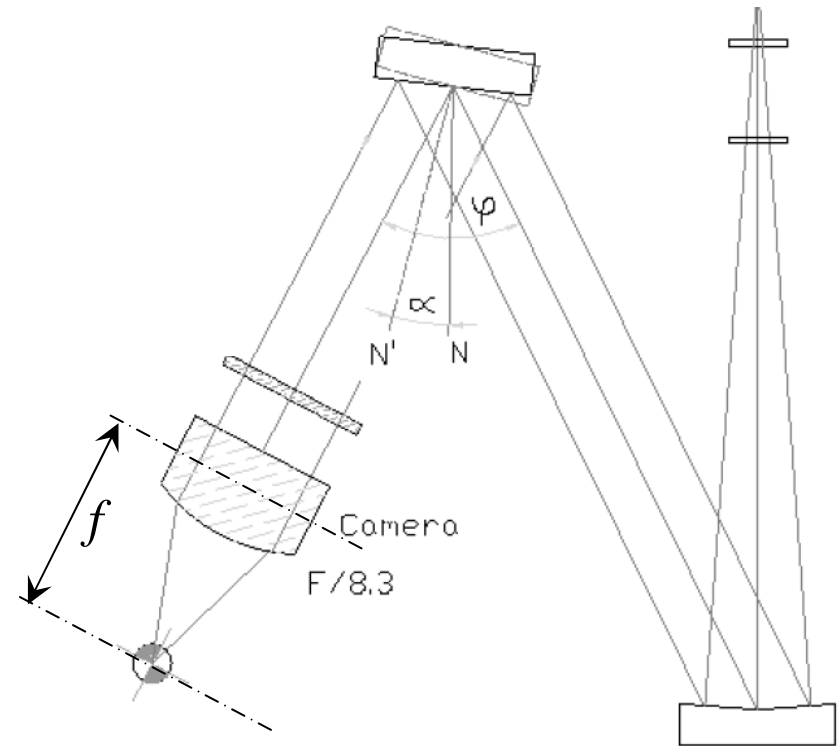
$$dx/f = d\theta$$

→ for fixed order m , $dx/d\lambda = \text{const}$
 ⇒ wavelength as function of detector pixel coordinates:
 approximately linear relation

Accurate representation obtained through *wavelength calibration* with spectral line lamps:

→ approximation of the exact dispersion formula through polynomial obtained from emission line wavelengths as supporting points for fit per each individual spectrum

reciprocal dispersion: $d\lambda/dx$, often given in [$\text{\AA}/\text{mm}$] or [$\text{\AA}/\text{pixel}$]



Spectral Resolution

spectral resolving power R: $R = \lambda/\Delta\lambda$

separation of two equally bright emission lines
→ Rayleigh criterion, first diffraction minimum
N: number of illuminated grating grooves

$$\Delta\theta = \frac{\lambda}{N \cdot a \cos \theta}$$

substituted in dispersion equation:

$$\frac{m \Delta\lambda}{a \cos \theta'} = \frac{\lambda}{N a \cos \theta'}$$

diffraction limited resolving power: $R = \lambda/\Delta\lambda = m N$

- independent of wavelength, depends for given order only on number of illuminated grooves N
- resolving power of real spectrographs is most of the time *not* diffraction limited, but determined by slit width and image quality of spectrograph optical system

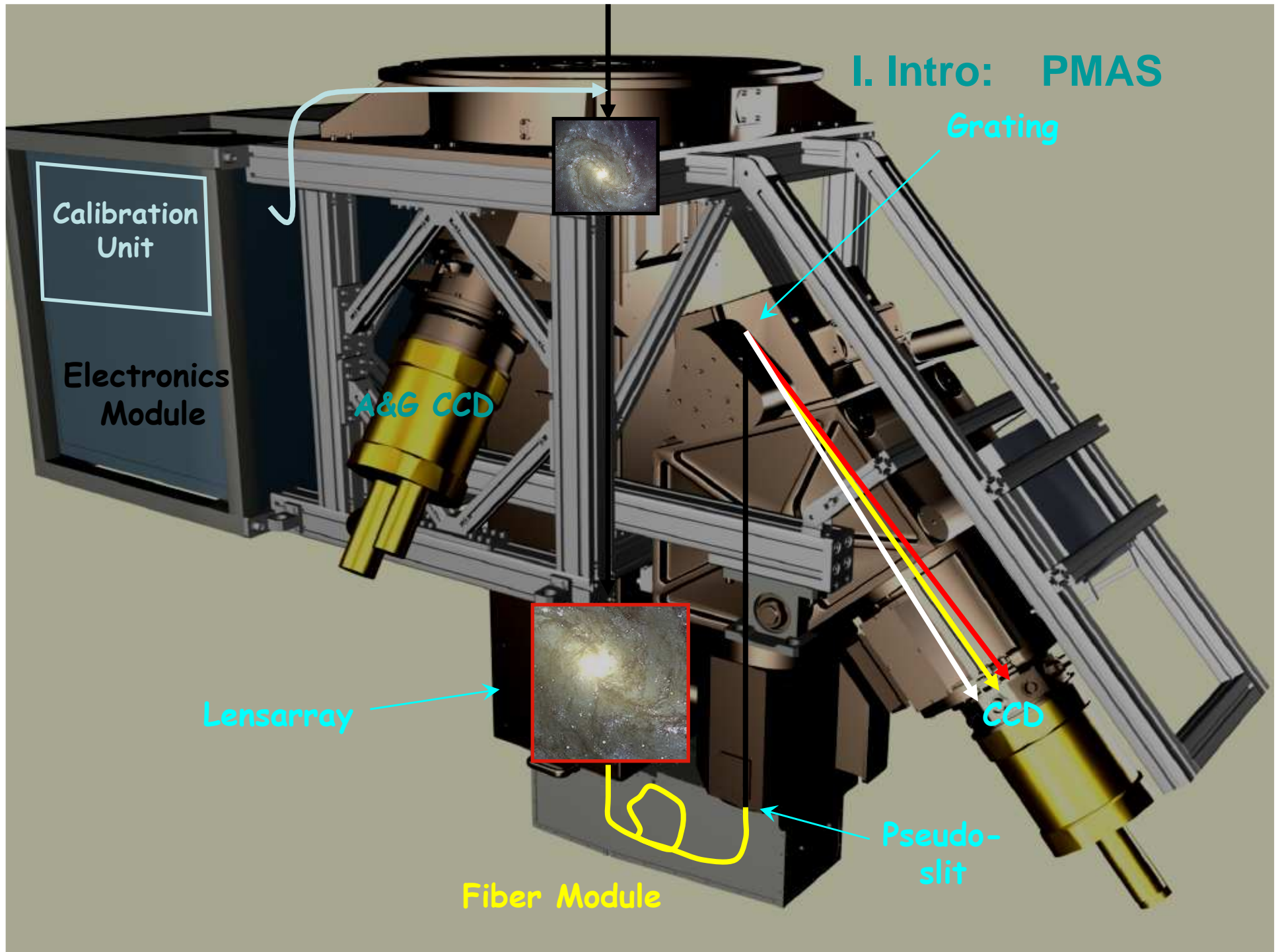
If projected slit width $s' >$ diffraction limit, i.e.:
(f_1, f_2 : collimator-/camera focal lengths)

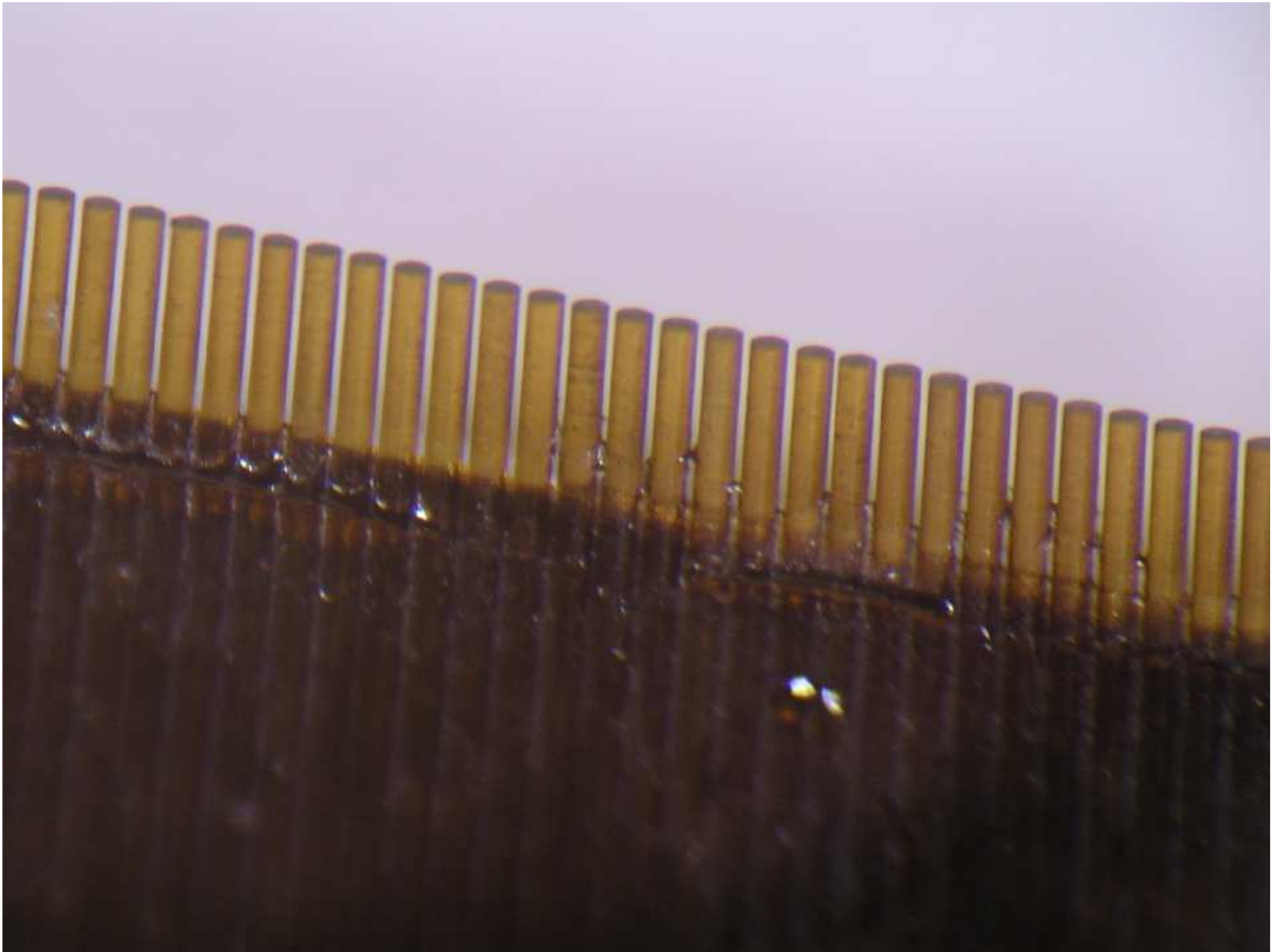
$$s' = s \cdot \frac{f_2}{f_1} > \frac{m f_2}{a \cos \theta'} \cdot \frac{\lambda}{R}$$

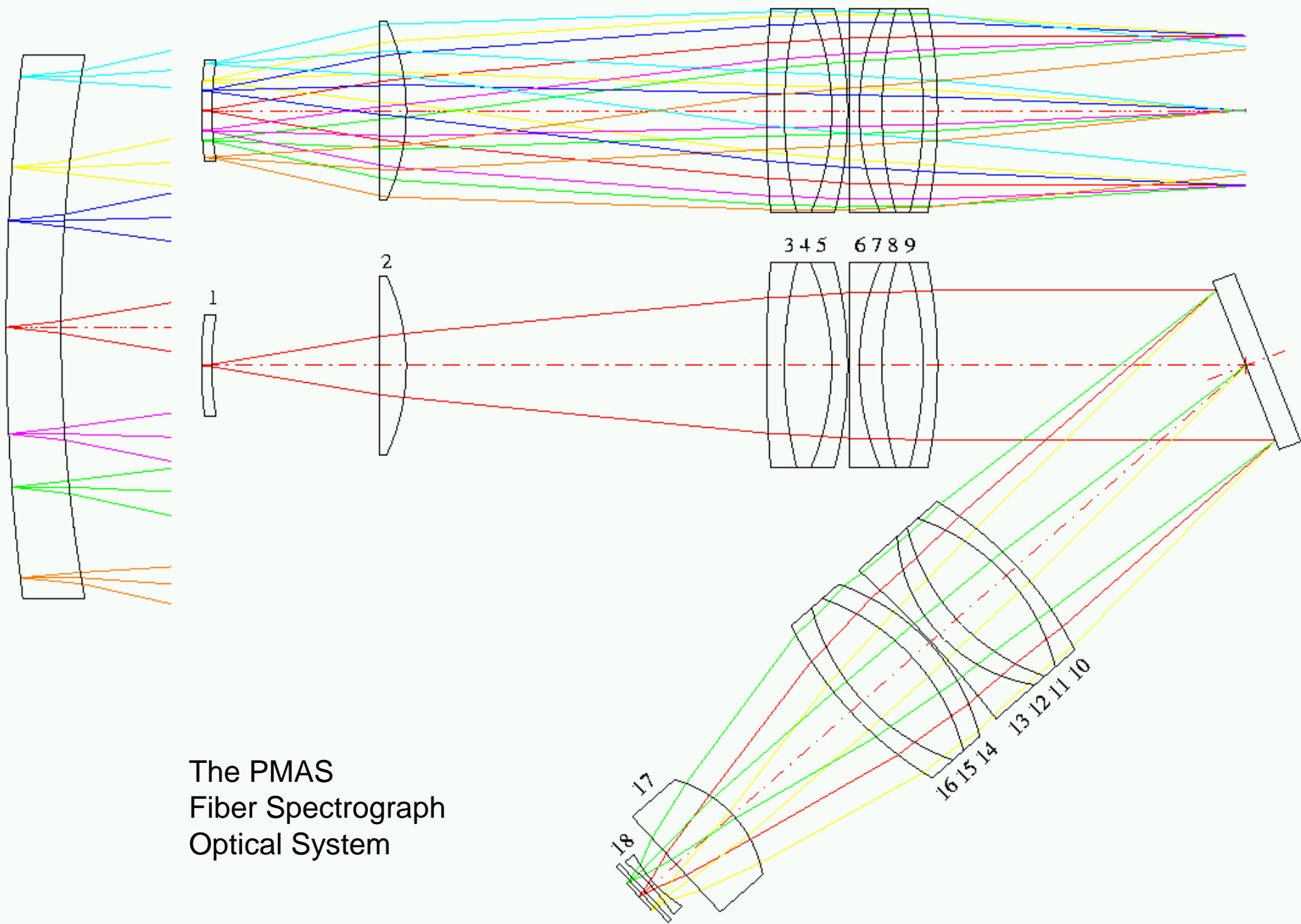
→ diffraction limit reached only if:

$$s \leq \frac{\lambda f_1}{N a \cos \theta'}$$

I. Intro: PMAS

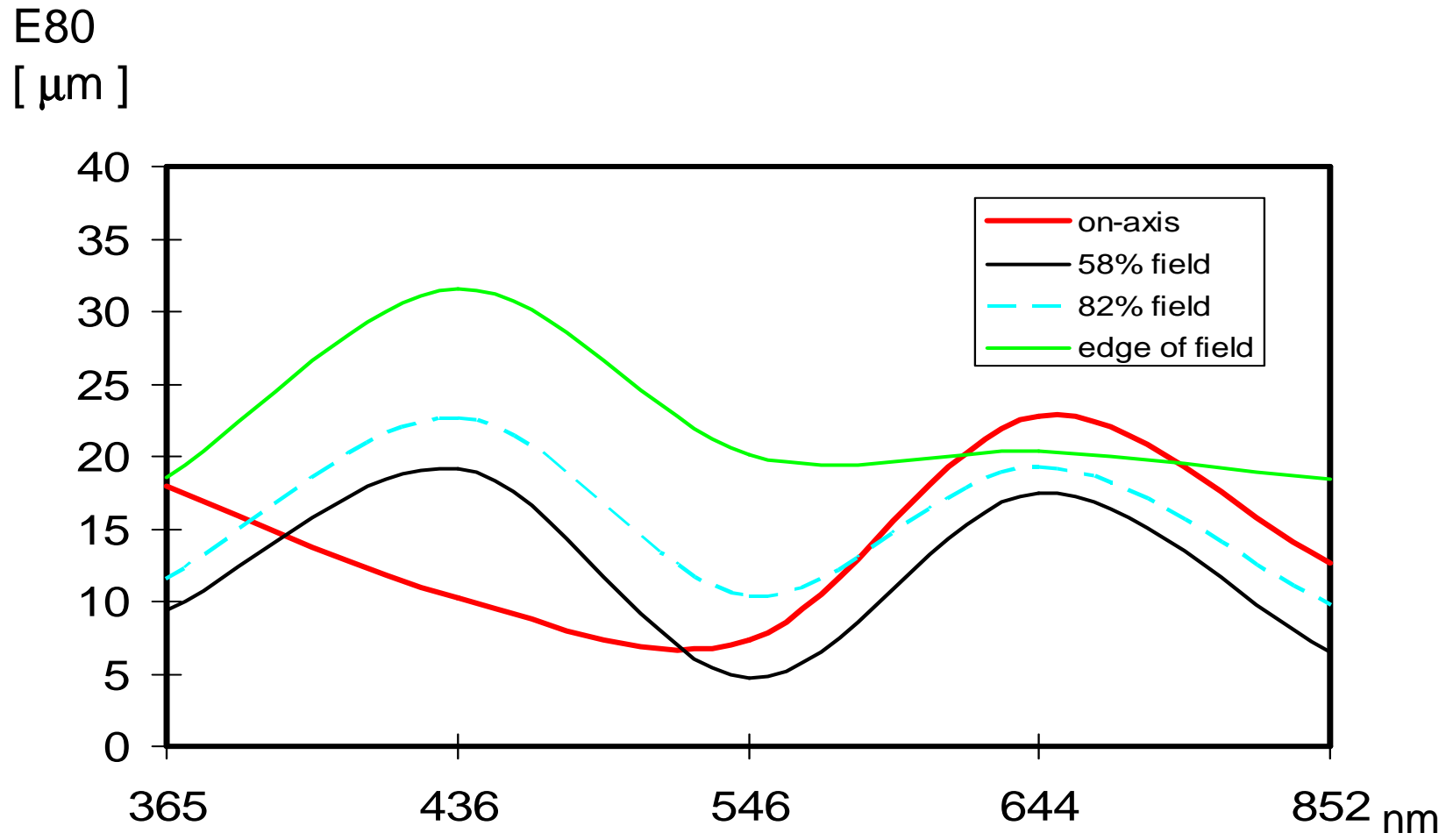






The PMAS
Fiber Spectrograph
Optical System

Image quality



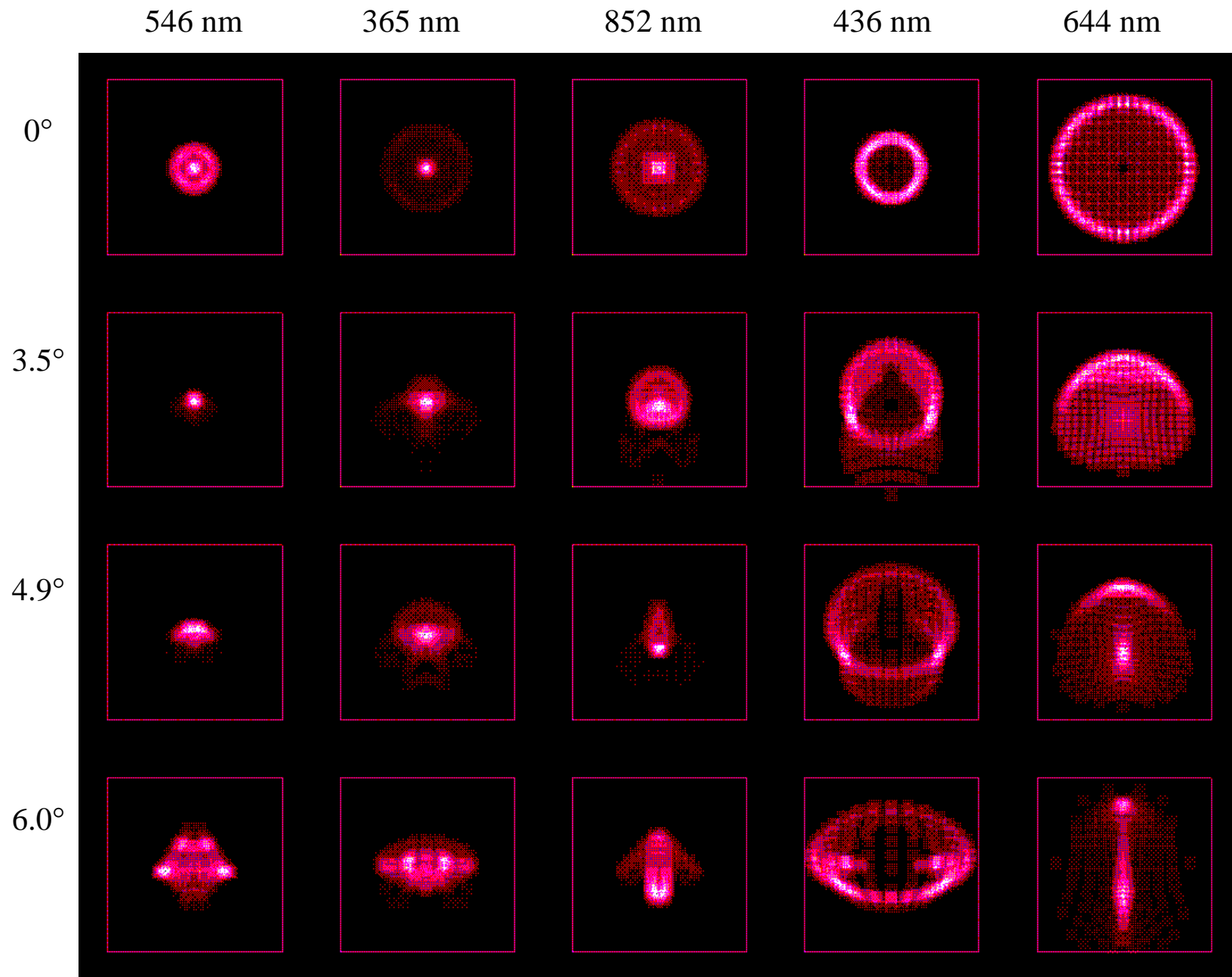


Image quality

365
nm



0 deg on-axis, monochromatic light (365 nm)

0 deg

546
nm



3.5 deg off-axis, monochromatic light (546 nm)

3.5 deg

647
nm



4.9 deg off-axis, monochromatic light (647 nm)

4.9 deg

SPECTEST (11, 17, & 18.07.2000) Focus series: focus steps: 0.125 mm, box size: 0.192 mm (8 pixel)

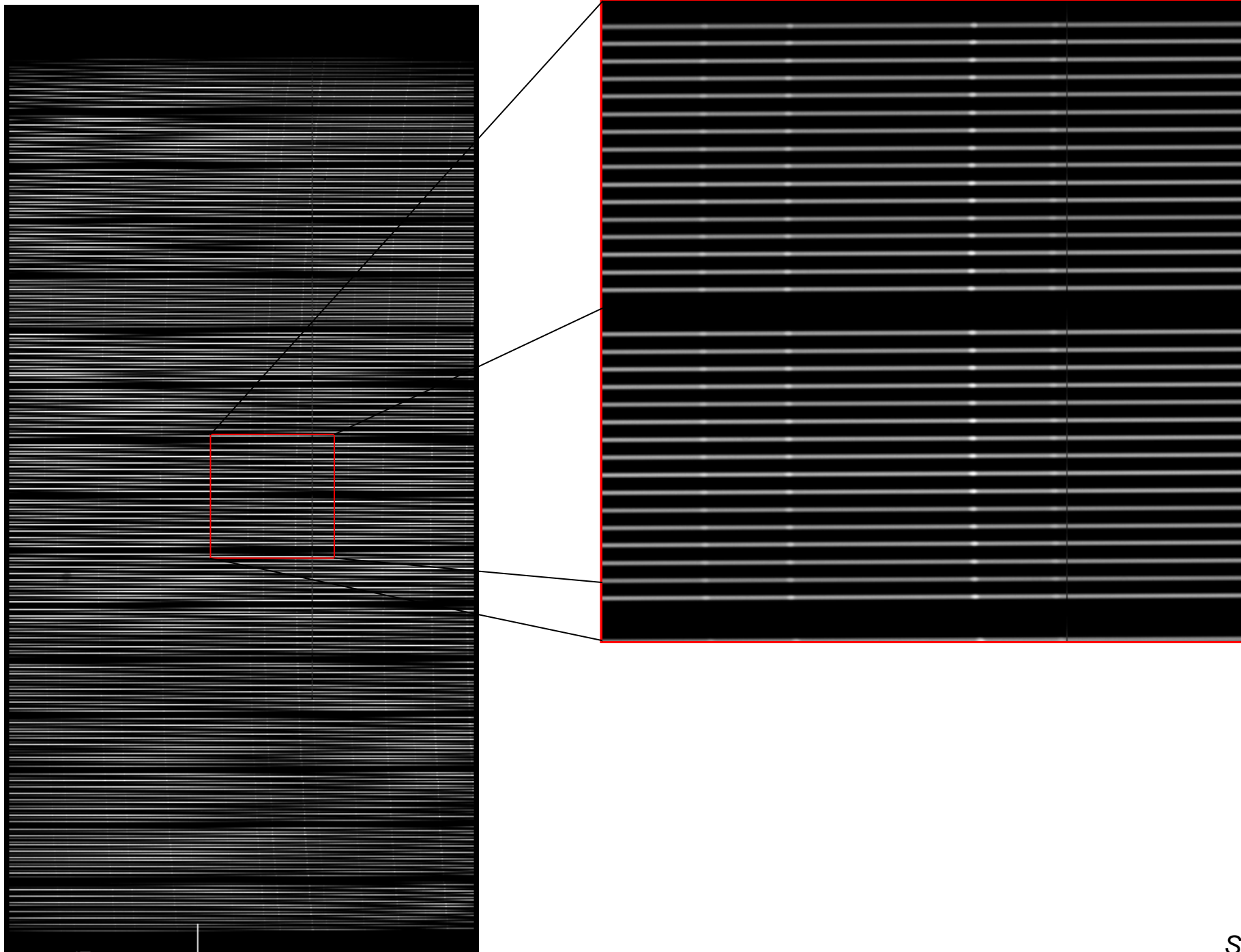
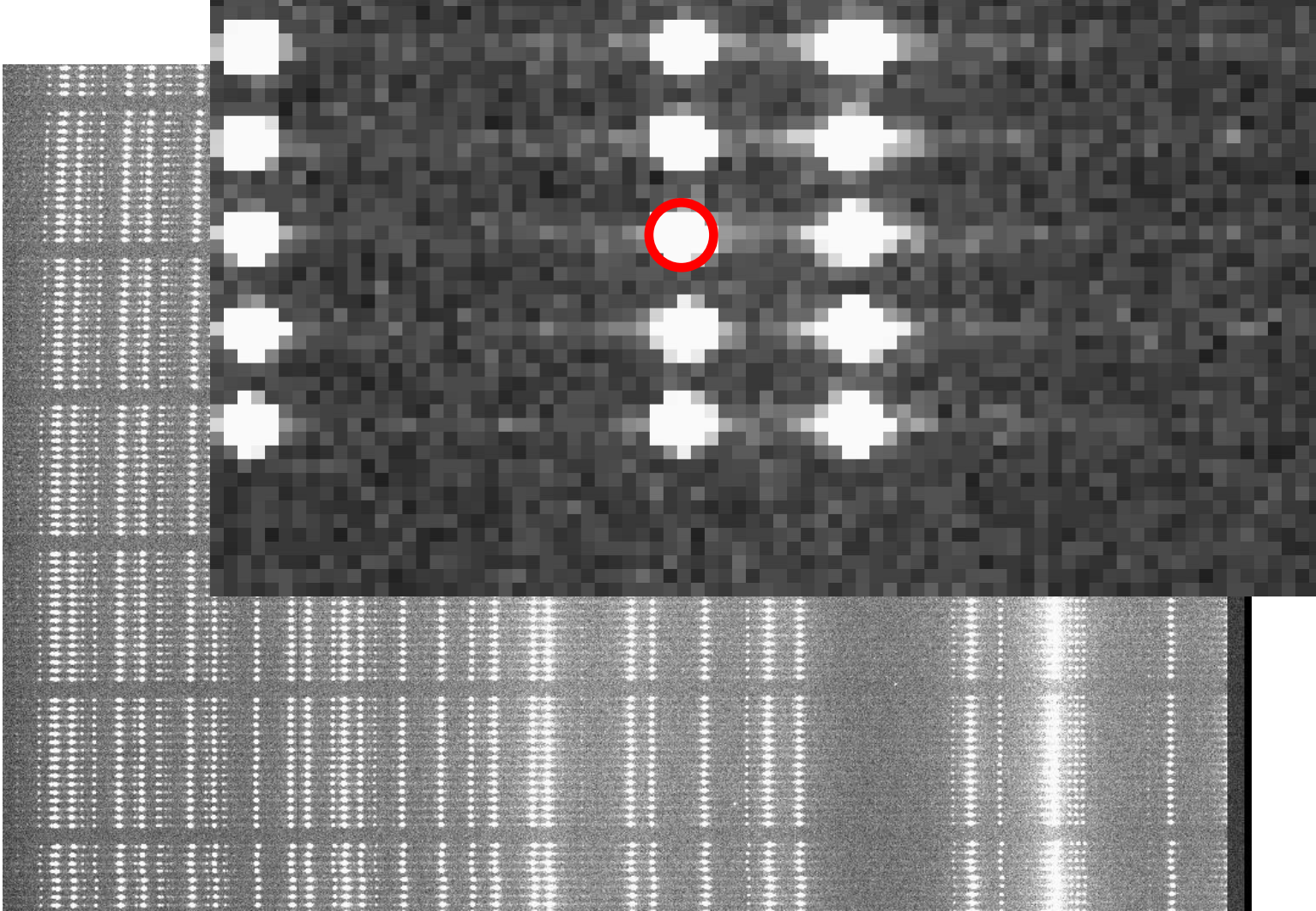
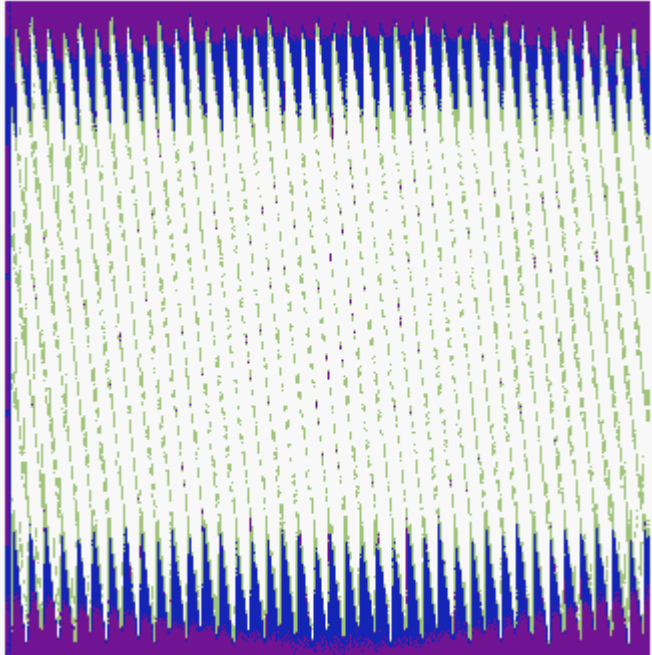


Image quality, spectral resolution



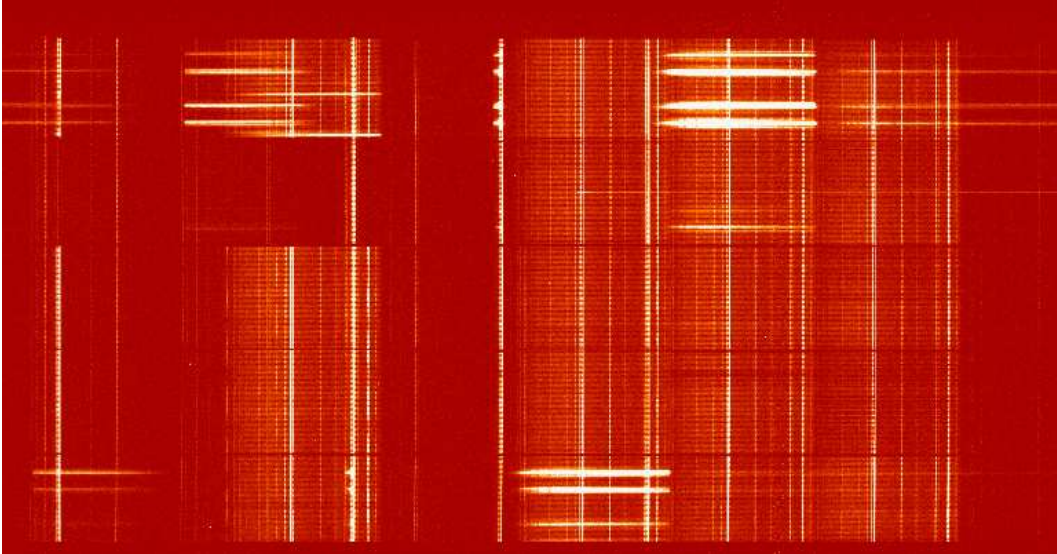
PMAS Ne Arc Exposure

Free spectral range



Sauron

VIMOS IFU

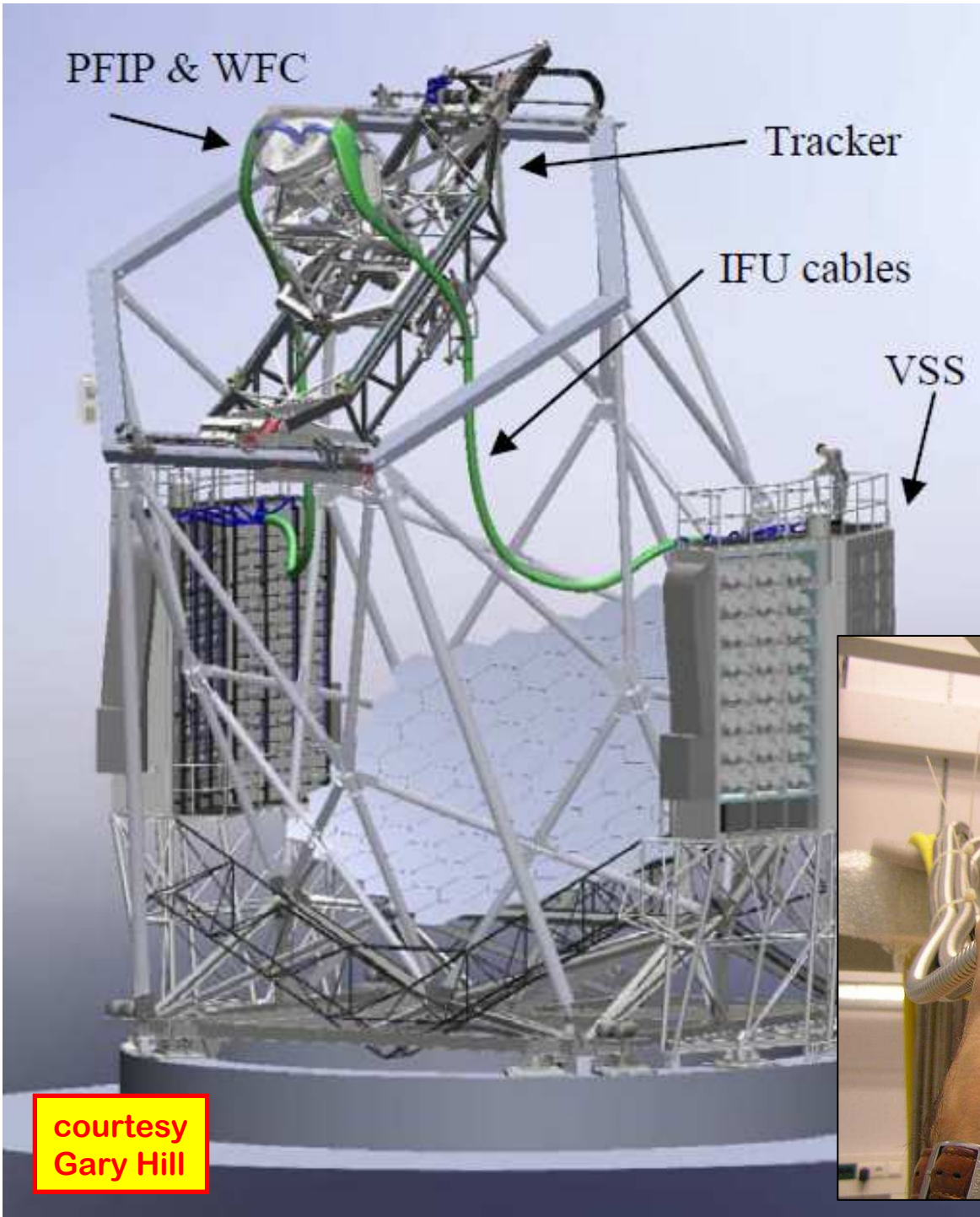


Hobby Eberly Telescope Dark Energy Experiment

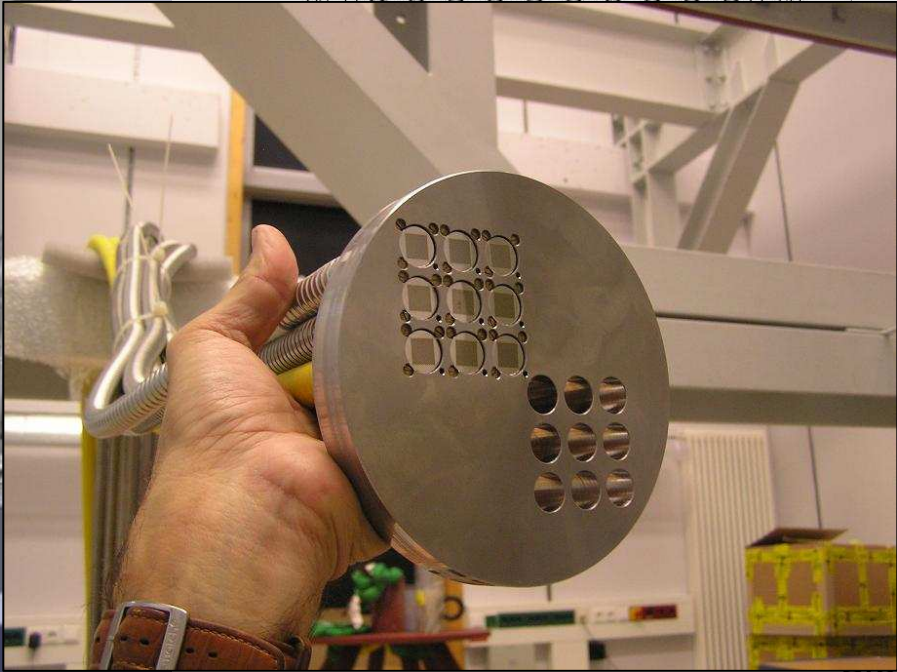
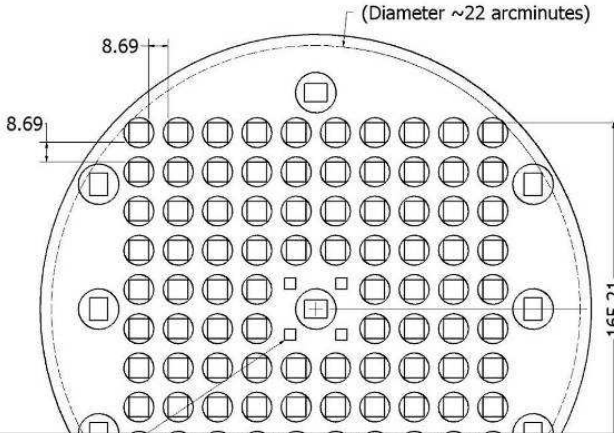
<http://hetdex.org>

- survey of more than a million galaxies up to redshift $z=3.5$
- constrains expansion history of the Universe
- direct detection on dark energy at $z\sim 2.5$
- evolution of dark energy ?

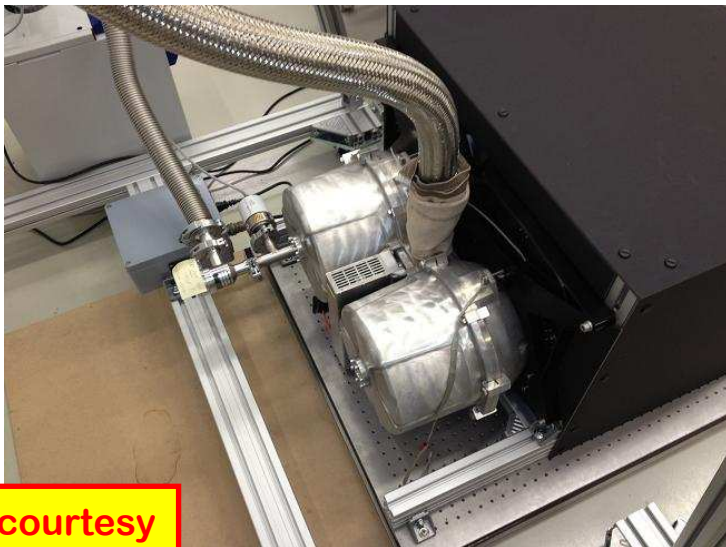
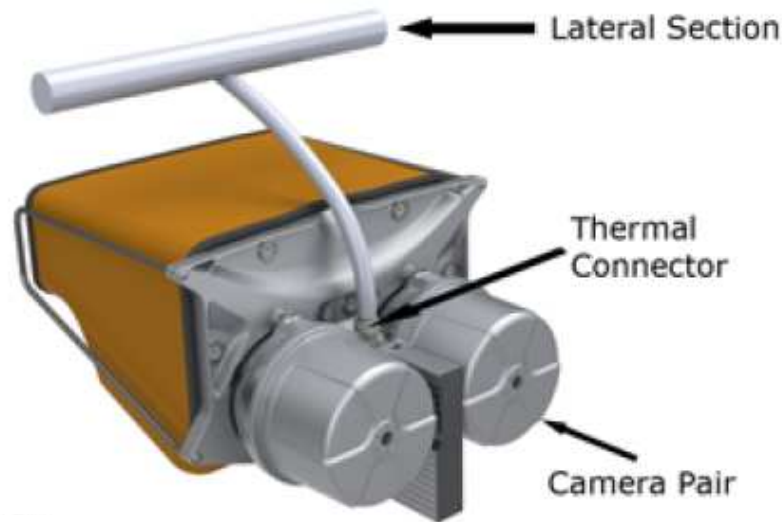




HETDEX



VIRUS: Visible Integral-field Replicable Unit Spectrograph



courtesy
Gary Hill

- unit spectrograph
 - 224 fibers, 1.5 arcsec on the sky
 - 350-550 nm wavelength range, $R=700$
- 2 unit spectrographs fed by one 50 arcsec x 50 arcsec IFU, 448 fibers
- 150 VIRUS cover
 - 33,600 spectra simultaneously
 - 12 million independent resolution elements per exposure
- Industrial replication concept
 - Massive replication of inexpensive unit spectrograph cuts costs and development time
 - Prototype development at AIP
 - R&D with local industry

VIRUS: Visible Integral-field Replicable Unit Spectrograph

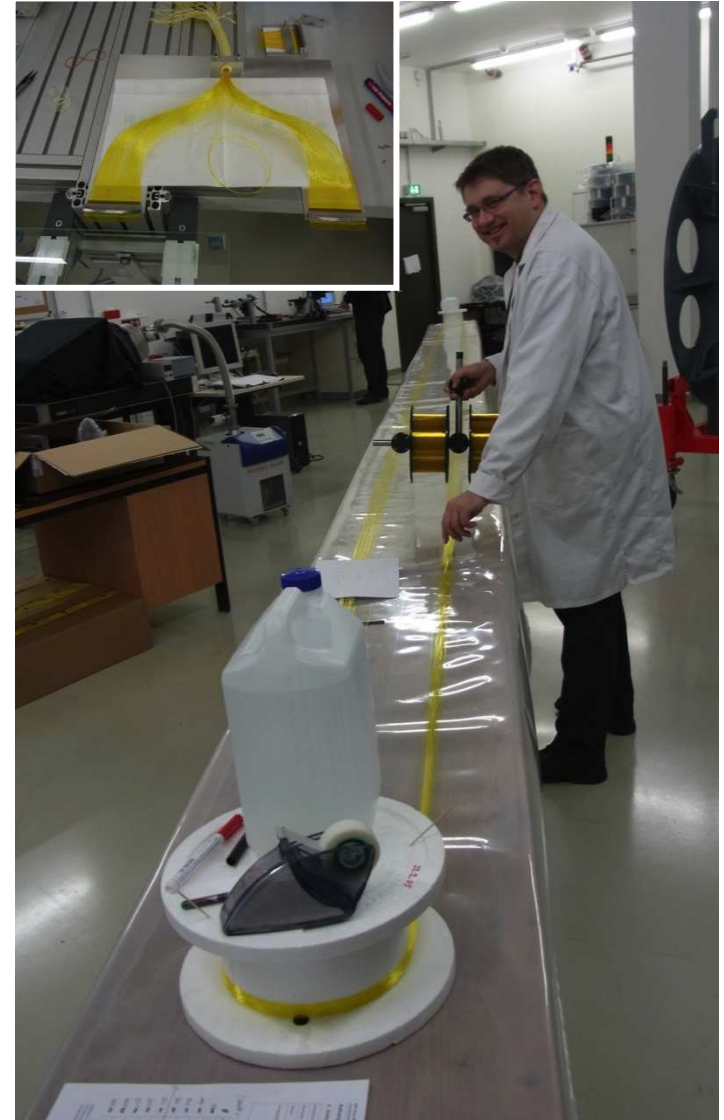


Completed spectrographs at UT Austin

Completed IFU fiber cable in Potsdam



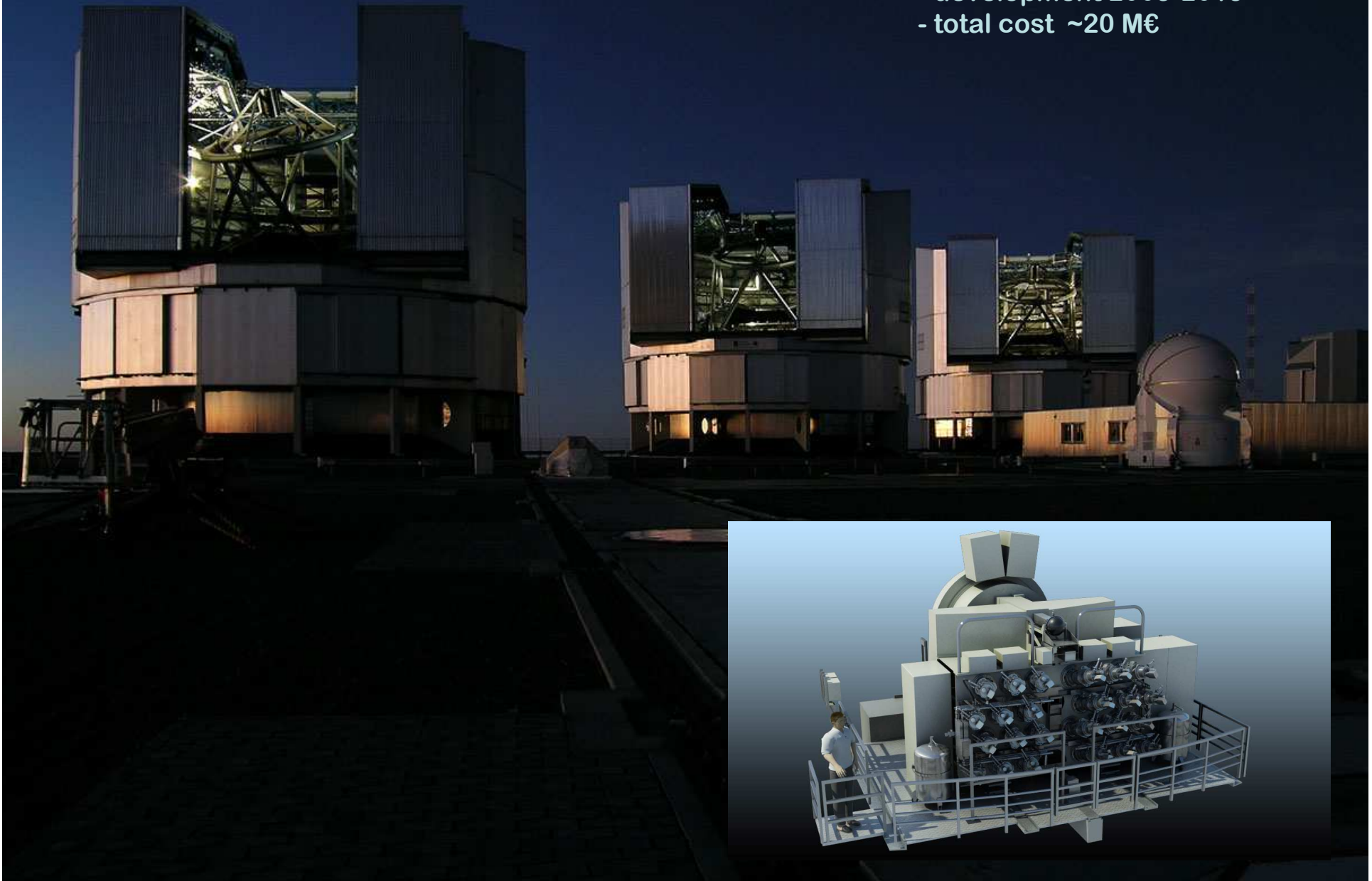
**courtesy
Gary Hill**



Laying out fibers at AIP

MUSE, 2nd Generation VLT Instrument

- development 2003-2013
- total cost ~20 M€

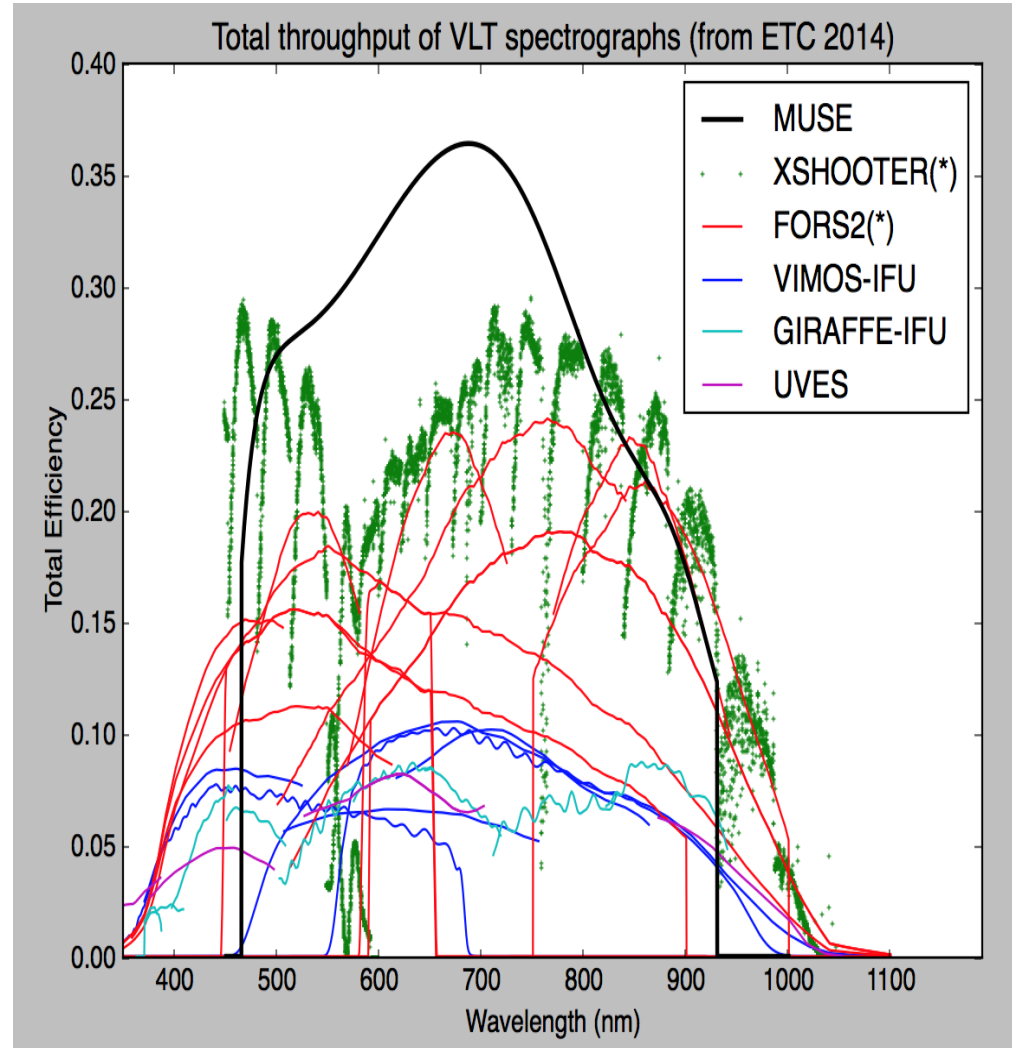
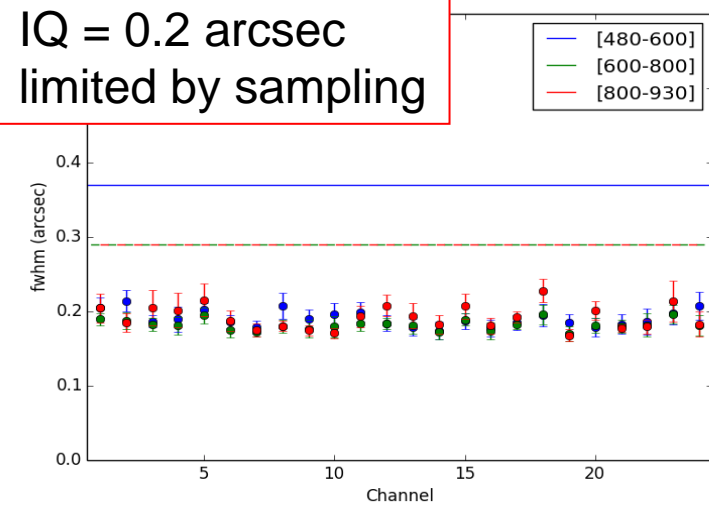
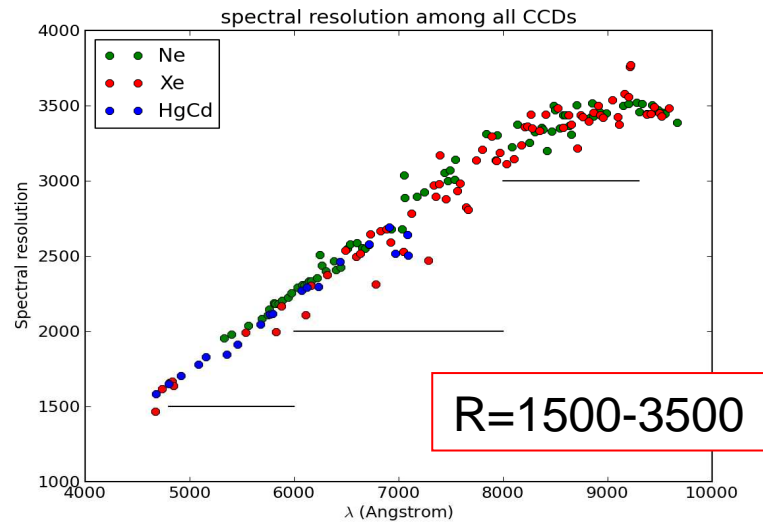


MUSE (ESO VLT)

Spectral range (simultaneous)	0.465-0.93 μm	4096 pixels
Resolving power	2000@0.46 μm	370 x 10 ⁶ pixels
	4000@0.93 μm	
Wide Field Mode (WFM)		90,000 spaxels
Field of view	1x1 arcmin ²	
Spatial sampling	0.2x0.2 arcsec ²	
Spatial resolution (FWHM)	0.3-0.4 arcsec	
Gain in ensquared energy within one pixel with respect to seeing	2	AO
Condition of operation with AO	70%-ile	Laser guide stars
Sky coverage with AO	70% at Galactic Pole	
Limiting magnitude in 80h	I _{AB} = 25.0 (R=3500)	High throughput
	I _{AB} = 26.7 (R=180)	
Limiting Flux in 80h	3.9 10 ⁻¹⁹ erg.s ⁻¹ .cm ⁻²	Stability

courtesy Roland Bacon

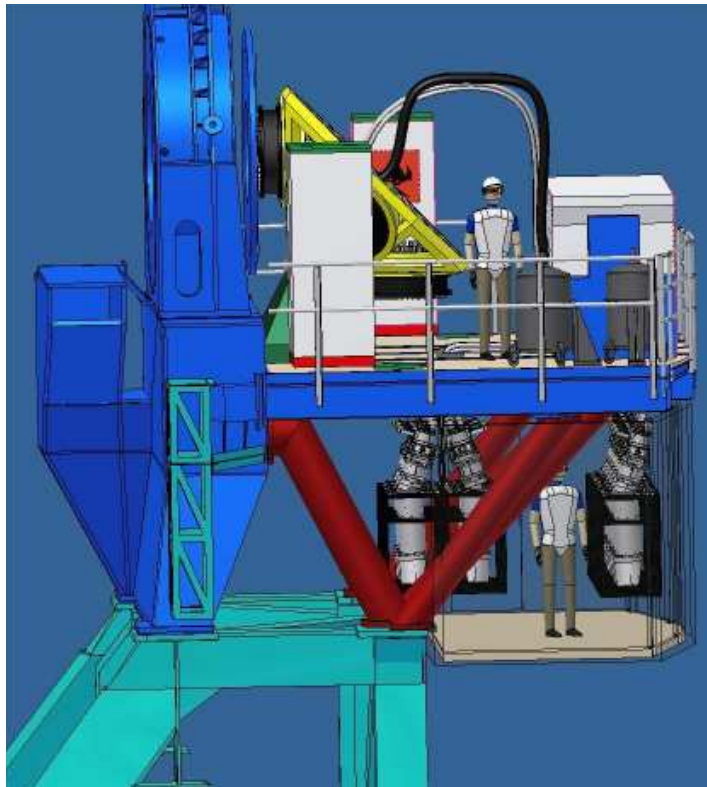
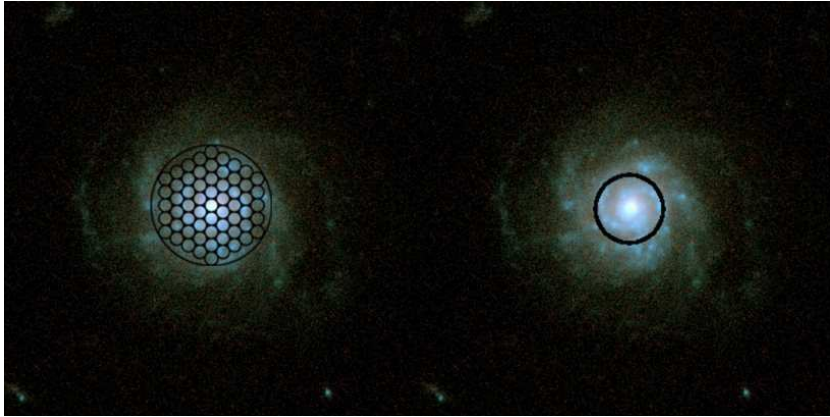
Instrument Performance



(*) 17% Slit loss included in FORS2 & XSHOOTER (e.g. 1 arcsec slit with 0.8 arcsec seeing)

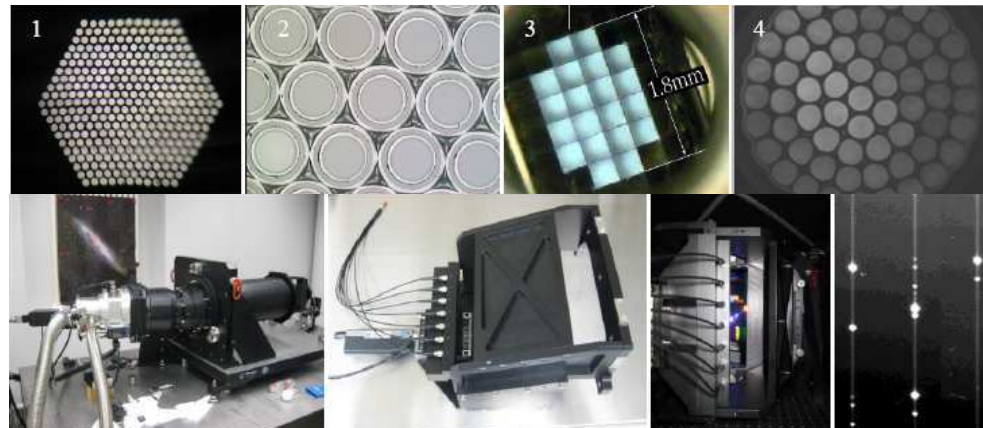
courtesy Roland Bacon

FIREBALL Concept 2011

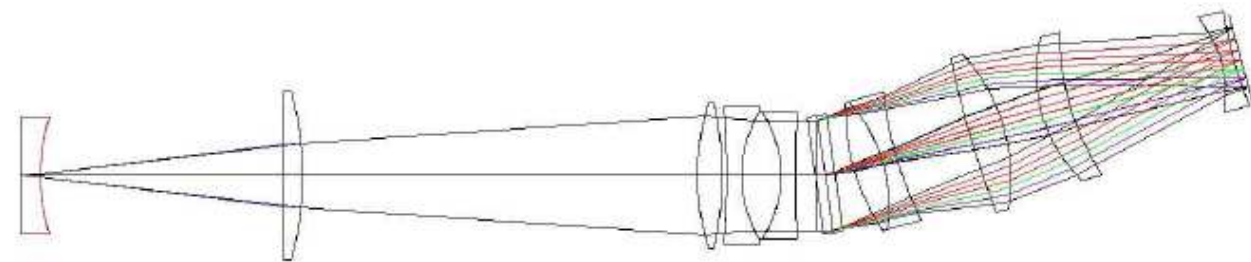
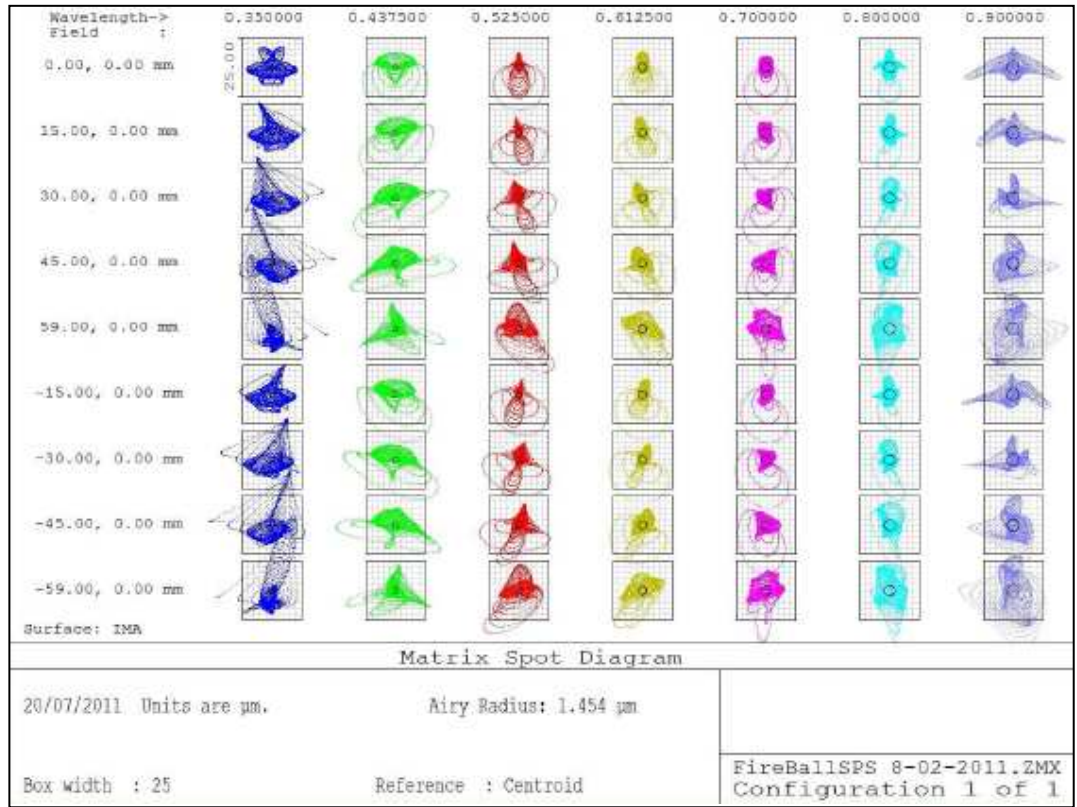


FIREBALL Baseline Parameters

- ▶ FLAMES OzPoz patrol field with 26' diameter FoV
- ▶ 90 hexabundle IFUs, each with ~5" diameter FoV
- ▶ Hexabundles: 61 fibres, 0.6" projected fibre core diameter
- ▶ 6 spectrographs, adapted for fibre-feed, $R \sim 1200-2100$
- ▶ free spectral range: 430-850nm (goal: blue extension)
- ▶ total throughput goal: 30%
- ▶ sensitivity: $R \sim 19.8$ survey limit, resulting in 100-160 galaxies per FLAMES field at median $z \sim 0.2$; typical half-light sizes for disk galaxies 2"-6" diameter
- ▶ detector head, NGC CCD controller, vacuum/cooling system adapted from MUSE
- ▶ individual spectrograph shutters
- ▶ no moving parts other than shutters + fibre positioner
- ▶ retain full existing facility and utilise as much FLAMES infrastructure as practical



Next Generation Fiber Spectrograph „Fireball“





► Miniature Spectrographs

IPS based on arrayed waveguide gratings (AWG)

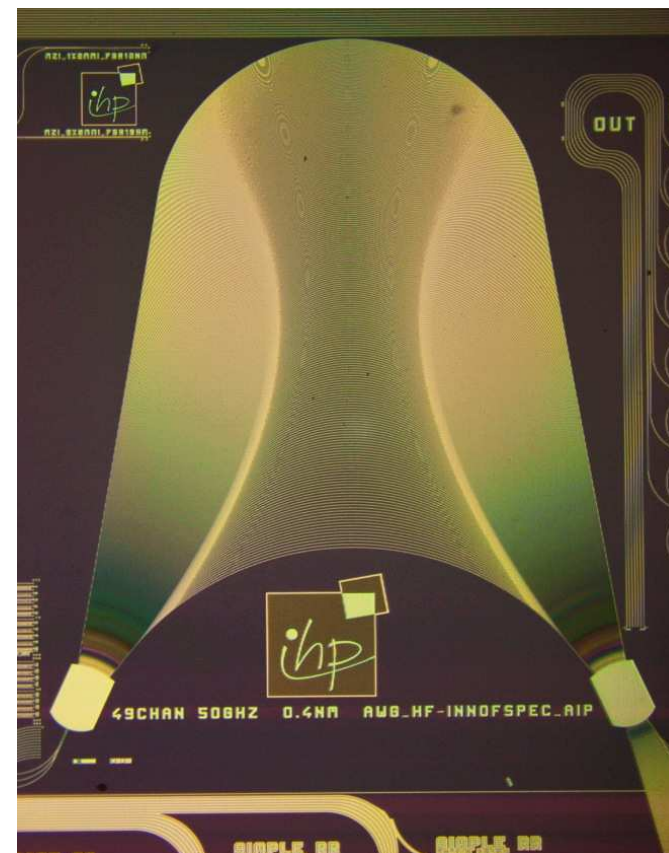
- Replicable, modular and robust
- Small and costs effective
- Astronomy, remote sensing, etc

Si_3N_4 and Silica-on-Silicon (SoS)

- $R > 60,000$
- $< 0.02\text{nm}$ spacing (Comm. $> 0.4\text{nm}$)
- $\lambda \sim 1.15 - 1.79\mu\text{m}$ (Comm. $1.53-15.6\mu\text{m}$)

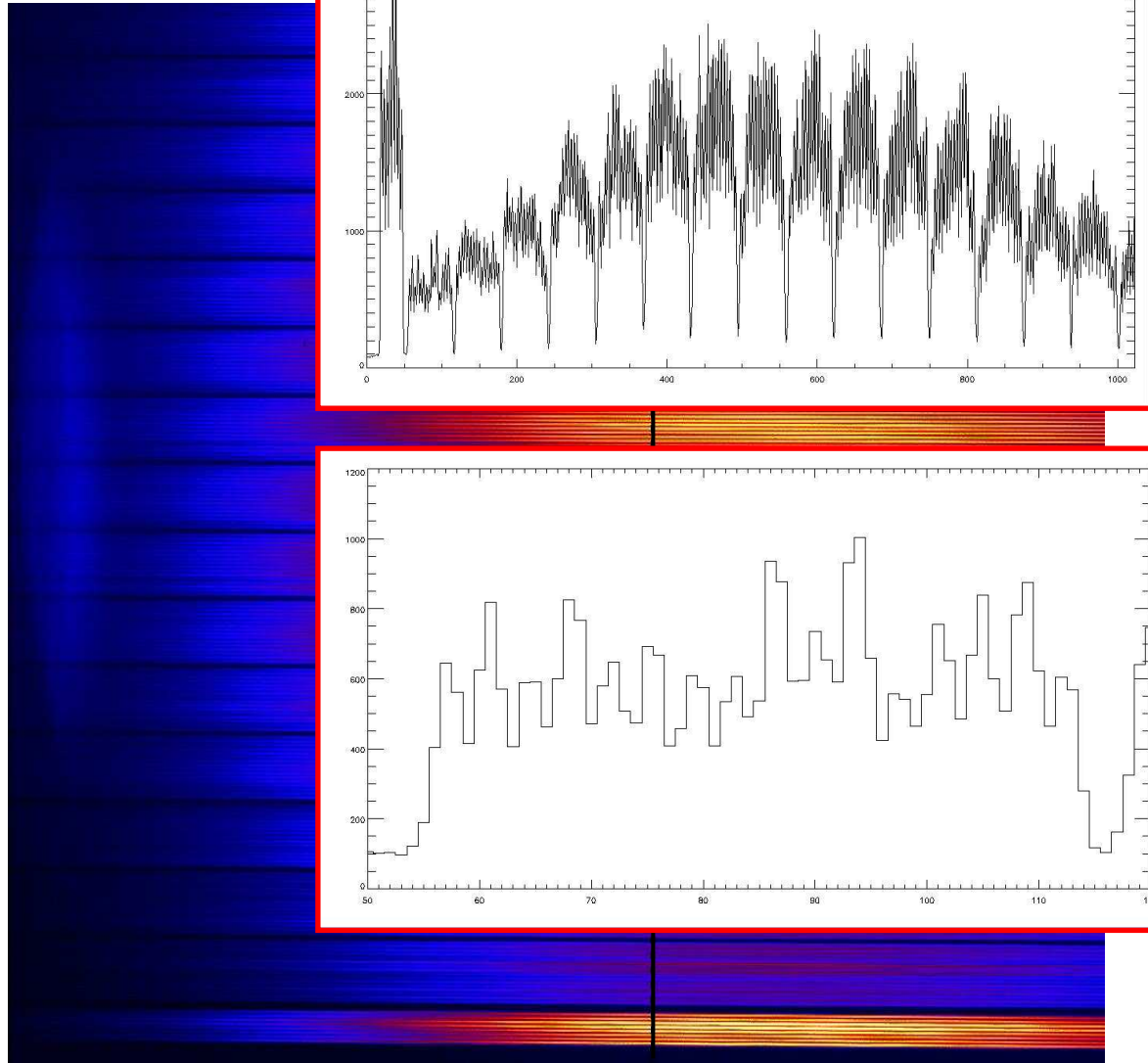
Collaborations:

- IHP Frankfurt Oder (Si_3N_4)
- MQ-Photonics and AAO (Astro)
- FBH Berlin (Space)
- ORC Southampton



(4) Data Reduction

Data reduction: extraction of spectra



PhD thesis
Thomas Becker
2002

→ p3d

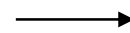
Data reduction: extraction of spectra

swath extraction:

Flux lost in wings ~10%

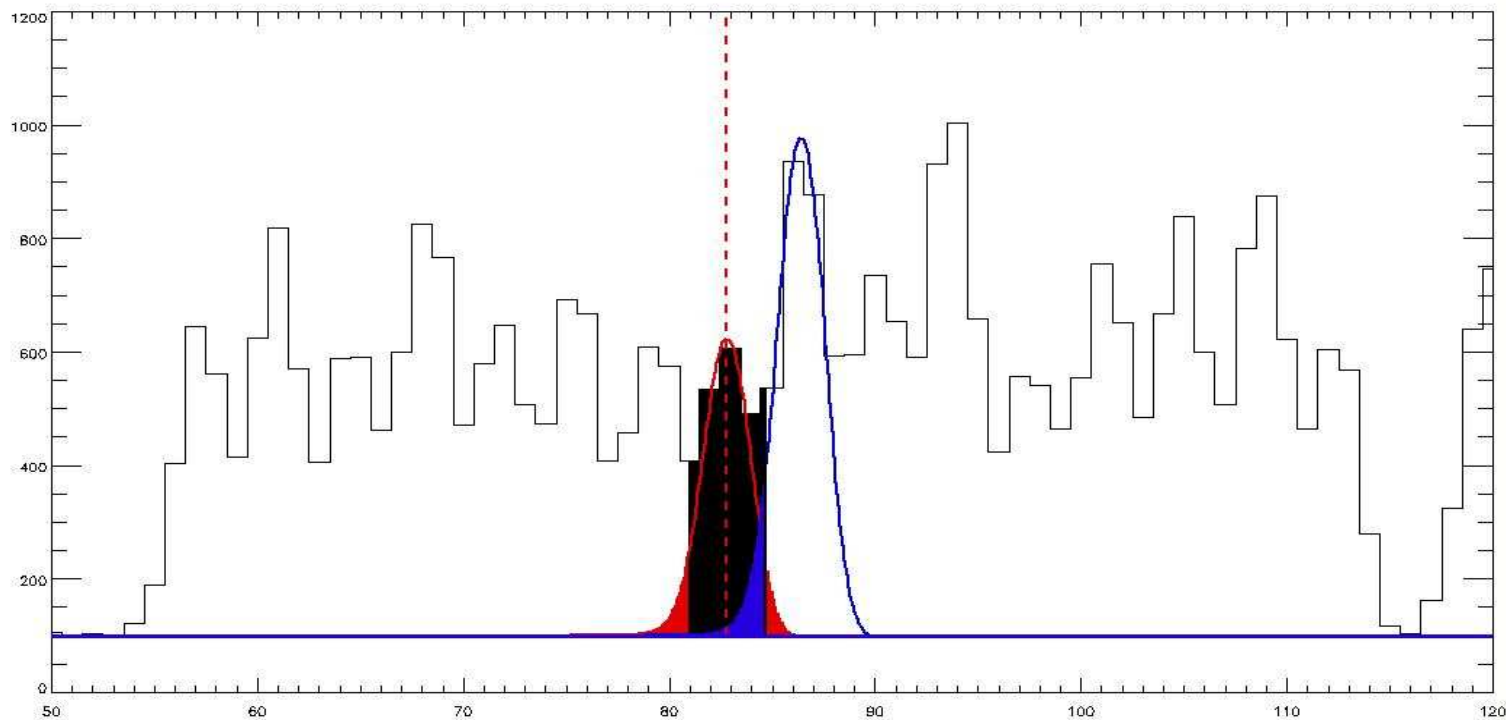
Crosstalk between adjacent spectra

Fiber throughput variation ~ 20%



9% systematic error.

optimal extraction : simultaneous fit of all spectra



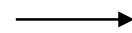
Data reduction: extraction of spectra

swath extraction:

Flux lost in wings ~10%

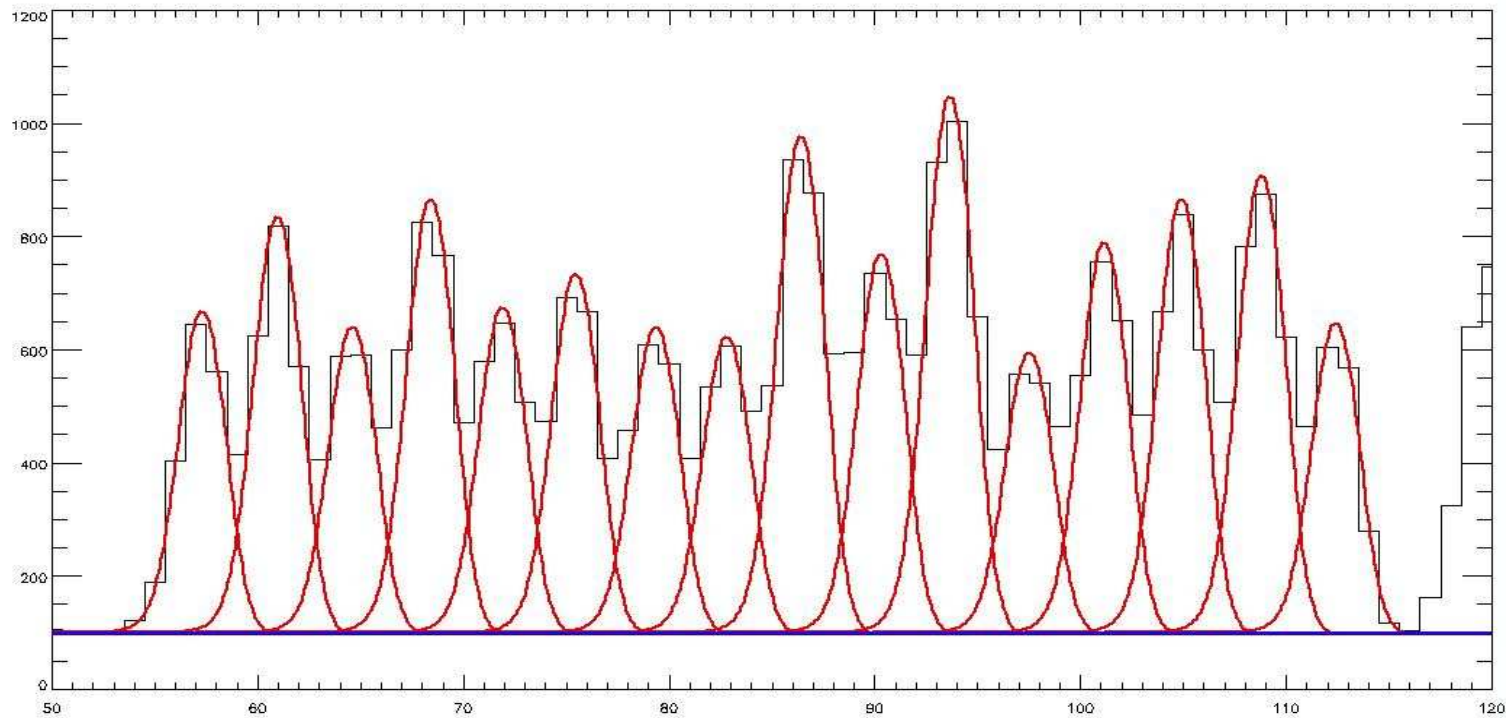
Crosstalk between adjacent spectra

Fiber throughput variation ~ 20%



9% systematic error.

optimal extraction : simultaneous fit of all spectra



Versatile data reduction package „P3d“

<http://p3d.sourceforge.net/>



a general data-reduction tool for fiber-fed integral-field spectrographs

About p3d

Readme

Download

Documentation

Links

Screenshots

Authors & Thanks

Sourceforge
project page

About p3d

Introduction

p3d is a general data-reduction tool that is intended to be used with data of fiber-fed integral field spectrographs (IFSSs). This tool can be useful for people who have access to astronomical data of such an instrument. Data-reduction tasks, which are performed by p3d, convert raw data of CCD detectors into extracted spectra that can thereafter be used for scientific purposes.

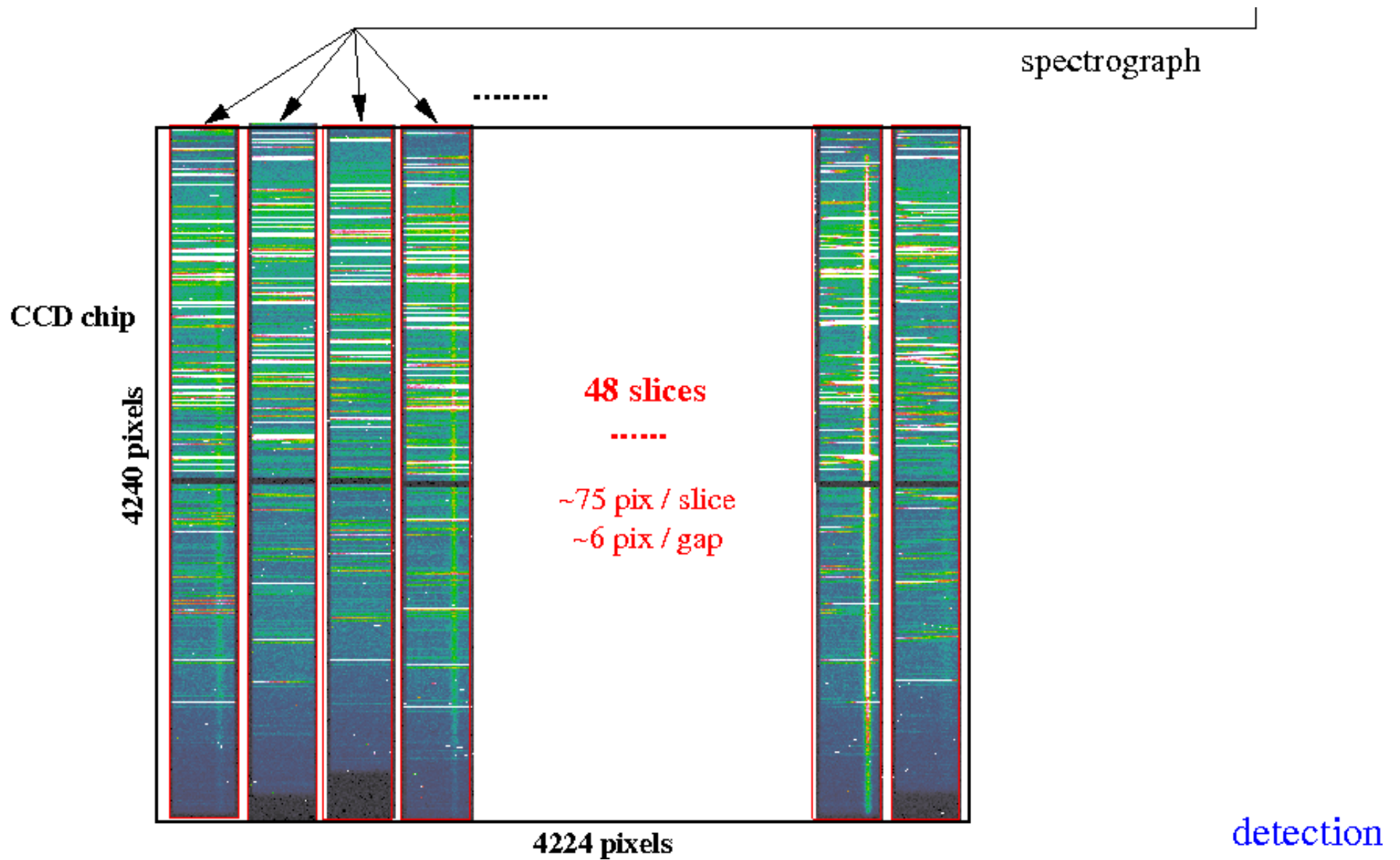
Please, have a look at the presentation of p3d – for use with the ESO instruments VIMOS and FLAMES – in the June 2011 issue of *The Messenger*; "p3d – A Data Reduction Tool for the Integral-field Modes of VIMOS and FLAMES" Sandin C., Weillbacher P., Streicher O., Walcher J., and Roth M.M. 2011, *The Messenger*, 144, 13–16.

In October 2010, at the STScI-workshop "IFUs in the era of JWST", Christer Sandin gave a presentation with the title "Integral-Field Spectroscopy Data Reduction Made Easy with p3d". You can watch the presentation if you follow this link.

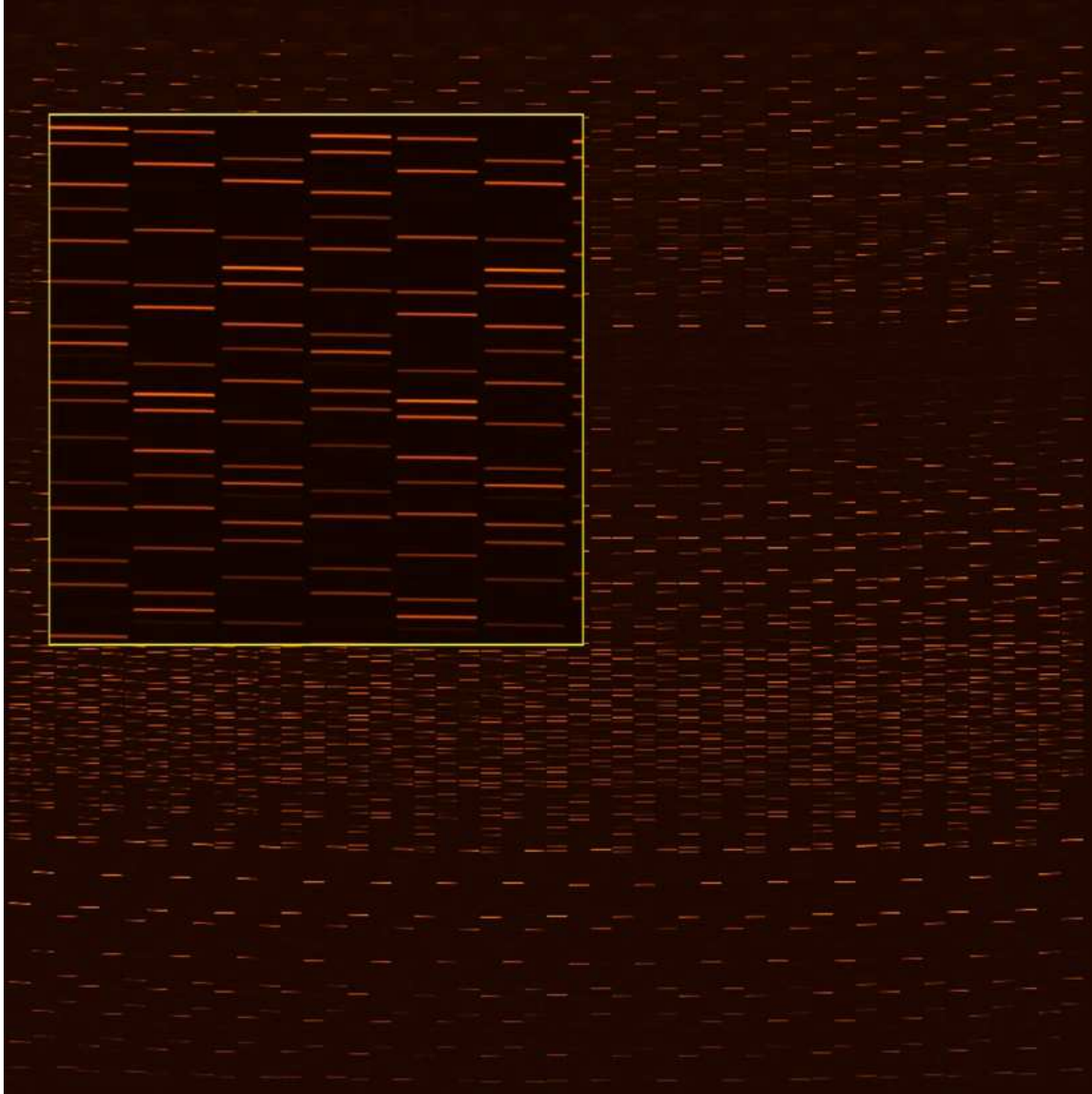
Version 2.1 was released on 24.11.2011.

Versions 2.0.3 (13.10.2011), 2.0.2 (28.9.2011), 2.0.1 (18.8.2011), 2.0 (17.6.2011), 2.0b (3.5.2011), 2.0a (26.01.2011), 1.1.1 (21.04.2010).

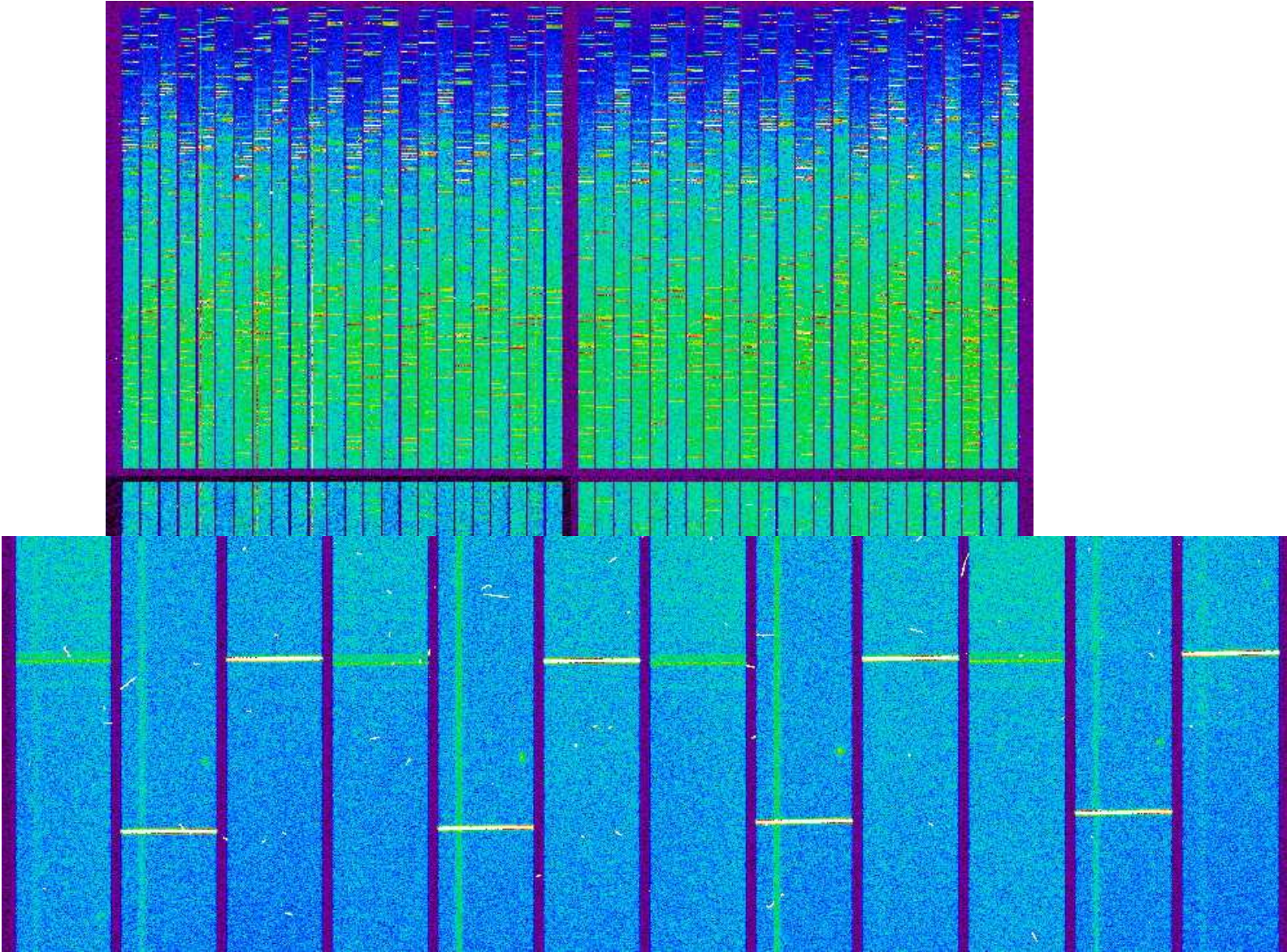
Data Reduction: MUSE pipeline



Data Reduction: MUSE pipeline



Data Reduction: MUSE pipeline



courtesy Peter Weilbacher

Data Reduction: MUSE pipeline

Data “reduction”: re-assemble recorded data into a *datacube* (two spatial axes plus wavelength)

High-level goals

- Speed and automation: reduce data without creating backlog
- Track bad pixels + error information: assess reality of detected objects
- Deliver scientifically usable data for most programs

Consequences

- Use parallelization
- Resample only once

Data Reduction: MUSE pipeline

ESO framework: written in C, based on ESO's CPL

“Recipes” (plugins, i.e. shared libraries) do the work

Possible callers (ESO): esorex / gasgano / Reflex (?)

Callers (consortium): data-management system based on AstroWISE, using a python-cpl interface

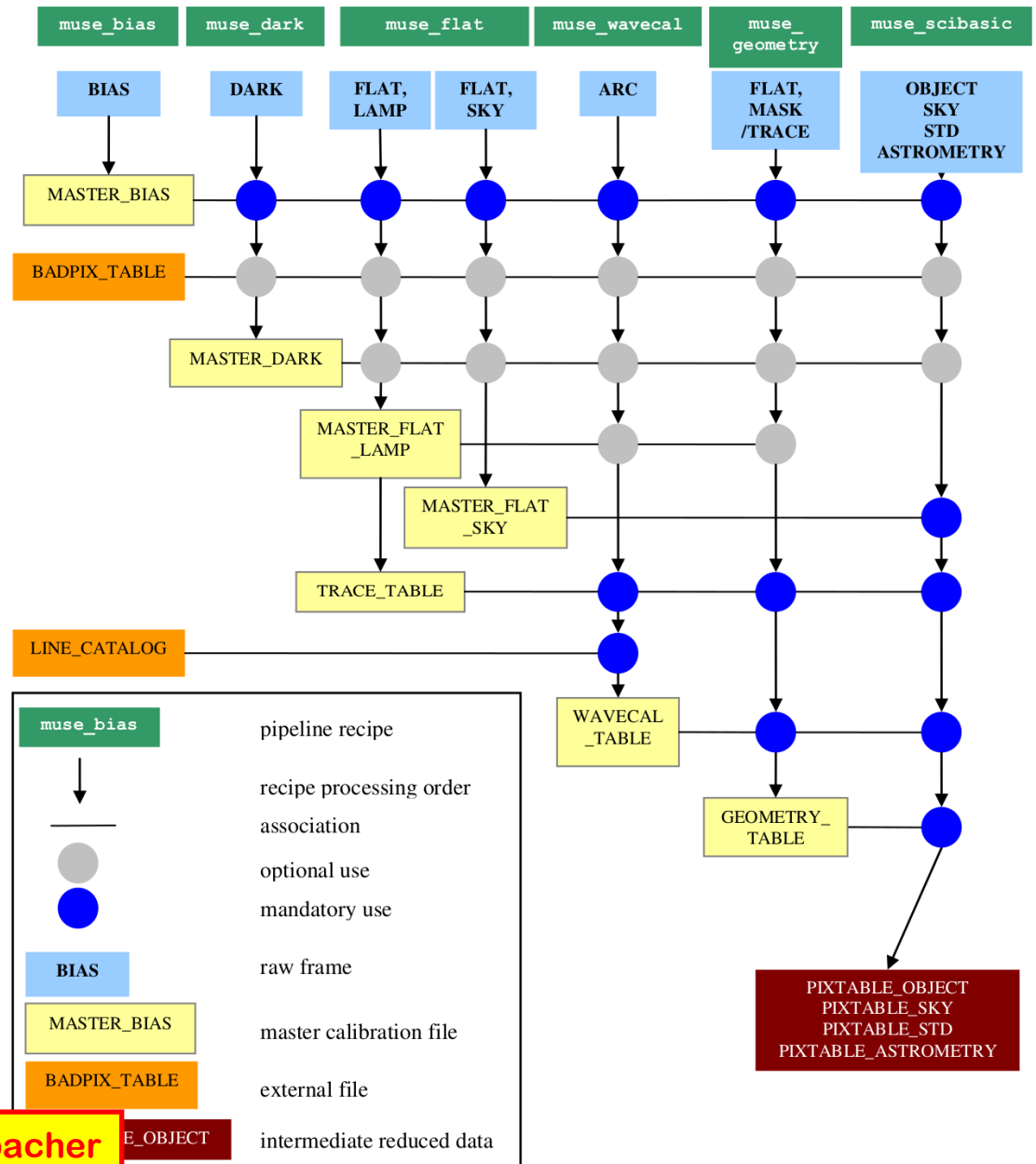
Big data: 800 MiB per raw exposure

Two processing levels

- Per IFU (CCD) calibration, basic reduction (bias, dark, flat, wavecal, and mask handling, **scibasic**)
- → trivially parallel
- Full instrument: on-sky calibrations, science processing (standard stars, sky, astrometry, **scipost**)
- → parallelized as far as possible (shared memory, OpenMP)

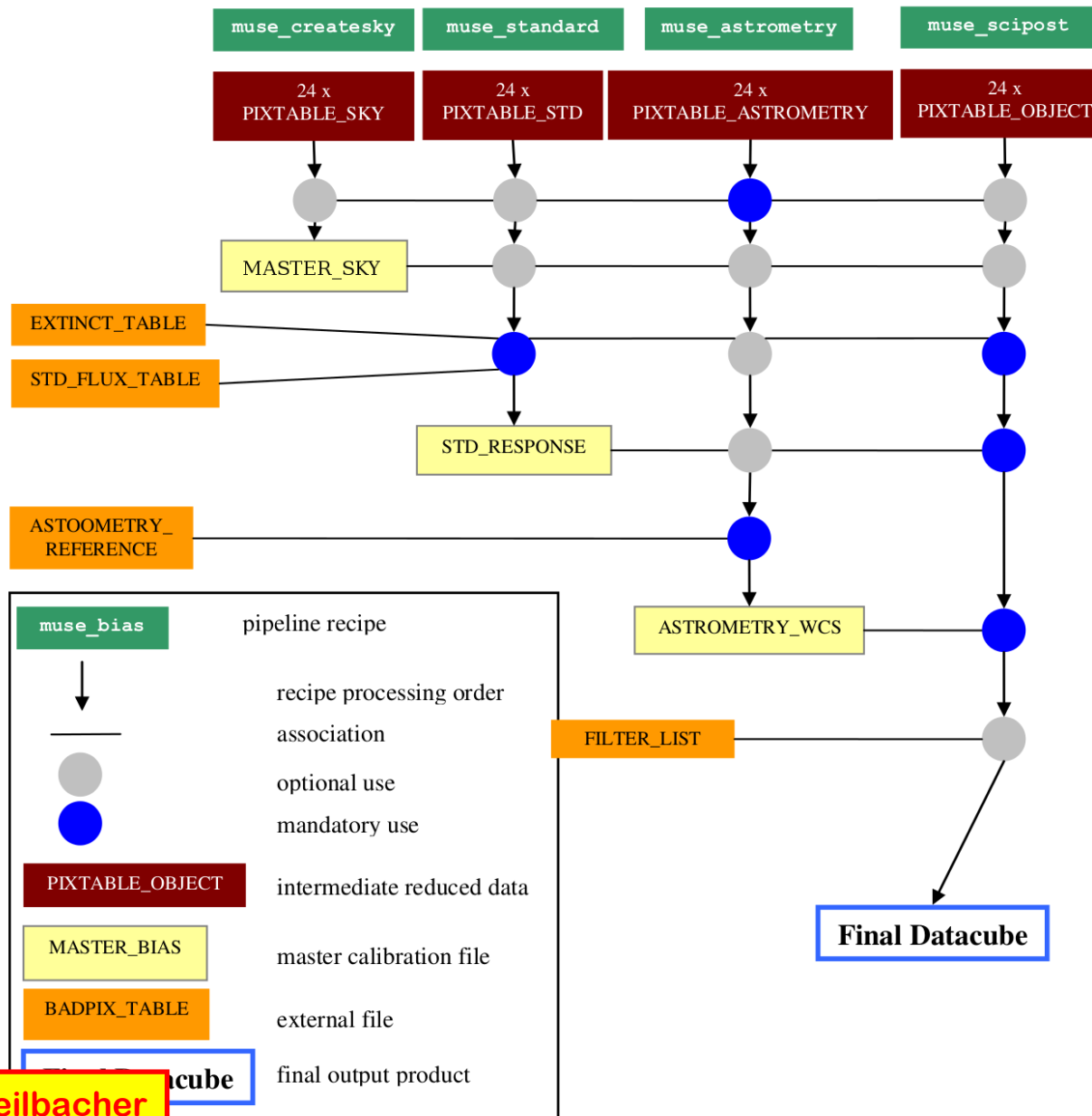
Use multi-core machines with lots of RAM and fast I/O

Data Reduction: MUSE pipeline



courtesy Peter Weilbacher

Data Reduction: MUSE pipeline

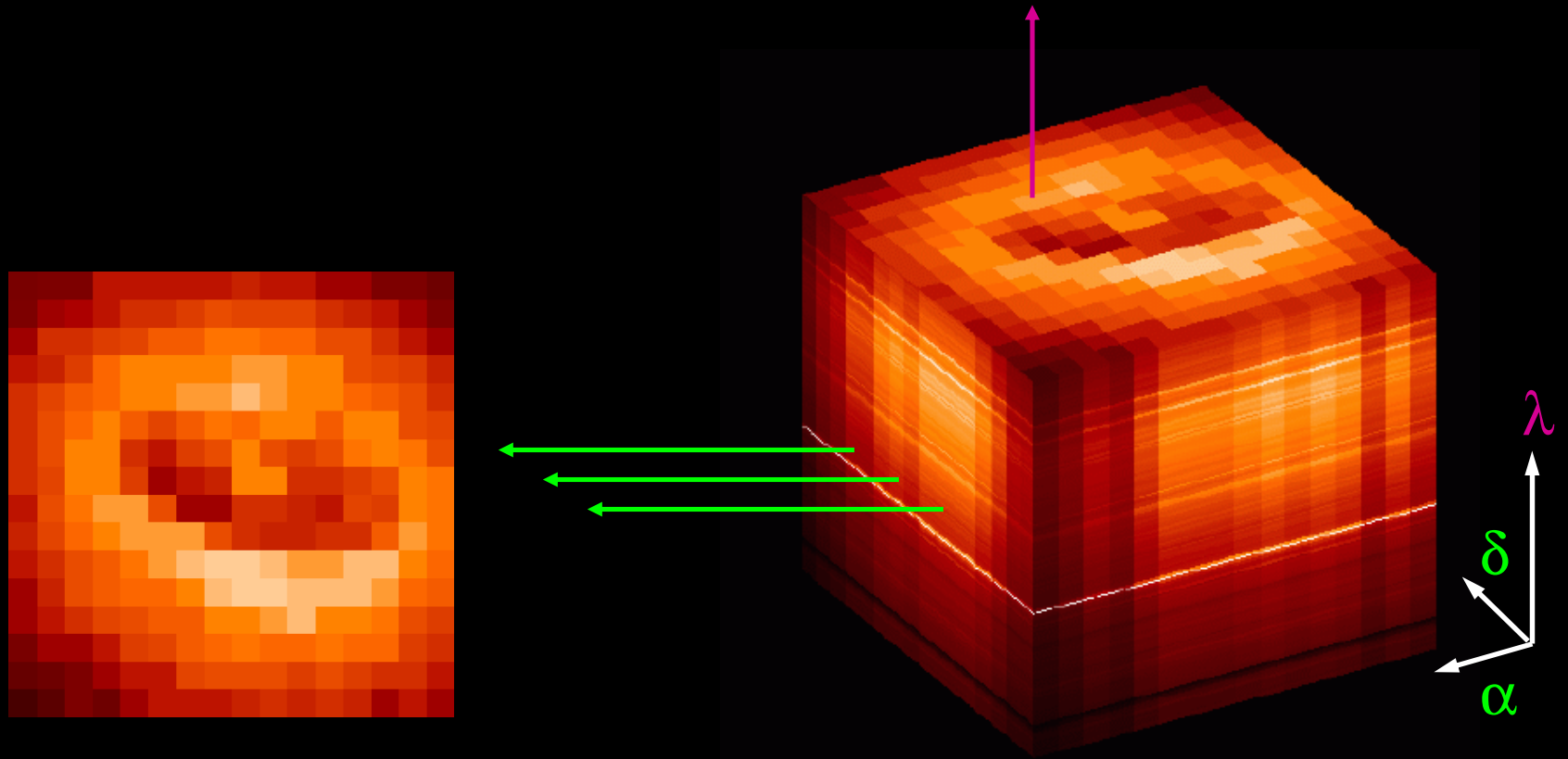
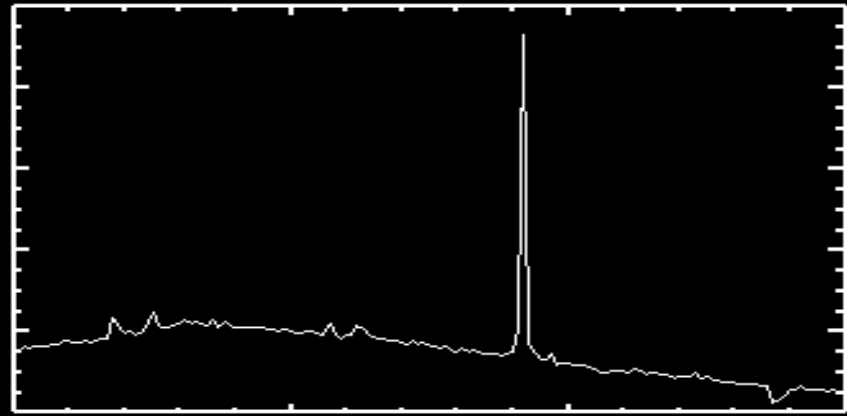


courtesy Peter Weilbacher

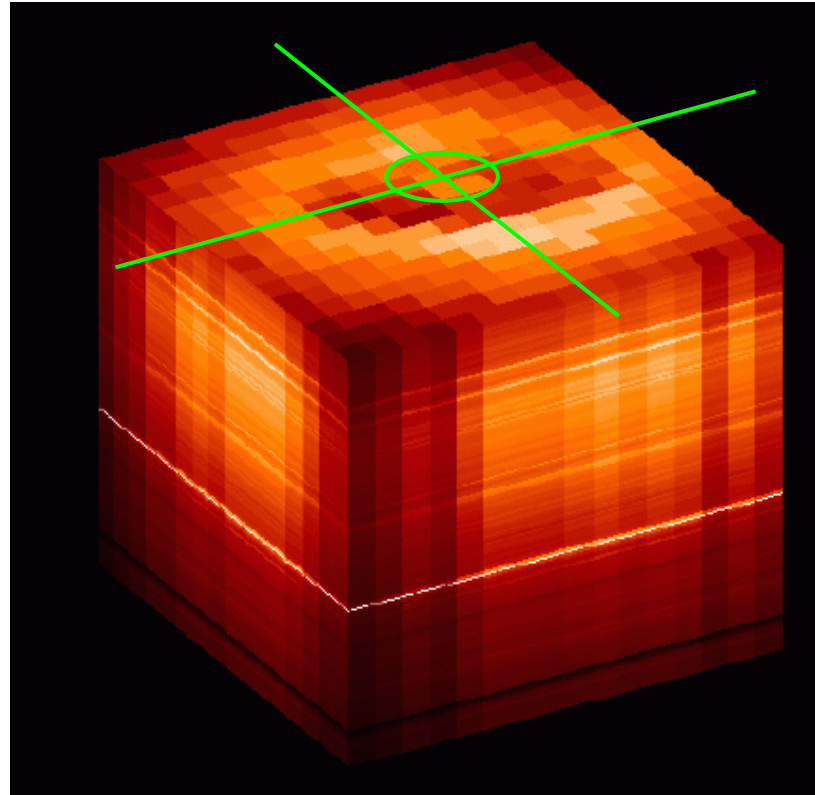
(5) Observation

Peculiar observational features of 3D spectroscopy

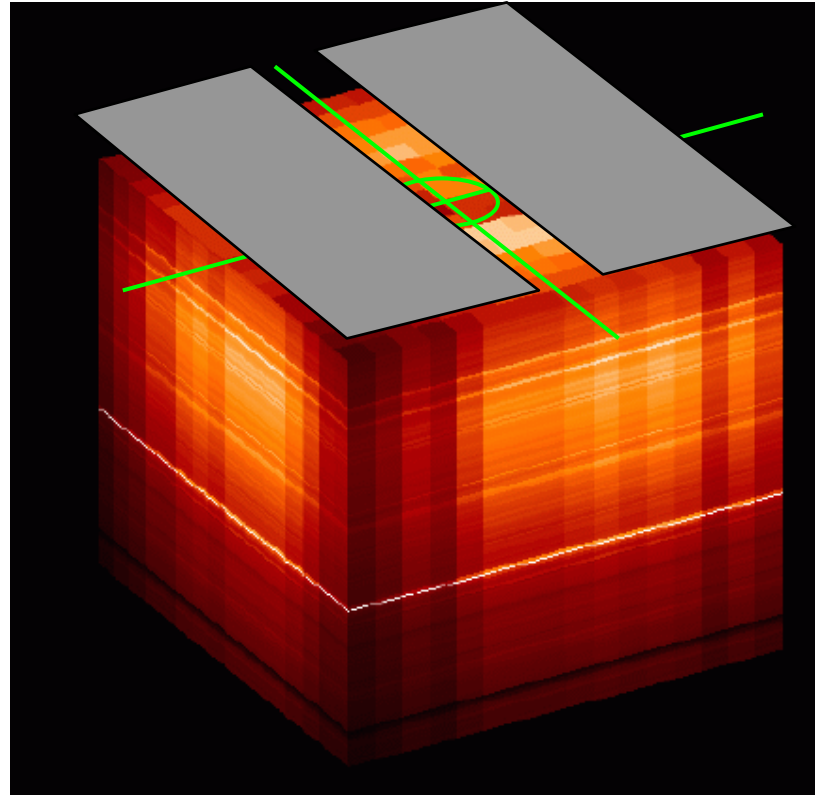
- „a posteriori“ advantage → Pointing
- slit effects
- atmospheric refraction
- spectrophotometry
- spatial binning (→ low surface brightness)
- „crowded field“ 3D spektroskopie
- ultra-deep „*faint object*“ 3D spectroscopy



3D-spectroscopy: „a posteriori“ advantage



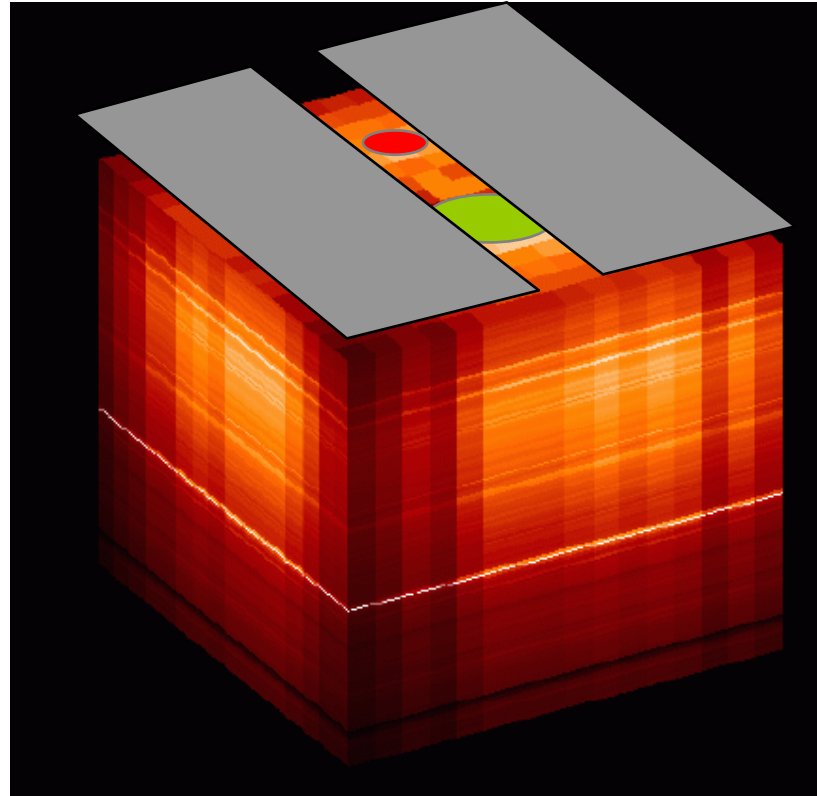
3D-spectroscopy: „a posteriori“ advantage



Slit effects

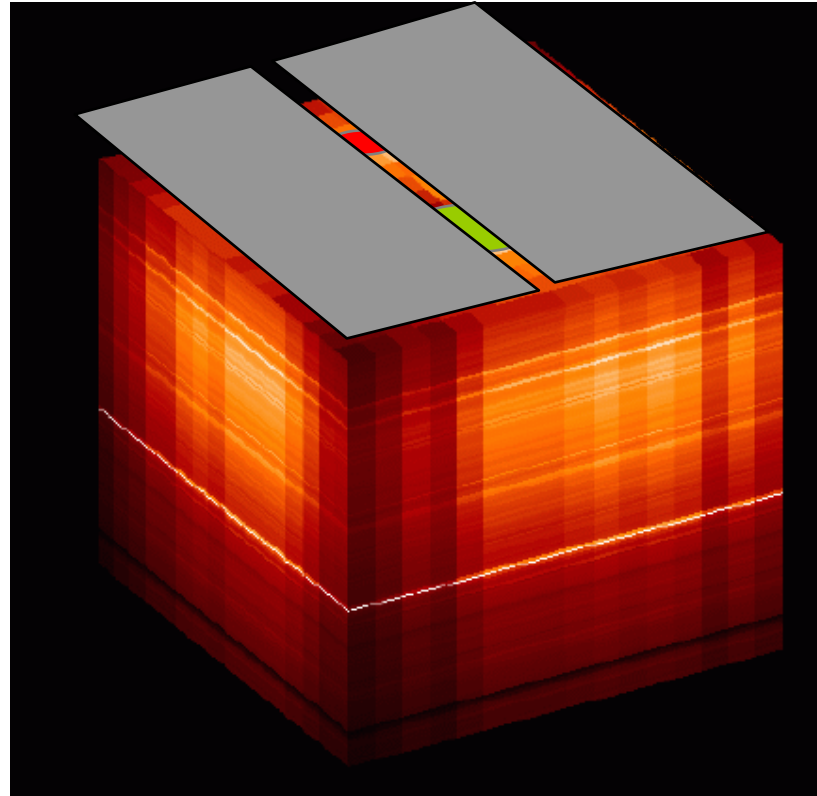
- Light lost on slit blades
- Line-spread function affected by position of image
- Seeing = $f(t)$
- Seeing = $f(\lambda)$
- Atmospheric refraction
- Source confusion in crowded fields

Slit effects



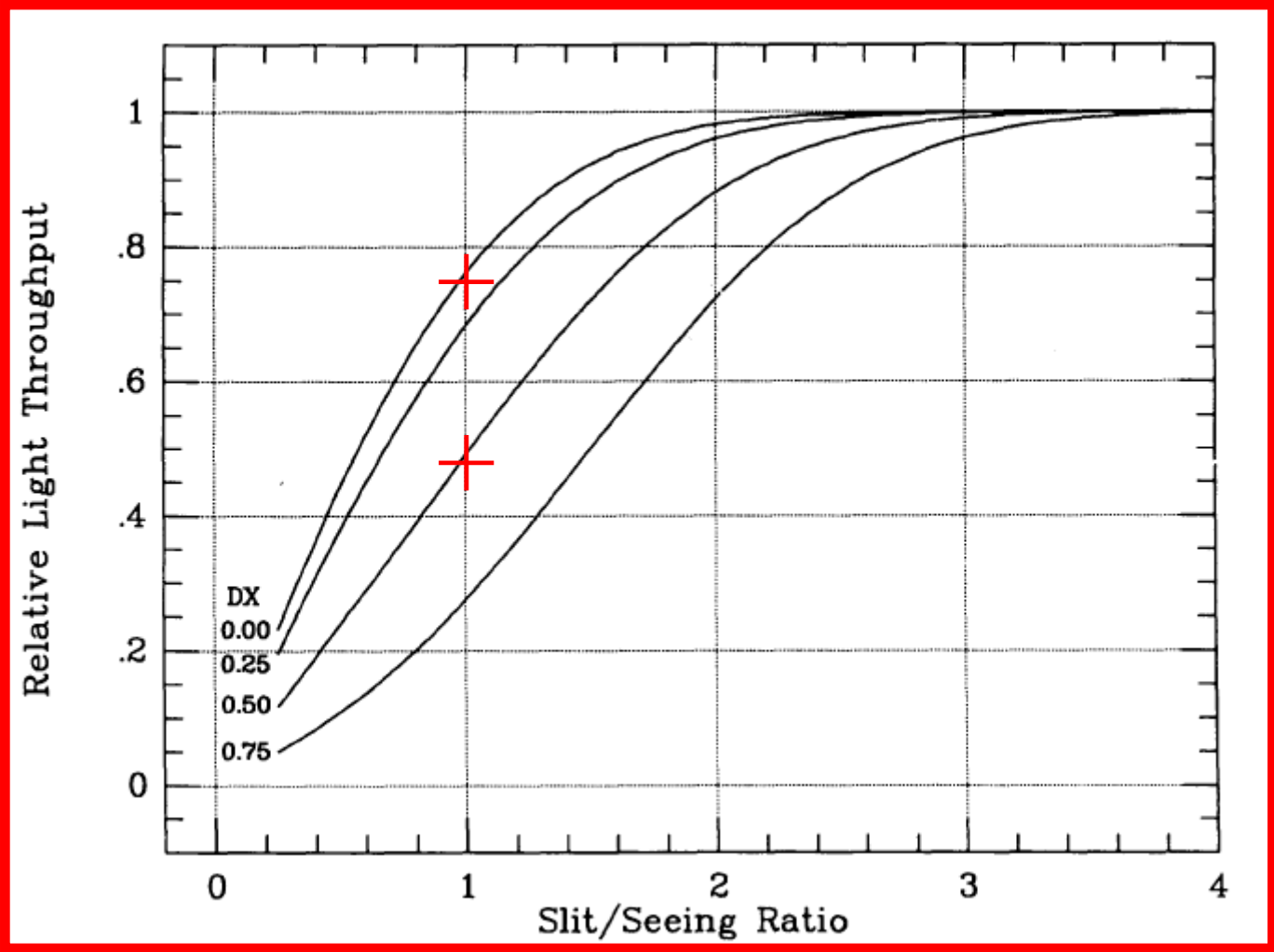
$$\text{fwhm} \sim (1/\lambda)^{0.2}$$

Slit effects



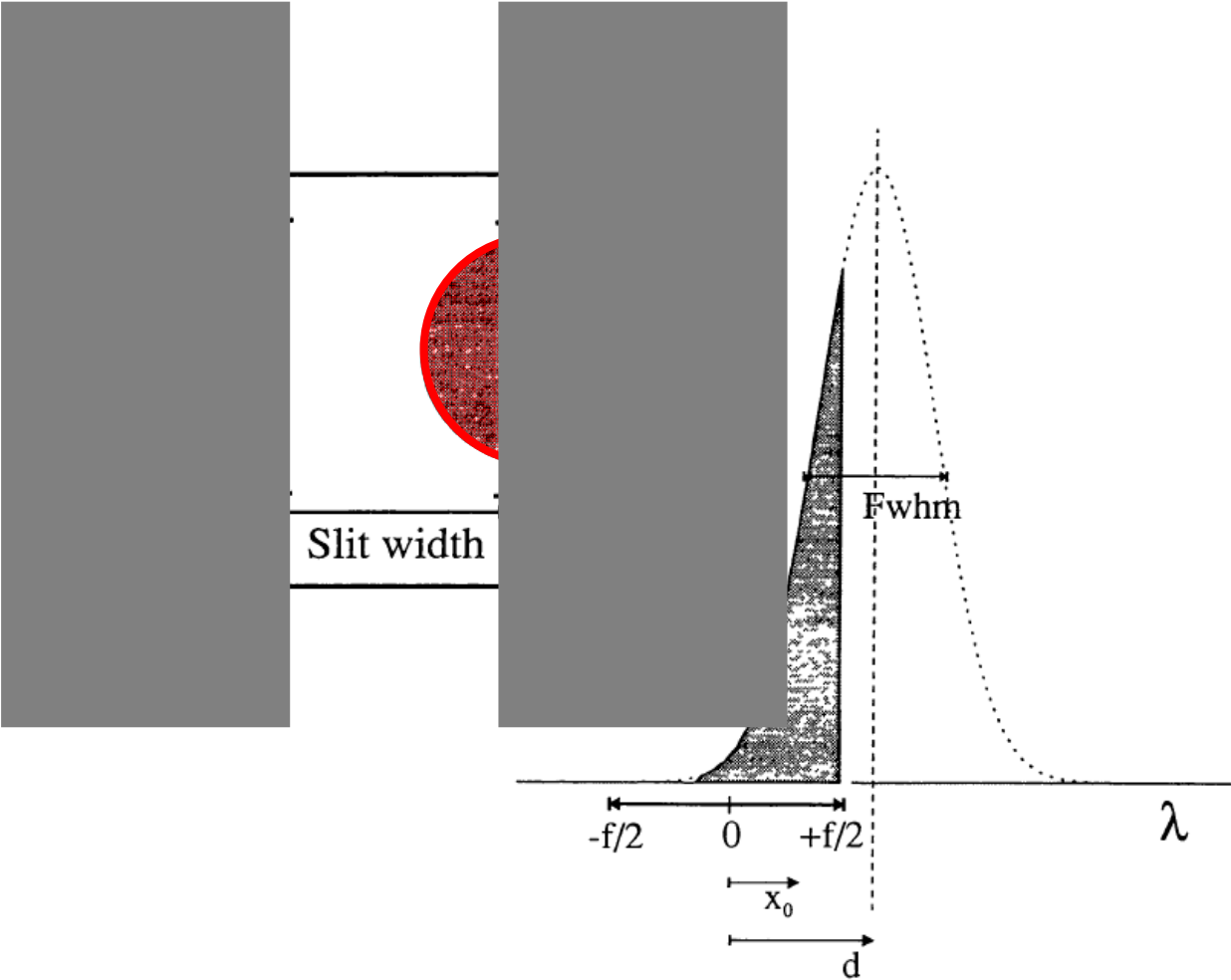
$$\text{fwhm} \sim (1/\lambda)^{0.2}$$

Slit effects



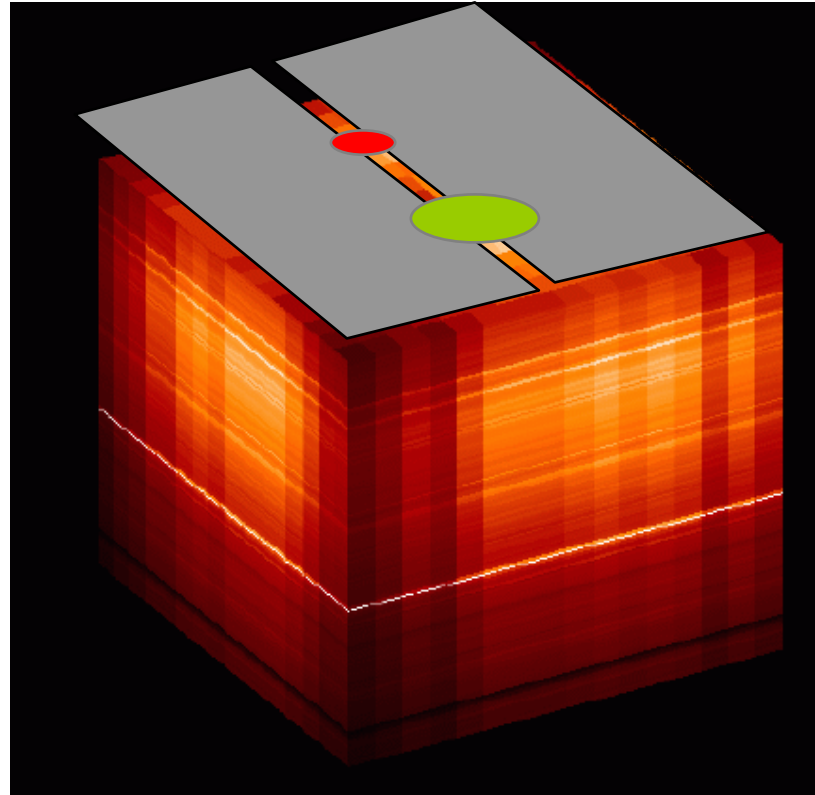
Jacoby & Kaler 1993, *ApJ* 417, 209

Slit effects

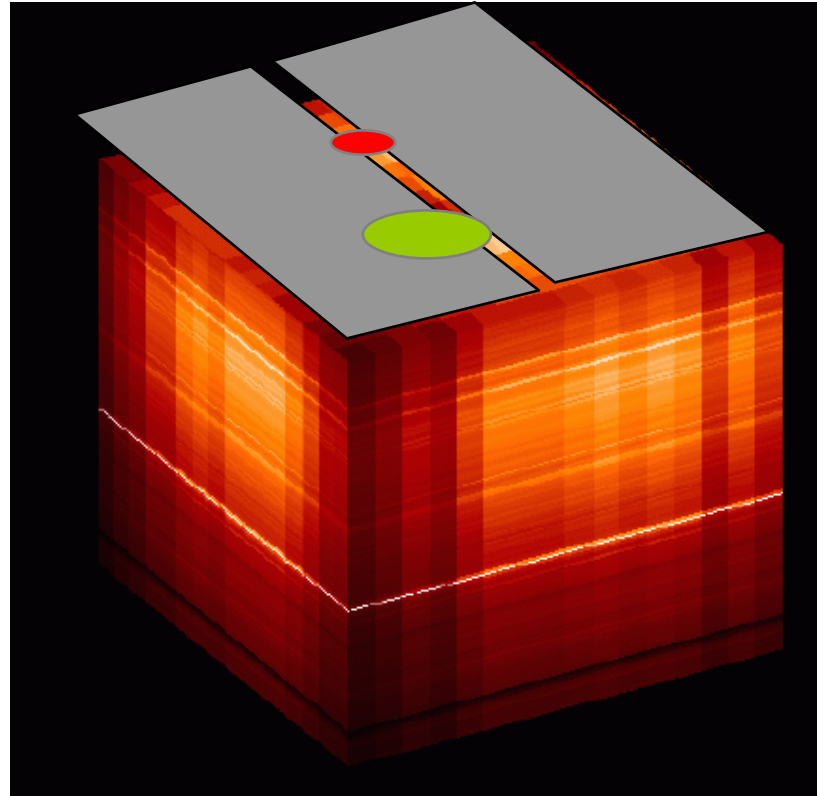


Bacon et al. 1995, A&ASuppl. 113, 347

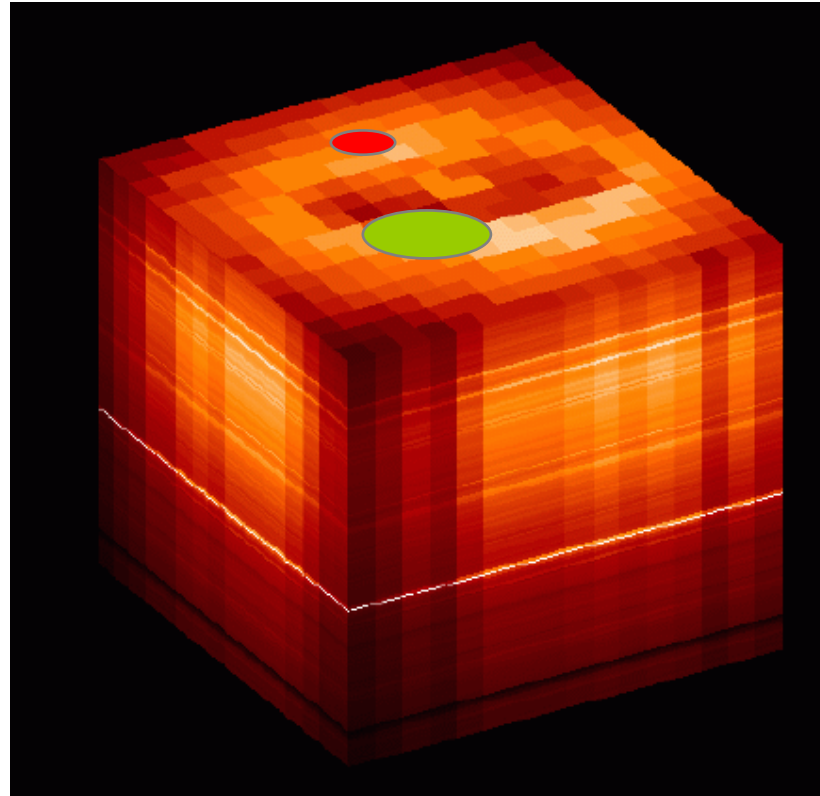
Atmospheric refraction



Atmospheric refraction



Atmospheric refraction



Atmospheric refraction

Filippenko, A. (1982)

„The importance of atmospheric differential refraction
in spectrophotometry“

PASP 94, 715

$$(n(\lambda)_{15,760} - 1)10^6 = 64.328 + \frac{29498.1}{146 - (1/\lambda)^2} + \frac{255.4}{41 - (1/\lambda)^2} \quad (1)$$

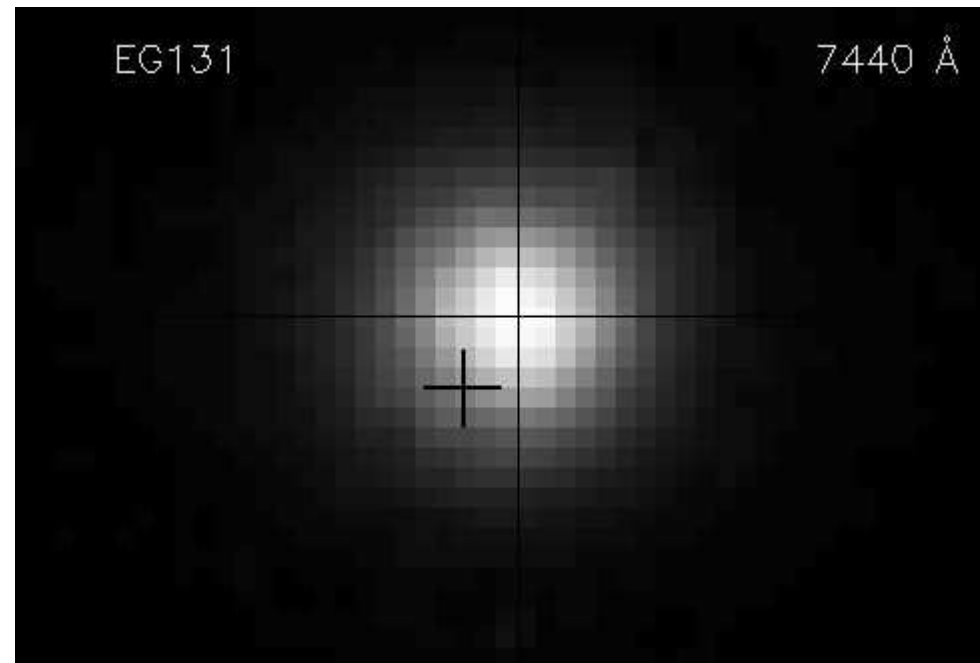
$$(n(\lambda)_{T,P} - 1) = (n(\lambda)_{15,760} - 1) \times \frac{P [1 + (1.049 - 0.0157 T)10^{-6}P]}{720.883(1 + 0.003661 T)} \quad (2)$$

$$\frac{0.0624 - 0.000680/\lambda^2}{1 + 0.003661T} f \quad (3)$$

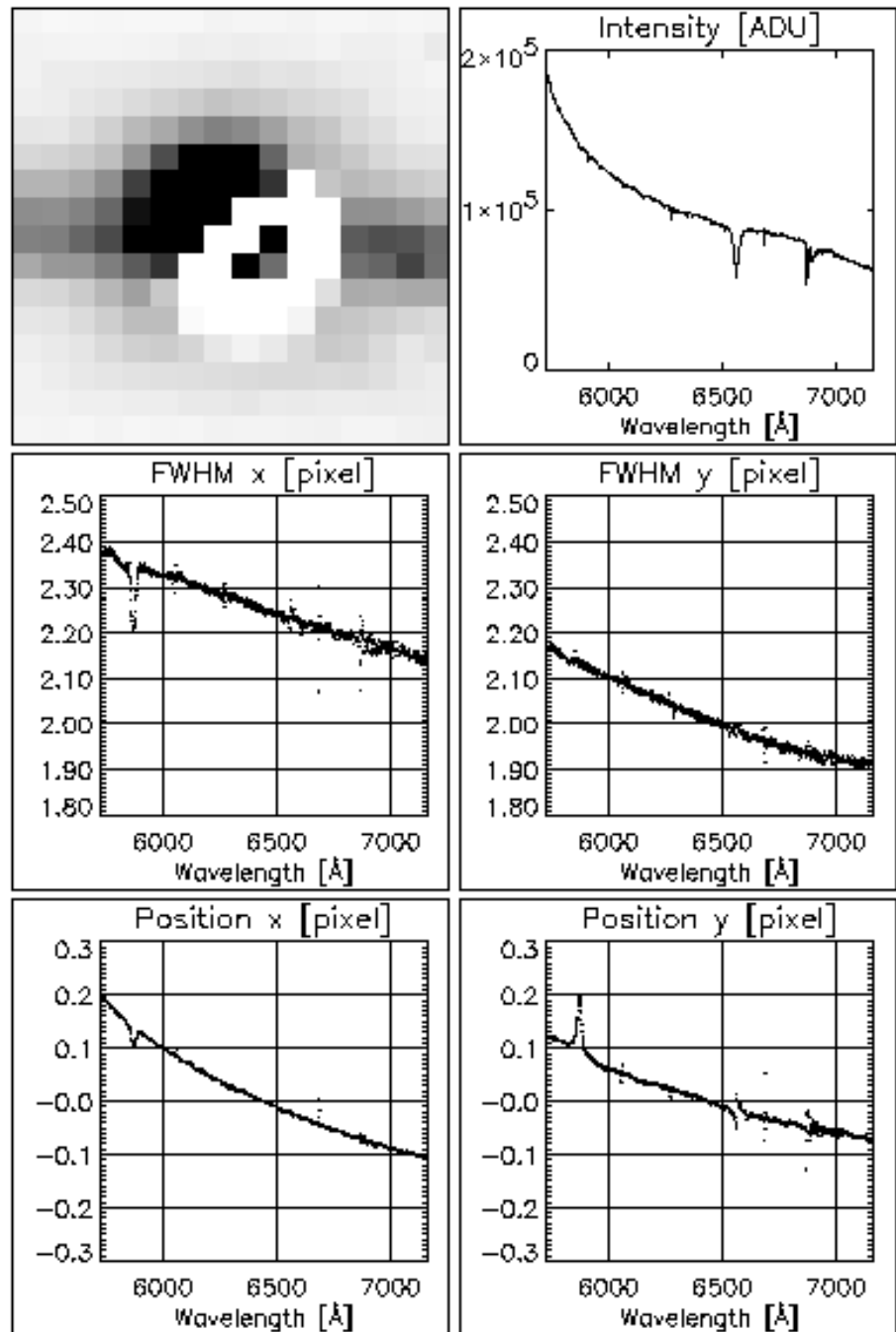
Atmospheric refraction

	3000	3500	4000	4500	5000	5500	6000	6500	7000	7500	8000	8500	9000	9500	10000
sec z															
1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1.05	0.68	0.38	0.20	0.08	0.00	-0.06	-0.11	-0.14	-0.17	-0.19	-0.21	-0.23	-0.24	-0.25	-0.26
1.10	0.97	0.55	0.29	0.12	0.00	-0.09	-0.15	-0.20	-0.24	-0.28	-0.30	-0.32	-0.34	-0.36	-0.37
1.15	1.20	0.68	0.36	0.15	0.00	-0.11	-0.19	-0.25	-0.30	-0.34	-0.38	-0.40	-0.42	-0.44	-0.46
1.20	1.40	0.80	0.42	0.17	0.00	-0.13	-0.22	-0.30	-0.35	-0.40	-0.44	-0.47	-0.50	-0.52	-0.54
1.25	1.59	0.90	0.48	0.20	0.00	-0.14	-0.25	-0.33	-0.40	-0.45	-0.50	-0.53	-0.56	-0.59	-0.61
1.30	1.76	1.00	0.53	0.22	0.00	-0.16	-0.28	-0.37	-0.44	-0.50	-0.55	-0.59	-0.62	-0.65	-0.67
1.35	1.92	1.09	0.58	0.24	0.00	-0.17	-0.30	-0.40	-0.48	-0.55	-0.60	-0.64	-0.68	-0.71	-0.73
1.40	2.07	1.18	0.62	0.26	0.00	-0.19	-0.33	-0.44	-0.52	-0.59	-0.65	-0.69	-0.73	-0.77	-0.79
1.45	0.22	1.26	0.67	0.28	0.00	-0.20	-0.35	-0.47	-0.56	-0.63	-0.69	-0.74	-0.79	-0.82	-0.85
1.50	2.37	1.34	0.71	0.29	0.00	-0.21	-0.37	-0.50	-0.60	-0.68	-0.74	-0.79	-0.84	-0.87	-0.91
1.55	2.51	1.42	0.75	0.31	0.00	-0.23	-0.40	-0.53	-0.63	-0.72	-0.78	-0.84	-0.89	-0.93	-0.96
1.60	2.64	1.50	0.80	0.33	0.00	-0.24	-0.42	-0.56	-0.67	-0.75	-0.83	-0.88	-0.93	-0.98	-1.01
1.65	2.78	1.58	0.84	0.34	0.00	-0.25	-0.44	-0.59	-0.70	-0.79	-0.87	-0.93	-0.98	-1.03	-1.06
1.70	2.91	1.65	0.88	0.36	0.00	-0.26	-0.46	-0.61	-0.73	-0.83	-0.91	-0.97	-1.03	-1.07	-1.11
1.75	3.04	1.73	0.92	0.38	0.00	-0.27	-0.48	-0.64	-0.77	-0.87	-0.95	-1.02	-1.07	-1.12	-1.16
1.80	3.17	1.80	0.95	0.39	0.00	-0.29	-0.50	-0.67	-0.80	-0.90	-0.99	-1.06	-1.12	-1.17	-1.21
1.85	3.29	1.87	0.99	0.41	0.00	-0.30	-0.52	-0.69	-0.83	-0.94	-1.03	-1.10	-1.16	-1.22	-1.26
1.90	3.42	1.94	1.03	0.42	0.00	-0.31	-0.54	-0.72	-0.86	-0.98	-1.07	-1.14	-1.21	-1.26	-1.31
1.95	3.54	2.01	1.07	0.44	0.00	-0.32	-0.56	-0.75	-0.89	-1.01	-1.11	-1.19	-1.25	-1.31	-1.36

Atmospheric refraction

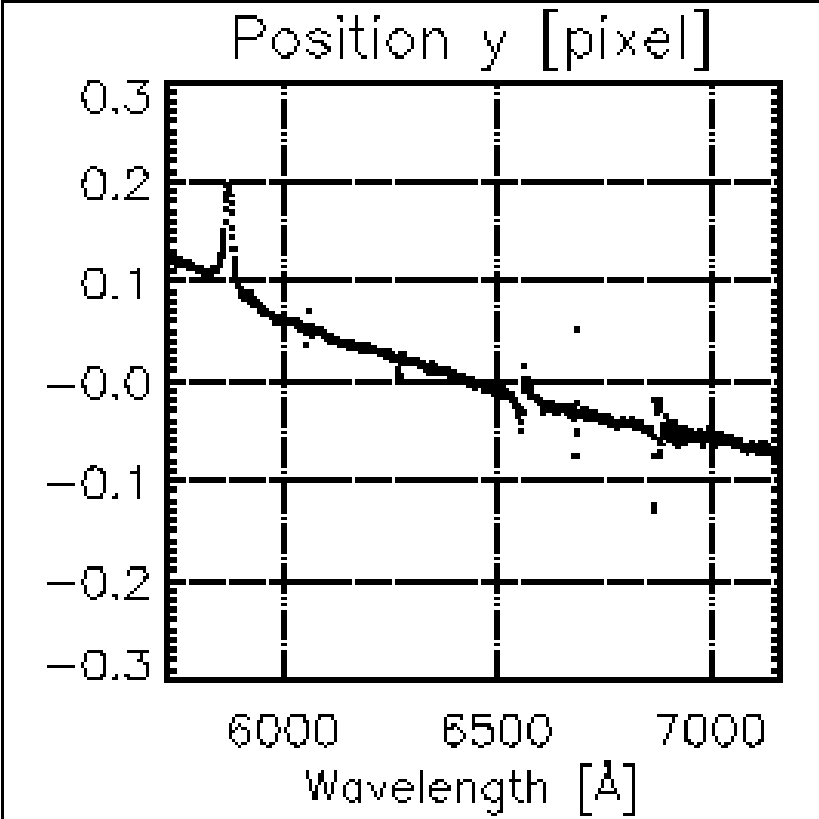
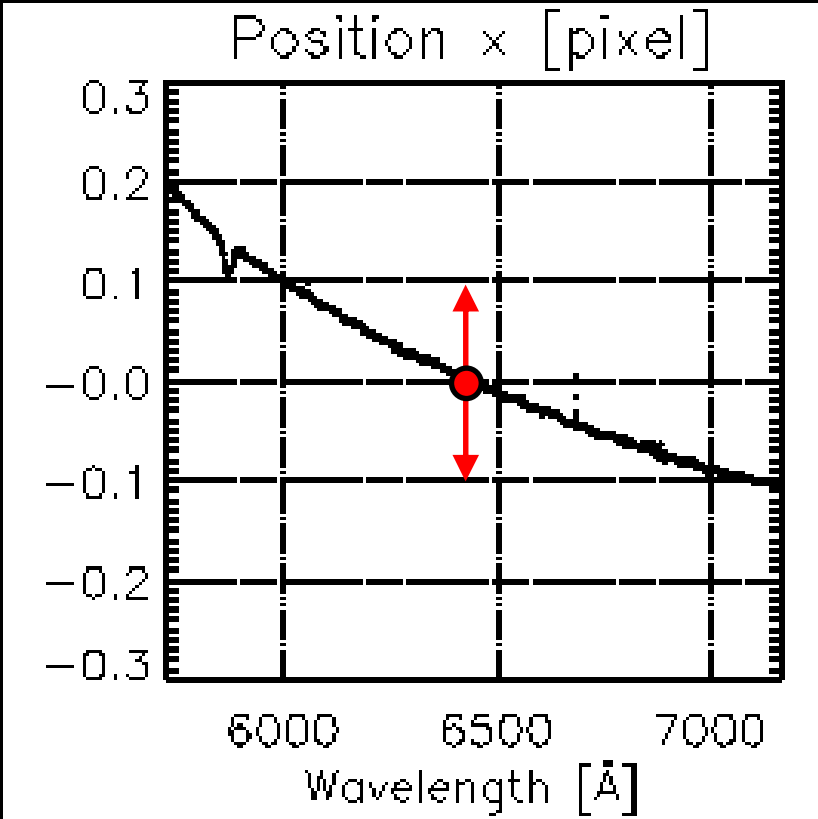


Atmospheric refraction



HR 1544

Atmospheric refraction



PSF-fitting crowded field 3D spectroscopy

Resolving stellar populations with crowded field 3D spectroscopy^{*,**}

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Received / Accepted

ABSTRACT

We describe a new method to extract spectra of stars from observations of crowded stellar fields with integral field spectrographs (IFS). Our approach extends the well-established concept of *crowded field photometry* in images into the domain of 3-dimensional spectroscopic datacubes. The main features of our algorithm are: (1) We assume that a high-fidelity input source catalogue already exists, e.g. from HST data, and that it is not needed to perform sophisticated source detection in the IFS data. (2) Source positions and properties of the point spread function (PSF) vary smoothly between spectral layers of the datacube, and these variations can be described by simple fitting functions. (3) The shape of the PSF can be adequately described by an analytical function. Even without isolated PSF calibrator stars we can therefore estimate the PSF by a model fit to the full ensemble of stars visible within the field of view. (4) By using sparse matrices to describe the sources, the problem of extracting the spectra of many stars simultaneously becomes computationally tractable. We present extensive performance and validation tests of our algorithm using realistic simulated datacubes that closely reproduce actual IFS observations of the central regions of Galactic globular clusters. We investigate the quality of the extracted spectra under the effects of crowding with respect to the resulting signal-to-noise ratios (S/N) and any possible changes in the continuum level, as well as with respect to absorption line spectral parameters, radial velocities and equivalent widths. The main effect of blending between two nearby stars is a decrease in the S/N in their spectra. The effect increases with the crowding in the field in a way that the maximum number of stars with useful spectra is always ~ 0.2 per spatial resolution element. This balance breaks down when exceeding a total source density of ~ 1 significantly detected star per resolution element. We also explore the effects of PSF mismatch and other systematics. We close with an outlook by applying our method to a simulated globular cluster observation with the upcoming MUSE instrument at the ESO-VLT.

Key words. Methods: data analysis - Techniques: imaging spectroscopy - Galaxy: globular clusters

Crowded field 3D spectroscopy: Kamann et al. 2013

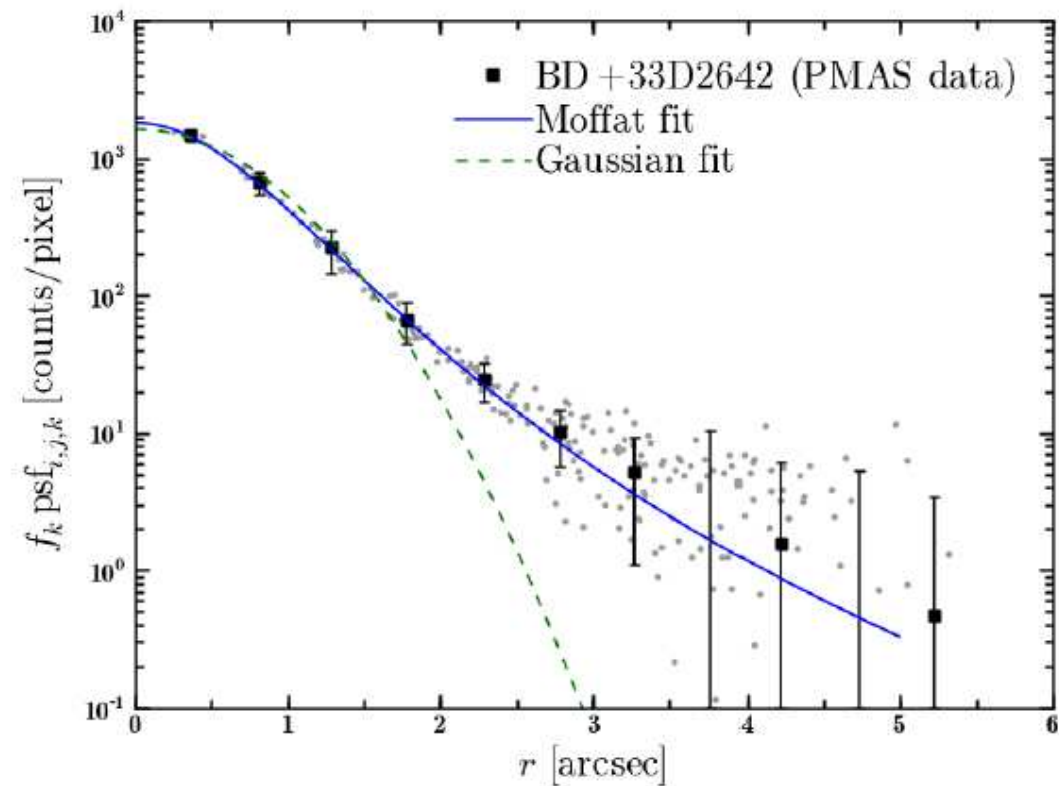
Point Spread Function (PSF):

$$\hat{x} = (x - x^n) \cos \theta - (y - y^n) \sin \theta,$$

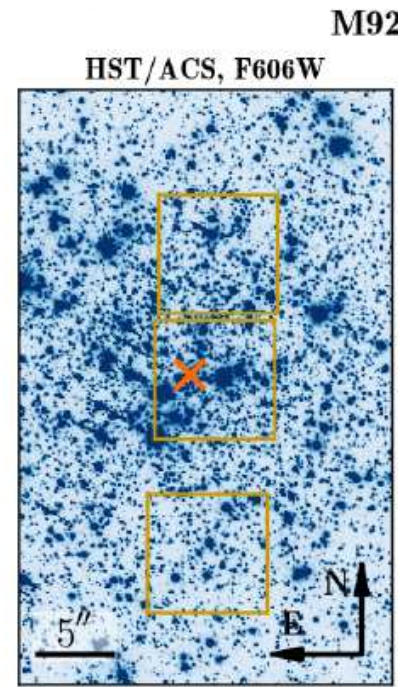
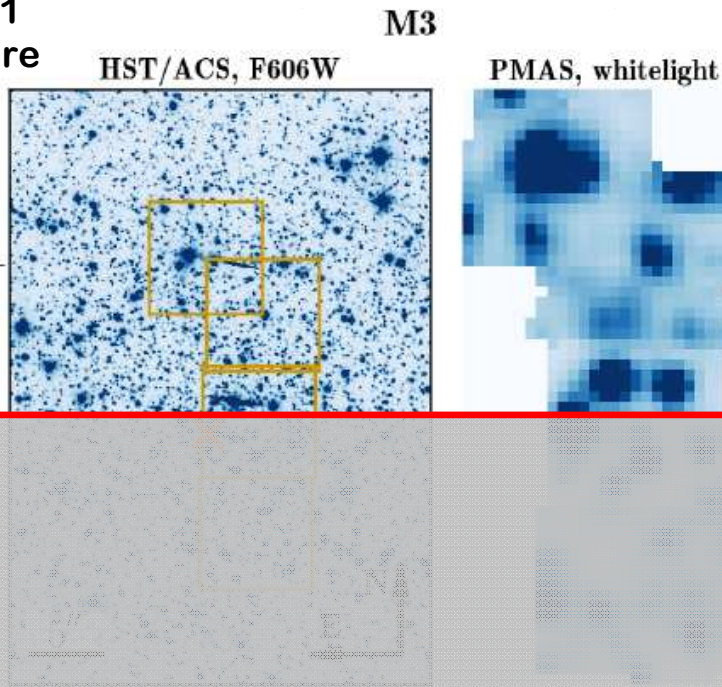
$$\hat{y} = (x - x^n) \sin \theta + (y - y^n) \cos \theta,$$

$$r(x, y) = \sqrt{\hat{x}^2 + \left(\frac{\hat{y}}{1 - e}\right)^2}$$

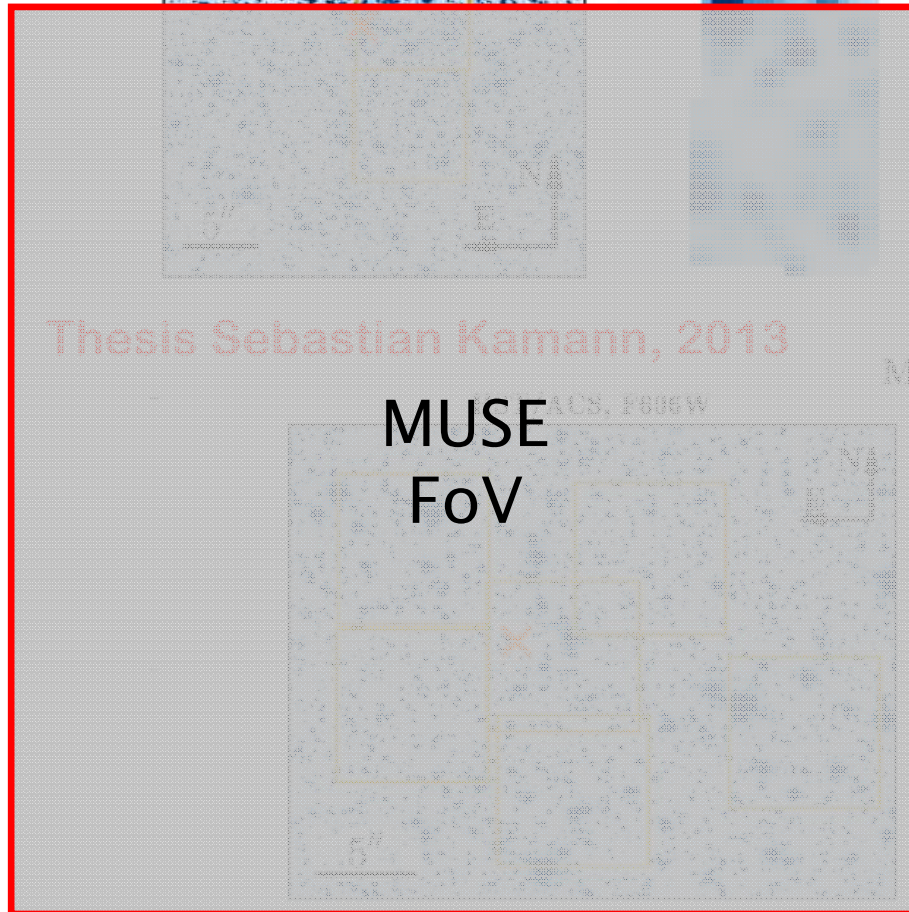
$$M(x, y) = \Sigma_0 \left(1 + \left(\frac{r(x, y)}{r_d}\right)^2\right)^{-\beta}$$



50 vrad + 161
from literature
17 excl.



77 vrad + 308,
23 excl.



M13

